Model and Calibration Risks for the Heston Model

Florence Guillaume, Wim Schoutens

3rd SMAI European Summer School in Financial Mathematics

August 24th, 2010
Motivation

- calibrated parameters: unstable and often unreasonable
- emergence of market data for vol derivatives ⇒ fix beforehand $v_0$, $\eta$ (from time series or market quotes) ⇒ calibration on a reduced parameter set

![VIX option monthly volume chart](chart.png)
Objectives

1. **market implied estimate** of the long run variance $\eta$
2. compare the different estimate of $\eta$ and $v_0$
3. compare the **calibration performance** of the Heston model by using
   - a fully free parameter set $\{v_0, \kappa, \eta, \lambda, \rho\}$;
   - a reduced parameter set $\{\kappa, \lambda, \rho\}$, using market data to fix $v_0$ and $\eta$
4. **calibration risk** arising from the different calibration procedures and objective functions: pricing of **exotics**
The Heston model

- Stock price process:

\[
\frac{dS_t}{S_t} = (r - q)dt + \sqrt{v_t}dW_t, \quad S_0 \geq 0
\]

Squared volatility process:

\[
dv_t = \kappa (\eta - v_t) dt + \lambda \sqrt{v_t}d\tilde{W}_t, \quad v_0 = \sigma^2_0 \geq 0,
\]

where \(W = \{W_t, t \geq 0\}\) and \(\tilde{W} = \{\tilde{W}_t, t \geq 0\}\) are correlated standard BM such that \(\text{Cov}(dW_t, d\tilde{W}_t) = \rho dt\).

- Parameters involved:
  - \(v_0 > 0\): initial variance
  - \(\kappa > 0\): mean reversion rate
  - \(\eta > 0\): long run variance
  - \(\lambda > 0\): volatility of variance
  - \(\rho\): correlation
Estimate of $v_0$

- $v_0 =$ square of the spot price of the VIX index expressed in units:
  
  $$v_0 = \left( \frac{\text{VIX}(t_0)}{100} \right)^2;$$
Estimate of $\eta$

- **moving window (MW) estimate:**

$$
\eta_{MW} = \frac{1}{T^{VIX}} \int_{-T^{VIX}}^{0} \left( \frac{VIX(t)}{100} \right)^2 \, dt = \text{mean}_{-T^{VIX} \leq t \leq 0} \left( \frac{VIX(t)}{100} \right)^2.
$$

(1)

- **exponentially weighted moving average (EWMA) estimate:**

$$
\eta_{EWMA} = (1 - \alpha) \sum_{i=1}^{N} \alpha^{N-i} \left( \frac{VIX(t_i)}{100} \right)^2
$$

(2)

- **mark-to-market (MTM) estimate:**

$$
P(K, T) - C(K, T) = \exp(-rT)(K - VIX_T)
$$

$$
\eta_{MTM} = \left( \frac{K^{ATM}}{100} \right)^2 = \left( \frac{LVIX(t_0)}{100} \right)^2.
$$

(3)
Evolution of the RMSE through time

Calibration performance

- calibration
- time series ($T^{VIX} = 0.5$)
- time series ($T^{VIX} = 3$)
- time series ($T^{VIX} = 5$)
- EWMA
- VIX option surface

02/01/2008 01/07/2008 02/01/2009 01/07/2009 30/10/2009
0 5 10 15 20 25 30 35 40
Trading day
RMSE
Calibration performance
- calibration
- time series ($T^{VIX} = 0.5$)
- time series ($T^{VIX} = 3$)
- time series ($T^{VIX} = 5$)
- EWMA
- VIX option surface

24/02/2006 03/07/2006 03/01/2007 02/07/2007 10/12/2007
0 2 4 6 8 10 12 14
Trading day
RMSE
Calibration performance
- calibration
- time series ($T^{VIX} = 0.5$)
- time series ($T^{VIX} = 3$)
- time series ($T^{VIX} = 5$)
- EWMA
- VIX option surface
Evolution of the estimate of $v_0$ and $\eta$ through time

