On existence and bubbles of Ramsey equilibrium with borrowing constraints

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Ramsey conjecture

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- This conjecture was proved by Robert Becker half a century later.

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- Extension with elastic labor supply (Le Van et al., 2007).

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  - Stationarity. At the steady state, impatient agents consume. The steady state vanishes without borrowing constraints (Le Van and Vailakis, 2003).
  - Monotonicity. Borrowing constraints promote persistent cycles (Becker, 1980).

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- Ramsey conjecture still holds in the case of financial constraints.
- But under other imperfections (distortionary taxes or market power), a non-degenerated distribution of capital in the long run is possible.

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- Bosi and Seegmuller (2010) give a local proof of existence with elastic labor supply (fixed point for the policy function).

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- Global argument of existence (differently from Bosi and Seegmuller, 2010).
- No bubbles in a productive economy with heterogeneous agents and imperfect markets.

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- This equilibrium is also an equilibrium of an unbounded truncated economy.
- Proof for an infinite-horizon economy as a limit of a sequence of truncated economies.

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- w_t, wage.

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  - consumption plans are optimal under the budget and the borrowing constraints.

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  - one unit of leisure per period: $\lambda_{it} = 1 - l_{it}$.

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Equilibrium of a finite-horizon economy

- Choose sufficiently large quantity bounds for individual capital and consumption, and aggregate inputs.

Theorem

Under the previous Assumptions, there exists an equilibrium $\left(\overline{\mathbf{p}}, \overline{\mathbf{r}}, \overline{\mathbf{w}}, \left(\overline{\mathbf{c}}_i, \overline{\mathbf{k}}_i, \overline{\lambda}_i\right)_{i=1}^{m}, \overline{\mathbf{K}}, \overline{\mathbf{L}}\right)$ *for the finite-horizon bounded economy.*

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- Define the price simplex:
  $\triangle \equiv \{(p, r, w) : p, r, w \geq 0,\ p + r + w = 1\}$ and the budget sets

$$C_i^T(\mathbf{p}, \mathbf{r}, \mathbf{w}) \equiv \left\{ \begin{array}{c} (\mathbf{c}_i, \mathbf{k}_i, \boldsymbol{\lambda}_i) \in X_i \times Y_i \times Z_i : \\ p_t \left[c_{it} + k_{it+1} - (1-\delta)\, k_{it}\right] \leq r_t k_{it} + w_t \left(1 - \lambda_{it}\right) \\ t = 0, \ldots, T \end{array} \right\}$$

and its interior

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- If $w_0 > 0$ and $r_t + w_t > 0$, for $t = 1, \ldots, T$, then the set $B_i^T(\mathbf{p}, \mathbf{r}, \mathbf{w})$ is nonempty and $C_i^T(\mathbf{p}, \mathbf{r}, \mathbf{w})$ is the closure of $B_i^T(\mathbf{p}, \mathbf{r}, \mathbf{w})$.

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- ε and k_{it} are the same good.

- Define the perturbed budget sets:

$$C_i^{T\varepsilon}(\mathbf{p},\mathbf{r},\mathbf{w}) \equiv$$
$$\left\{ \begin{array}{c} (\mathbf{c}_i, \mathbf{k}_i, \boldsymbol{\lambda}_i) \in X_i \times Y_i \times Z_i : \\ p_t\left(c_{it} + k_{it+1}\right) \leq p_t\varepsilon + \left[p_t\left(1-\delta\right) + r_t\right]\left(k_{it} + \varepsilon\right) + w_t\left(1 - \lambda_{it}\right) \\ t = 0, \ldots, T \end{array} \right.$$

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- $B_i^{T\varepsilon}(\mathbf{p},\mathbf{r},\mathbf{w})$ is nonempty and $C_i^{T\varepsilon}(\mathbf{p},\mathbf{r},\mathbf{w}) = \bar{B}_i^{T\varepsilon}(\mathbf{p},\mathbf{r},\mathbf{w})$. Moreover the correspondence $B_i^{T\varepsilon}$ is lower semicontinuous.

