0. INTRODUCTION

0.A Wigner + he Neumaun 50's
40's

Complex Nuclear spectra

\( \lambda \)

\( \gamma \)

Numerical analysis

\[ H \hat{x} = \hat{y} \]

\( \sigma = \frac{\text{Max}}{\text{Min}} \) : Conditioning factor

Random tests: \( H_{ij} = H_{ji} = \pm 4 \)

\( L \sim \text{pdf of } \sigma \)

What if \( H \) is a random matrix?

Hope: universality

RMT: statistics of \{ eigenvalues \} of random matrices

Go's: Dyson, Beta [Brownian motion model; interacting charged model]

Go's: MacKenzie [free random matrix]
0.8 Micro vs. macro universality - Bulk vs. Edge

H: IID RV with \( P(H) \sim \frac{A}{|H|^{1+\mu}} \) as \(|H| \to \infty\)

**Macro-universality**

- IID Sums
  \[
  \frac{\mu > 2}{\sum \frac{1}{N} \sum H_i} \to \text{Gauß}
  \]
  \[
  \frac{\mu < 2}{\sum \frac{1}{N^{1/\mu}} H_i} \to \text{Lévy}
  \]

- IID Extremes
  \[
  \mu = \infty \text{ Gauß}
  \]
  \[
  \mu < \infty \text{ Fréchet}
  \]

**Micro-universality**

- IID Sym. Mat.
  \[
  H_{ij} = H_{ji}
  \]
  \[
  \sum_{N \times N} \to \infty
  \]
  \[
  \frac{\mu > 2}{\sum \frac{1}{N} H_i^2 = \lambda^2} \to \text{Wigner}
  \]
  \[
  \frac{\mu < 2}{\sum \frac{1}{N^{1/\mu}} H_i^2 = \lambda^2} \to \text{Lévy}
  \]

- IID Emp. Correlation
  \[
  H_{ij} \to \text{Mareenko-Laskin} \quad \frac{\mu > 2}{\text{Markenko-Laskin}}
  \]
  \[
  \frac{\mu < 2}{\text{Lévy}}
  \]

**Extreme eigenvalue**

- \( \mu < 4 \)
  \[
  \mu < 4 \text{ Fréchet}
  \]

\[ H_{sym}, \mu > 2 \]

**Density fluctuations**

- \( \langle N^2 \rangle \sim \langle N \rangle^2 \sim L \Delta \)

- Strong correlations may change \( p(x) \) but not \( l(s) \)

- \( l(s) \) sensitive to symmetry, e.g. \( H_{ij} = H_{ij}^T \)

- \( l(s) \) sensitive to Lévy ??
O.C. Tracy - Widow & Directed Polymers

TW: \( \lambda_{\text{max}} - 2 = \frac{5}{N^{2/3}} \)

\[ f(\frac{3}{N}) \xrightarrow{T_W} e^{-\frac{3}{N}} \]

\[ E_i = \sum_{\text{paths}} e_{ij} \]

\[ E_0 = \min_{\text{paths}} E_i \]

\[ E_0 \sim N + \frac{5}{3} N^{1/3} \]

O.D. Applications of RMT

- Nuclear spectra
- Quantum transport
- Data analysis
- Finance
- Wireless communications
- Random zeros
- Random landscape