Evolution of $v_0$ through time

Evolution of $\eta$ through time

**Calibration**
- time series ($T^{VIX} = 0.5$)
- time series ($T^{VIX} = 3$)
- time series ($T^{VIX} = 5$)
- EWMA
- VIX option surface

**Market Implied**
- calibration

Trading day
Evolution of the calibrated parameters through time

- **Evolution of $\lambda$ through time**
  - Calibration
  - Time series ($T^{VIX} = 0.5$)
  - Time series ($T^{VIX} = 3$)
  - Time series ($T^{VIX} = 5$)
  - EWMA
  - VIX option surface

- **Evolution of $\kappa$ through time**
  - Calibration
  - Time series ($T^{VIX} = 0.5$)
  - Time series ($T^{VIX} = 3$)
  - Time series ($T^{VIX} = 5$)
  - EWMA
  - VIX option surface
Evolution of the calibrated parameters through time (Cont.)
Objective functions

Objective functions:

• root mean squared error

\[
\text{RMSE} = \sqrt{\frac{1}{N} \sum_{j=1}^{N} (P_j - \hat{P}_j)^2}
\]

• average absolute error as a percentage of the mean price

\[
\text{APE} = \frac{1}{\text{mean}_j \hat{P}_j} \sum_{j=1}^{N} \frac{|P_j - \hat{P}_j|}{N}
\]

• average relative error

\[
\text{ARPE} = \frac{1}{N} \sum_{j=1}^{N} \frac{|P_j - \hat{P}_j|}{\hat{P}_j}
\]
## Daily variation of the model parameters

### RMSE

<table>
<thead>
<tr>
<th></th>
<th>calibration</th>
<th>MW ($T^{VIX} = 0.5$)</th>
<th>MW ($T^{VIX} = 3$)</th>
<th>MW ($T^{VIX} = 5$)</th>
<th>EWMA</th>
<th>market-implied</th>
</tr>
</thead>
<tbody>
<tr>
<td>average precision</td>
<td>2.439650</td>
<td>4.502293</td>
<td>4.834824</td>
<td>4.654399</td>
<td>5.530451</td>
<td>3.099928</td>
</tr>
<tr>
<td>average $\Delta v_0$</td>
<td>0.006793</td>
<td>0.009460</td>
<td>0.009460</td>
<td>0.009460</td>
<td>0.009460</td>
<td>0.009460</td>
</tr>
<tr>
<td>average $\Delta \eta$</td>
<td>0.022588</td>
<td>0.000567</td>
<td>8.864555e-05</td>
<td>5.796916e-05</td>
<td>0.001152</td>
<td>0.001986</td>
</tr>
<tr>
<td>average $\Delta \lambda$</td>
<td>0.080715</td>
<td>0.082411</td>
<td>0.050275</td>
<td>0.045892</td>
<td>0.134313</td>
<td>0.098315</td>
</tr>
<tr>
<td>average $\Delta \kappa$</td>
<td>0.413670</td>
<td>1.264941</td>
<td>0.631513</td>
<td>0.570401</td>
<td>2.030938</td>
<td>1.179841</td>
</tr>
<tr>
<td>average $\Delta \rho$</td>
<td>0.052782</td>
<td>0.014224</td>
<td>0.012173</td>
<td>0.015022</td>
<td>0.017561</td>
<td>0.038741</td>
</tr>
</tbody>
</table>