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- Agent $i = 0$ (the "additional" agent):
$$\varphi_0\left(\mathbf{p}, \mathbf{r}, \mathbf{w}, \left(\mathbf{c}_h, \mathbf{k}_h, \boldsymbol{\lambda}_h\right)_{h=1}^{m}, \mathbf{K}, \mathbf{L}\right) \equiv$$
$$\left\{\begin{array}{c} \left(\tilde{\mathbf{p}}, \tilde{\mathbf{r}}, \tilde{\mathbf{w}}\right) \in P : \\ \sum_{t=0}^{T}\left(\tilde{p}_t - p_t\right) \\ \left(\sum_i \left[c_{it} + k_{it+1} - (1-\delta)\, k_{it}\right] - m\varepsilon - m(1-\delta)\,\varepsilon - F\left(K_t, L_t\right)\right) \\ + \sum_{t=0}^{T}\left(\tilde{r}_t - r_t\right)\left(K_t - m\varepsilon - \sum_{i=1}^{m} k_{it}\right) \\ + \sum_{t=0}^{T}\left(\tilde{w}_t - w_t\right)\left(L_t - m + \sum_{i=1}^{m} \lambda_{it}\right) > 0 \end{array}\right\}$$

- Agents $i = 1, \ldots, m$ (consumers-workers):

$$\varphi_i\left(\mathbf{p}, \mathbf{r}, \mathbf{w}, (\mathbf{c}_h, \mathbf{k}_h, \boldsymbol{\lambda}_h)_{h=1}^m, \mathbf{K}, \mathbf{L}\right) \equiv$$
$$\left\{ \begin{array}{c} B_i^{T\varepsilon}(\mathbf{p}, \mathbf{r}, \mathbf{w}) \text{ if } (\mathbf{c}_i, \mathbf{k}_i, \boldsymbol{\lambda}_i) \notin C_i^{T\varepsilon}(\mathbf{p}, \mathbf{r}, \mathbf{w}) \\ B_i^{T\varepsilon}(\mathbf{p}, \mathbf{r}, \mathbf{w}) \cap [P_i(\mathbf{c}_i, \boldsymbol{\lambda}_i) \times Y_i] \text{ if } (\mathbf{c}_i, \mathbf{k}_i, \boldsymbol{\lambda}_i) \in C_i^{T\varepsilon}(\mathbf{p}, \mathbf{r}, \mathbf{w}) \end{array} \right\}$$

where P_i is the ith agent's set of strictly preferred allocations.

- Agent $i = m + 1$ (the firm):

$$\varphi_{m+1}\left(\mathbf{p}, \mathbf{r}, \mathbf{w}, (\mathbf{c}_h, \mathbf{k}_h, \boldsymbol{\lambda}_h)_{h=1}^{m}, \mathbf{K}, \mathbf{L}\right) \equiv \left\{ \begin{array}{c} \left(\tilde{\mathbf{K}}, \tilde{\mathbf{L}}\right) \in Y \times Z : \\ \sum_{t=0}^{T}\left[p_t F\left(\tilde{K}_t, \tilde{L}_t\right) - r_t \tilde{K}_t - w_t \tilde{L}_t\right] \\ > \sum_{t=0}^{T}\left[p_t F\left(K_t, L_t\right) - r_t K_t - w_t L_t\right] \end{array} \right\}$$

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- $(\mathbf{c}_i, \mathbf{k}_i, \boldsymbol{\lambda}_i) \notin P_i(\mathbf{c}_i, \boldsymbol{\lambda}_i) \times Y_i$ implies that $(\mathbf{c}_i, \mathbf{k}_i, \boldsymbol{\lambda}_i) \notin \varphi_i(\mathbf{v})$ for $i = 1, \ldots, m$.

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- By definition of φ_{m+1} (the inequality is also strict): $(\mathbf{K}, \mathbf{L}) \notin \varphi_{m+1}(\mathbf{v})$.
- Then, for $i = 0, \ldots, m+1$, $\mathbf{v}_i \notin \varphi_i(\mathbf{v})$.

- Apply Gale and Mas-Colell (1975) fixed-point theorem. There exists $\bar{\mathbf{v}} \in \Phi$ such that $\varphi_i(\bar{\mathbf{v}}) = \varnothing$ for $i = 0, \ldots, m+1$, that is, there exists $\bar{\mathbf{v}} \in \Phi$ such that the following holds.

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- Focus on "agent" $i = 0$. For every $(\mathbf{p}, \mathbf{r}, \mathbf{w}) \in P$,

$$
\begin{aligned}
& \sum_{t=0}^{T} (p_t - \bar{p}_t) \\
& \left(\sum_{i=1}^{m} \left[\bar{c}_{it} + \bar{k}_{it+1} - (1-\delta) \bar{k}_{it} \right] - m\varepsilon - m(1-\delta)\varepsilon - F(\bar{K}_t, \bar{L}_t) \right) \\
& + \sum_{t=0}^{T} (r_t - \bar{r}_t) \\
& \left(\bar{K}_t - m\varepsilon - \sum_{i=1}^{m} \bar{k}_{it} \right) + \sum_{t=0}^{T} (w_t - \bar{w}_t) \left(\bar{L}_t - m + \sum_{i=1}^{m} \bar{\lambda}_{it} \right) \\
\leq \quad & 0
\end{aligned}
$$