### ARPE

<table>
<thead>
<tr>
<th></th>
<th>calibration</th>
<th>MW ($T^{VIX} = 0.5$)</th>
<th>MW ($T^{VIX} = 3$)</th>
<th>MW ($T^{VIX} = 5$)</th>
<th>EWMA</th>
<th>market-implied</th>
</tr>
</thead>
<tbody>
<tr>
<td>average precision</td>
<td>0.235445</td>
<td>0.358760</td>
<td>0.359595</td>
<td>0.348677</td>
<td>0.324854</td>
<td>0.305351</td>
</tr>
<tr>
<td>average $\Delta v_0$</td>
<td>0.009716</td>
<td>0.009460</td>
<td>0.009460</td>
<td>0.009460</td>
<td>0.009460</td>
<td>0.009460</td>
</tr>
<tr>
<td>average $\Delta \eta$</td>
<td>0.044100</td>
<td>0.000567</td>
<td>8.864555e-05</td>
<td>5.796916e-05</td>
<td>0.001152</td>
<td>0.001986</td>
</tr>
<tr>
<td>average $\Delta \lambda$</td>
<td>0.111124</td>
<td>0.062548</td>
<td>0.049030</td>
<td>0.051594</td>
<td>0.079298</td>
<td>0.082063</td>
</tr>
<tr>
<td>average $\Delta \kappa$</td>
<td>0.684585</td>
<td>0.634506</td>
<td>0.578065</td>
<td>0.522025</td>
<td>0.686207</td>
<td>0.691249</td>
</tr>
<tr>
<td>average $\Delta \rho$</td>
<td>0.053700</td>
<td>0.035944</td>
<td>0.048639</td>
<td>0.038836</td>
<td>0.034663</td>
<td>0.038442</td>
</tr>
</tbody>
</table>

### APE

<table>
<thead>
<tr>
<th></th>
<th>calibration</th>
<th>MW ($T^{VIX} = 0.5$)</th>
<th>MW ($T^{VIX} = 3$)</th>
<th>MW ($T^{VIX} = 5$)</th>
<th>EWMA</th>
<th>market-implied</th>
</tr>
</thead>
<tbody>
<tr>
<td>average precision</td>
<td>0.019201</td>
<td>0.032250</td>
<td>0.035712</td>
<td>0.034686</td>
<td>0.038651</td>
<td>0.022346</td>
</tr>
<tr>
<td>average $\Delta v_0$</td>
<td>0.007801</td>
<td>0.009460</td>
<td>0.009460</td>
<td>0.009460</td>
<td>0.009460</td>
<td>0.009460</td>
</tr>
<tr>
<td>average $\Delta \eta$</td>
<td>0.027242</td>
<td>0.000567</td>
<td>8.864555e-05</td>
<td>5.796916e-05</td>
<td>0.001152</td>
<td>0.001986</td>
</tr>
<tr>
<td>average $\Delta \lambda$</td>
<td>0.094447</td>
<td>0.078535</td>
<td>0.059901</td>
<td>0.049769</td>
<td>0.126614</td>
<td>0.101588</td>
</tr>
<tr>
<td>average $\Delta \kappa$</td>
<td>0.537634</td>
<td>1.198459</td>
<td>0.931575</td>
<td>0.708374</td>
<td>1.879946</td>
<td>1.139020</td>
</tr>
<tr>
<td>average $\Delta \rho$</td>
<td>0.060817</td>
<td>0.030710</td>
<td>0.021987</td>
<td>0.016877</td>
<td>0.037403</td>
<td>0.043229</td>
</tr>
</tbody>
</table>
### Objective function and calibration time

<table>
<thead>
<tr>
<th>Date</th>
<th>Period</th>
<th>RMSE</th>
<th>ARPE</th>
<th>APE</th>
<th>RMSE</th>
<th>ARPE</th>
<th>APE</th>
<th>RMSE</th>
<th>ARPE</th>
<th>APE</th>
<th>RMSE</th>
<th>ARPE</th>
<th>APE</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/01/2007</td>
<td>the low volatility regime period</td>
<td>1.29784</td>
<td>0.23482</td>
<td>0.013068</td>
<td>1.268454</td>
<td>0.240736</td>
<td>0.012538</td>
<td>8.600138</td>
<td>0.41969</td>
<td>0.065222</td>
<td>1.632820</td>
<td>0.288468</td>
<td>0.013605</td>
</tr>
<tr>
<td>01/10/2007</td>
<td>the credit crisis period</td>
<td>2.510712</td>
<td>0.256275</td>
<td>0.014204</td>
<td>5.239480</td>
<td>0.308869</td>
<td>0.033527</td>
<td>2.215681</td>
<td>0.242762</td>
<td>0.017981</td>
<td>2.305956</td>
<td>0.270390</td>
<td>0.023710</td>
</tr>
<tr>
<td>21/11/2008</td>
<td>the panic wave period</td>
<td>5.928921</td>
<td>0.296433</td>
<td>0.022584</td>
<td>7.720376</td>
<td>0.271187</td>
<td>0.034957</td>
<td>8.268856</td>
<td>0.297219</td>
<td>0.037614</td>
<td>23.210741</td>
<td>0.611025</td>
<td>0.122339</td>
</tr>
<tr>
<td>30/10/2009</td>
<td>the end of the credit crisis</td>
<td>1.648008</td>
<td>0.265229</td>
<td>0.016606</td>
<td>4.033707</td>
<td>0.339502</td>
<td>0.028112</td>
<td>4.032063</td>
<td>0.339012</td>
<td>0.028055</td>
<td>2.361227</td>
<td>0.262819</td>
<td>0.016775</td>
</tr>
</tbody>
</table>

### Calibration time

<table>
<thead>
<tr>
<th>Date</th>
<th>RMSE</th>
<th>ARPE</th>
<th>APE</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/01/2007</td>
<td>136.179</td>
<td>140.2031</td>
<td>92.2969</td>
</tr>
<tr>
<td>01/10/2007</td>
<td>40.8281</td>
<td>55.2656</td>
<td>36.1563</td>
</tr>
<tr>
<td>21/11/2008</td>
<td>41.179</td>
<td>53.9063</td>
<td>37.3438</td>
</tr>
<tr>
<td>30/10/2009</td>
<td>22.6406</td>
<td>29.8125</td>
<td>22.9531</td>
</tr>
</tbody>
</table>
Option surface fit: 03/01/2007

Heston calibration – S&P500 – 03/01/2007 ($T_1 = 0.0466$)

Heston calibration – S&P500 – 03/01/2007 ($T_2 = 0.1233$)

Heston calibration – S&P500 – 03/01/2007 ($T_3 = 0.2000$)

Heston calibration – S&P500 – 03/01/2007 ($T_4 = 0.4493$)

Heston calibration – S&P500 – 03/01/2007 ($T_5 = 0.7178$)

Heston calibration – S&P500 – 03/01/2007 ($T_6 = 0.9671$)

Heston calibration – S&P500 – 03/01/2007 ($T_7 = 1.4658$)

Heston calibration – S&P500 – 03/01/2007 ($T_8 = 1.9644$)

Heston calibration – S&P500 – 03/01/2007 ($T_9 = 2.9616$)
Option surface fit: 21/11/2008
Calibration risk: difference in the value of the calibrated parameters arising from the different specifications of the objective function (Detlefsen and Hardle (2007)).

- maximum absolute value of the difference between the optimal parameter $p^* \in \{v_0^*, \kappa^*, \eta^*, \lambda^*, \rho^*\}$ obtained with the different objective functions:
  \[
  \max (|p^*_{\text{RMSE}} - p^*_{\text{APE}}|, |p^*_{\text{RMSE}} - p^*_{\text{ARPE}}|, |p^*_{\text{APE}} - p^*_{\text{ARPE}}|)
  \]

- Exotic prices
Calibration risk: $v_0$ and $\eta$
Calibration risk: $\lambda$ and $\kappa$

- $\lambda$ calibration risk plots for different time series ($T^{VIX} = 0.5$, $T^{VIX} = 3$, $T^{VIX} = 5$) and EWMA VIX option surface.