- Consider $i = 1, \ldots, m$. $\left(\overline{\mathbf{c}}_i, \overline{\mathbf{k}}_i, \overline{\boldsymbol{\lambda}}_i\right) \in C_i^{T\varepsilon}\left(\overline{\mathbf{p}}, \overline{\mathbf{r}}, \overline{\mathbf{w}}\right)$ and $B_i^{T\varepsilon}\left(\overline{\mathbf{p}}, \overline{\mathbf{r}}, \overline{\mathbf{w}}\right) \cap \left[P_i\left(\overline{\mathbf{c}}_i, \overline{\boldsymbol{\lambda}}_i\right) \times Y_i\right] = \varnothing$ for $i = 1, \ldots, m$. Then, for $i = 1, \ldots, m$, $\left(\mathbf{c}_i, \mathbf{k}_i, \boldsymbol{\lambda}_i\right) \in C_i^{T\varepsilon}\left(\overline{\mathbf{p}}, \overline{\mathbf{r}}, \overline{\mathbf{w}}\right) = \overline{B}_i^{T\varepsilon}\left(\overline{\mathbf{p}}, \overline{\mathbf{r}}, \overline{\mathbf{w}}\right)$ implies

$$\sum_{t=0}^{T} \beta_i^t u_i\left(c_{it}, \lambda_{it}\right) \leq \sum_{t=0}^{T} \beta_i^t u_i\left(\bar{c}_{it}, \bar{\lambda}_{it}\right)$$

- Consider $i = 1, \ldots, m$. $\left(\overline{\mathbf{c}}_i, \overline{\mathbf{k}}_i, \overline{\boldsymbol{\lambda}}_i\right) \in C_i^{T\varepsilon}\left(\overline{\mathbf{p}}, \overline{\mathbf{r}}, \overline{\mathbf{w}}\right)$ and $B_i^{T\varepsilon}\left(\overline{\mathbf{p}}, \overline{\mathbf{r}}, \overline{\mathbf{w}}\right) \cap \left[P_i\left(\overline{\mathbf{c}}_i, \overline{\boldsymbol{\lambda}}_i\right) \times Y_i\right] = \varnothing$ for $i = 1, \ldots, m$. Then, for $i = 1, \ldots, m$, $\left(\mathbf{c}_i, \mathbf{k}_i, \boldsymbol{\lambda}_i\right) \in C_i^{T\varepsilon}\left(\overline{\mathbf{p}}, \overline{\mathbf{r}}, \overline{\mathbf{w}}\right) = \overline{B}_i^{T\varepsilon}\left(\overline{\mathbf{p}}, \overline{\mathbf{r}}, \overline{\mathbf{w}}\right)$ implies

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- Focus on the firm $i = m + 1$. For $t = 0, \ldots, T$ and for every $(\mathbf{K}, \mathbf{L}) \in Y \times Z$, we have $\sum_{t=0}^{T}\left[\bar{p}_t F\left(K_t, L_t\right) - \bar{r}_t K_t - \bar{w}_t L_t\right] \leq \sum_{t=0}^{T}\left[\bar{p}_t F\left(\bar{K}_t, \bar{L}_t\right) - \bar{r}_t \bar{K}_t - \bar{w}_t \bar{L}_t\right]$.

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  - $\bar{r}_t > 0$, $\bar{w}_t > 0$.

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  - $\bar{K}_t - m\varepsilon - \sum_i \bar{k}_{it} = 0,$
  - $\bar{L}_t - \sum_i \left(1 - \bar{\lambda}_{it}\right) = 0.$

- We take the limit of the perturbed economy as ε tends to zero and we show that the limit

$$\begin{aligned}
&\left(\overline{\mathbf{p}}, \overline{\mathbf{r}}, \overline{\mathbf{w}}, \left(\overline{\mathbf{c}}_i, \overline{\mathbf{k}}_i, \overline{\boldsymbol{\lambda}}_i\right)_{i=1}^{m}, \overline{\mathbf{K}}, \overline{\mathbf{L}}\right) \\
= \; &\lim_{\varepsilon \to 0} \left(\overline{\mathbf{p}}(\varepsilon), \overline{\mathbf{r}}(\varepsilon), \overline{\mathbf{w}}(\varepsilon), \left(\overline{\mathbf{c}}_i(\varepsilon), \overline{\mathbf{k}}_i(\varepsilon), \overline{\boldsymbol{\lambda}}_i(\varepsilon)\right)_{i=1}^{m}, \overline{\mathbf{K}}(\varepsilon), \overline{\mathbf{L}}(\varepsilon)\right)
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satisfies the definition of Walrasian equilibrium (prices positivity, optimal plans and market clearing).