- $\kappa$ calibration risk plots for different time series ($T^{VIX} = 0.5$, $T^{VIX} = 3$, $T^{VIX} = 5$) and EWMA VIX option surface.
Calibration risk: $\rho$

Left graph:
- Calibration risk
- Time series ($T^{VIX}_1 = 0.5$)
- Time series ($T^{VIX}_3 = 3$)
- Time series ($T^{VIX}_5 = 5$)
- EWMA
- VIX option surface

Right graph:
- Calibration risk
- Time series ($T^{VIX}_1 = 0.5$)
- Time series ($T^{VIX}_3 = 3$)
- Time series ($T^{VIX}_5 = 5$)
- EWMA
- VIX option surface

Trading day:
- 24/02/2006
- 03/07/2006
- 03/01/2007
- 02/07/2007
- 10/12/2007

Trading day:
- 02/01/2008
- 01/07/2008
- 02/01/2009
- 01/07/2009
- 30/10/2009
Asian options

\[ AC = \exp(-rT)\mathbb{E}_Q[(\text{mean}_{0 \leq t \leq T} S_t - K)^+] \]
Lookback options

\[ LC = \exp(-rT) \mathbb{E}_Q[(S_T - m^S_T)^+] , \]

\[ m^X_t = \inf \{X_s, 0 \leq s \leq t\} \quad \text{and} \quad M^X_t = \sup \{X_s, 0 \leq s \leq t\} . \]

<table>
<thead>
<tr>
<th>calibration</th>
<th>03/01/2007</th>
<th>01/10/2007</th>
<th>30/10/2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>calibration</td>
<td>270.0157</td>
<td>343.0431</td>
<td>307.3771</td>
</tr>
<tr>
<td>MW - ( T^{VIX} = 0.5 )</td>
<td>231.8865</td>
<td>328.3097</td>
<td>308.3829</td>
</tr>
<tr>
<td>MW - ( T^{VIX} = 3 )</td>
<td>249.7935</td>
<td>304.9651</td>
<td>320.9077</td>
</tr>
<tr>
<td>MW - ( T^{VIX} = 5 )</td>
<td>271.4342</td>
<td>316.5518</td>
<td>299.2174</td>
</tr>
<tr>
<td>EWMA</td>
<td>221.4847</td>
<td>349.5726</td>
<td>299.4474</td>
</tr>
<tr>
<td>VIX</td>
<td>272.2177</td>
<td>340.0185</td>
<td>312.2275</td>
</tr>
</tbody>
</table>
One-touch barrier option

- **knock-out barrier**

\[
DOBC = \exp(-rT)\mathbb{E}_Q[(S_T - K)^+1(m_T^S > H)]
\]

\[
UOBC = \exp(-rT)\mathbb{E}_Q[(S_T - K)^+1(M_T^S < H)];
\]
One-touch barrier option (Cont.)

- **knock-in barrier**

\[
DIBC = \exp(-rT)E_Q[(S_T - K)^+1(m_T^S \leq H)]
\]

\[
UIBC = \exp(-rT)E_Q[(S_T - K)^+1(M_T^S \geq H)].
\]
Cliquet option

\[
\text{Cliquet} = \exp(-rT)\mathbb{E}_Q \left[ \min \left( \text{cap}^G, \max \left( \text{floor}^G, \sum_{i=1}^{N} \min \left( \text{cap}^L, \max \left( \text{floor}^L, \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}} \right) \right) \right) \right) \right].
\]
Calibration risk: the choice of the calibration procedure

Evolution of the Asian Call price through time ($K = S_0$, $T = 2$)

Evolution of the Lookback Call price through time ($T = 2$)

Evolution of the Cliquet price through time (local floor = $-0.03$, global floor = 0, local cap = 0.05, global cap = $+\infty$, $N = 8$, $T = 2$, $t_i = i/4$)
Calibration risk: the choice of the calibration procedure (Cont.)