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- We prove that any equilibrium of $\mathcal{E}^T$ is an equilibrium for the finite-horizon unbounded economy.
- Simply, consider a convex combination within the bounds of the equilibrium of the bounded economy with a candidate outside the bounds and derive a contradiction.

Equilibrium of a finite-horizon economy

Theorem

Under the assumptions of the model, there exists an equilibrium in the infinite-horizon economy with endogenous labor supply and borrowing constraints.

- We denote by $\left(\overline{\mathbf{p}}\left(T\right),\overline{\mathbf{r}}\left(T\right),\overline{\mathbf{w}}\left(T\right),\left(\overline{\mathbf{c}}_i\left(T\right),\overline{\mathbf{k}}_i\left(T\right),\overline{\lambda}_i\left(T\right)\right)_{i=1}^m,\overline{\mathbf{K}}\left(T\right),\overline{\mathbf{L}}\left(T\right)\right)$ an equilibrium for the truncated economy and by $\left(\hat{\mathbf{p}},\hat{\mathbf{r}},\hat{\mathbf{w}},\left(\hat{\mathbf{c}}_i,\hat{\mathbf{k}}_i,\hat{\lambda}_i\right)_{i=1}^m,\hat{\mathbf{K}},\hat{\mathbf{L}}\right)$ the limit for $T\longrightarrow\infty$ for the product topology.

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- The rest follows.

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Non-existence of bubbles

- Consider the equilibrium of an infinite-horizon economy. Take $\bar{p}_t = 1$ with $\bar{r}_t, \bar{w}_t > 0$.
- We introduce a market discount factor reflecting the marginal rate of substitution between t and $t+1$:

$$\bar{q}_{t+1} \equiv \max_i \frac{\beta_i \left(\partial u_i / \partial c\right) \left(\bar{c}_{it+1}, \bar{\lambda}_{it+1}\right)}{\left(\partial u_i / \partial c\right) \left(\bar{c}_{it}, \bar{\lambda}_{it}\right)} = \frac{1}{1 - \delta + \bar{r}_{t+1}}$$

- Let $\bar{Q}_0 \equiv 1$ and $\bar{Q}_t \equiv \prod_{s=1}^{t} \bar{q}_s$ for $t > 0$, that is $\bar{Q}_t = \prod_{s=1}^{t} \left(1 - \delta + \bar{r}_s\right)^{-1}$ for $t > 0$.

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- By induction, $1 = \bar{Q}_0 = \bar{Q}_T \left(1 - \delta\right)^T + \sum_{t=1}^{T} \bar{Q}_t \bar{r}_t \left(1 - \delta\right)^{t-1}$.

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- At period 2, $1 - \delta$ unit of capital will give back $(1 - \delta)^2$ unit of capital and $(1 - \delta)\,\bar{r}_2$ as its dividend.

- Definition of the Fundamental Value of capital:

$$\text{FV} \equiv \sum_{t=1}^{\infty} \bar{Q}_t \left(1-\delta\right)^{t-1} \bar{r}_t$$

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- The economy is said to experience a bubble if $\lim_{T\to\infty} \bar{Q}_T \left(1-\delta\right)^T > 0$. Otherwise ($\lim_{T\to\infty} \bar{Q}_T \left(1-\delta\right)^T = 0$), there is no bubble.

Lemma

If the economy experiences a bubble, then $\bar{r}_t$ converges to zero.

- Assume that $\bar{r}_t$ does not converge to zero. There is $\rho > 0$ and a strictly increasing sequence $(t_i)_{i=1}^{\infty}$ such that $\bar{r}_{t_i} \geq \rho$ for $i = 1, 2, \ldots$ For $T > t_n$, we get

$$\bar{Q}_T (1-\delta)^T = \prod_{s=1}^{T} \frac{1-\delta}{1-\delta+\bar{r}_s} \leq \prod_{i=1}^{n} \frac{1-\delta}{1-\delta+\bar{r}_{t_i}} \leq \left(\frac{1-\delta}{1-\delta+\rho}\right)^n$$

and

$$0 \leq \lim \sup_{T\to\infty} \bar{Q}_T (1-\delta)^T \leq \lim_{n\to\infty} \left(\frac{1-\delta}{1-\delta+\rho}\right)^n = 0$$

That is there are no bubbles.

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Under the assumptions of the model (Inada included), our productive economy experiences no bubbles.

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- The last (laborious) part of the proof consists in proving that $\bar{r}_t$ does not converge to zero.

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  - We show that, if the economy experiences a bubble, then the interest rate converges to zero.
  - We prove that our assumptions prevent the interest rate from going to zero.