Evolution of the DOBC price through time ($K = S_0$, $H = 0.75 S_0$, $T = 2$)

Evolution of the DIBC price through time ($K = S_0$, $H = 0.75 S_0$, $T = 2$)

Evolution of the UOBC price through time ($K = S_0$, $H = 1.25 S_0$, $T = 2$)

Evolution of the UIBC price through time ($K = S_0$, $H = 1.25 S_0$, $T = 2$)
Calibration risk: the choice of the objective function

Evolution of the Asian Call price through time \((K = S_0, T = 2)\) − full calibration

Evolution of the Asian Call price through time \((K = S_0, T = 2)\) − market implied reduced calibration

Evolution of the Lookback Call price through time \((T = 2)\) − full calibration

Evolution of the Lookback Call price through time \((T = 2)\) − market implied reduced calibration

Evolution of the Cliquet price through time \((\text{local floor} = -0.03, \text{global floor} = 0, \text{local cap} = 0.05, \text{global cap} = +\infty, N = 8, T = 2, t_i = i/4)\) − full calibration

Evolution of the Cliquet price through time \((\text{local floor} = -0.03, \text{global floor} = 0, \text{local cap} = 0.05, \text{global cap} = +\infty, N = 8, T = 2, t_i = i/4)\) − market implied reduced calibration
Calibration risk: the choice of the objective function (Cont.)
Global model risk measured by $\frac{\text{std}(\text{price})}{\text{mean}(\text{price})}$
## Global model risk

<table>
<thead>
<tr>
<th>Calibration</th>
<th>AC</th>
<th>AP</th>
<th>LC</th>
<th>LP</th>
<th>Cliquet 1</th>
<th>Cliquet 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calibration risk arising from the calibration procedure</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.033908</td>
<td>0.048530</td>
<td>0.060000</td>
<td>0.064051</td>
<td>0.085506</td>
<td>0.112690</td>
</tr>
<tr>
<td>APE</td>
<td>0.038819</td>
<td>0.054284</td>
<td>0.066131</td>
<td>0.080649</td>
<td>0.071326</td>
<td>0.095247</td>
</tr>
<tr>
<td>ARPE</td>
<td>0.066253</td>
<td>0.088669</td>
<td>0.096187</td>
<td>0.140164</td>
<td>0.047125</td>
<td>0.063327</td>
</tr>
<tr>
<td><strong>Calibration risk arising from the objective function</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MW ($T^{VIX} = 0.5$)</td>
<td>0.020271</td>
<td>0.225636</td>
<td>0.520212</td>
<td>0.425640</td>
<td>1.727657</td>
<td>1.726586</td>
</tr>
<tr>
<td>MW ($T^{VIX} = 3$)</td>
<td>0.022378</td>
<td>0.252357</td>
<td>0.506769</td>
<td>0.380589</td>
<td>1.727545</td>
<td>1.726371</td>
</tr>
<tr>
<td>MW ($T^{VIX} = 5$)</td>
<td>0.037969</td>
<td>0.256160</td>
<td>0.492920</td>
<td>0.378976</td>
<td>1.727664</td>
<td>1.726532</td>
</tr>
<tr>
<td>EWMA</td>
<td>0.014861</td>
<td>0.246158</td>
<td>0.506096</td>
<td>0.381877</td>
<td>1.727508</td>
<td>1.726304</td>
</tr>
<tr>
<td>market-implied</td>
<td>0.018896</td>
<td>0.229911</td>
<td>0.513259</td>
<td>0.407050</td>
<td>1.727500</td>
<td>1.726191</td>
</tr>
<tr>
<td><strong>Calibration risk arising from both the calibration procedure and the objective function</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALL</td>
<td>0.049028</td>
<td>0.213804</td>
<td>0.434454</td>
<td>0.352613</td>
<td>1.453121</td>
<td>1.452149</td>
</tr>
</tbody>
</table>
## Global model risk

<table>
<thead>
<tr>
<th>calibration</th>
<th>DOBC</th>
<th>DOBP</th>
<th>UOBC</th>
<th>UOBP</th>
<th>DIBC</th>
<th>DIBP</th>
<th>UIBC</th>
<th>UIBP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration risk arising from the calibration procedure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.039281</td>
<td>0.198357</td>
<td>0.135771</td>
<td>0.067489</td>
<td>0.524566</td>
<td>0.123583</td>
<td>0.087288</td>
<td>0.357493</td>
</tr>
<tr>
<td>APE</td>
<td>0.045615</td>
<td>0.185915</td>
<td>0.152006</td>
<td>0.075266</td>
<td>0.528947</td>
<td>0.130241</td>
<td>0.118739</td>
<td>0.465904</td>
</tr>
<tr>
<td>ARPE</td>
<td>0.077479</td>
<td>0.208932</td>
<td>0.254173</td>
<td>0.115150</td>
<td>0.642981</td>
<td>0.182209</td>
<td>0.212589</td>
<td>0.688647</td>
</tr>
<tr>
<td>Calibration risk arising from the objective function</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MW ($T_{VIX} = 0.5$)</td>
<td>0.274359</td>
<td>1.243682</td>
<td>0.814589</td>
<td>0.134273</td>
<td>1.302822</td>
<td>0.199536</td>
<td>0.217512</td>
<td>1.318924</td>
</tr>
<tr>
<td>MW ($T_{VIX} = 3$)</td>
<td>0.256693</td>
<td>1.205416</td>
<td>0.747263</td>
<td>0.168450</td>
<td>1.374050</td>
<td>0.285221</td>
<td>0.207119</td>
<td>1.318154</td>
</tr>
<tr>
<td>MW ($T_{VIX} = 5$)</td>
<td>0.246357</td>
<td>1.153557</td>
<td>0.692015</td>
<td>0.149320</td>
<td>1.486095</td>
<td>0.242757</td>
<td>0.172117</td>
<td>1.427167</td>
</tr>
<tr>
<td>EWMA</td>
<td>0.251869</td>
<td>1.173680</td>
<td>0.731435</td>
<td>0.112777</td>
<td>1.470301</td>
<td>0.168577</td>
<td>0.181764</td>
<td>1.442246</td>
</tr>
<tr>
<td>market-implied</td>
<td>0.256144</td>
<td>1.196937</td>
<td>0.760032</td>
<td>0.167349</td>
<td>1.377761</td>
<td>0.284868</td>
<td>0.224092</td>
<td>1.306372</td>
</tr>
<tr>
<td>Calibration risk arising from both the calibration procedure and the objective function</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALL</td>
<td>0.226264</td>
<td>1.011796</td>
<td>0.644223</td>
<td>0.147501</td>
<td>1.174243</td>
<td>0.227528</td>
<td>0.211867</td>
<td>1.135486</td>
</tr>
</tbody>
</table>
Conclusion

- market implied estimate of the long run variance of stochastic volatility models (Put-Call parity of long maturity options on the VIX)
- market implied estimate: same trend as calibrated parameter $\eta$ but typically lower
- optimal reduced calibration procedure: market implied estimate of $\eta$ (except during huge investors fear period)
- reduced calibration procedure: significant reduction of calibration computation time and calibration risk (except for $\kappa$)
- reduced calibration: more stable $\eta$ and $\rho$ through time, counterbalanced by lost of stability for $\kappa$.
- price of a wide range of exotic options: significantly $\neq$ under the 2 calibration settings (especially for cliquet and barrier options)
- Model risk (choice of calibration procedure) vs calibration risk (specifications of the objective function)
- Even within a particular model, model risk and calibration risk are present.
Thank you for your attention!