2009 Progress Report
Chair "FINANCIAL RISKS"
of the Risk Foundation.

Synthèse

L'année 2009 est une année exceptionnelle pour la chaire "Risques financiers". Grâce au renforcement du nombre de chercheurs permanents, notamment à l'Ecole Polytechnique pendant l'année 2008, l'activité de recherche a été très intense dans de nombreux domaines détaillés ci-dessous.

Des problèmes nouveaux, des idées nouvelles ont été introduits orientant en particulier les thèmes de recherche de nos doctorants, actuellement au nombre de 14. Cela représente une augmentation notable par rapport à ces dernières années.

La finance parisienne est devenue très attractive, et grâce aux bonnes conditions de travail et de salaire que nous pouvons offrir, le recrutement de post-docs de grande valeur n'est plus un problème. Deux post-docs supplémentaires nous ont rejoints cette année.

Nous avons de nombreux visiteurs de qualité, et participons activement comme organisateurs ou conférenciers invités à de nombreux congrès internationaux. Deux d'entre nous sont invités à donner des cours prestigieux à Toronto cette année, lors du semestre consacré à la finance.

Peu d'équipes dans le monde peuvent se vanter d'avoir autant de publications dans les meilleures revues de mathématiques financières ou de quantitative finance, et la liste des préprints est longue.

Depuis la rentrée, un nouveau thème de recherche est présent dans la chaire, sous l'impulsion de E. Bacry and M. Rosenbaum : la finance statistique avec une forte composante sur les données hautes fréquences et la microstructure: tous des thèmes très à la mode dans les marchés.

Plusieurs thèmes de recherche ont été approfondis à la suite de la crise, comme le risque de liquidité, le mimétisme, les mesures de risque, comme détaillé ci-dessous. Un groupe de travail sur la simulation des événements rares s'est tenu réguliérement cette année.

Un colloque organisé l'année dernière sur les méthodes numériques par l'ENPC, un autre est prévu pour début Janvier 2011, pour fêter à la fois la chaire, et les vingt ans du Master2 "Probabilités et Finances" de Paris VI et Ecole Polytechnique. Le Comité de pilotage ENPC/X/SG y est très impliqué. Nous souhaitons en faire un grand événement.

Dans cette dynamique ainsi créée, nous souhaitons renforcer les collaborations européennes.

Contents

Introduction

The goal of this report is to describe the impact of the Chair "Financial risks" on the research activities of the financial mathematics team of CMAP, and in particular its effect on international visibility, invitations of foreign researchers, publications, conferences, as well as the local impact on the hiring policy.

The presence of the Chair has not only brought the necessary funding to invite prestigious foreign visitors. More importantly, it has increased the role of CMAP as an important research center in financial mathematics, which allows us to compete with the best universities worldwide for the recruitment of the most talented PhD students, post-docs and professors. We have also been able to attract several top-level researchers as permanent part-time collaborators. Last but not least, regular discussions with quants from Société Générale during the joint seminars of the Chair allow us to remain in contact with the professional community and create a strong interest and motivation towards research on more applied subjects of interest to the banks.

CMAP is host of several chairs and it is difficult to attribute some activities or publications to a single chair. Our aim in this report is mainly to show how the presence of additional funding has created a strong and coherent research dynamics.

1 Research team

The finance team of Ecole Polytechnique continues to grow with the arrival of new doctoral students and post-docs.

1.1 Permanent members

There were no changes this year. In particular, senior researchers/professors, who may supervise PhD's students are 3 at ENPC and "$3\frac{1}{2}$" at Polytechnique.

- ENPC:

  - A. Alfonsi, Researcher at CERMICS, ENPC
  - J.-F. Delmas, Senior Researcher at ENPC, Part-time assistant professor at Ecole Polytechnique
  - B. Jourdain, Senior Professor at ENPC, Part-time assistant professor at Ecole Polytechnique
  - B. Lapeyre, Senior Professor at ENPC,

- Ecole Polytechnique:

  - Emmanuel Bacry, Senior CNRS researcher at Ecole Polytechnique
  - Nicole El Karoui, Senior Researcher at Ecole Polytechnique, Professor at Paris VI University
  - Alfred Galichon, Assistant professor at Ecole Polytechnique (Economics department)
  - Caroline Hillairet, Assistant professor at Ecole Polytechnique
  - Mathieu Rosenbaum, Assistant professor at Ecole Polytechnique
  - Peter Tankov, Assistant professor at Ecole Polytechnique
  - Nizar Touzi, Senior Professor at Ecole Polytechnique.

1.2   Doctoral students

PhD defended in October 2009   *Mohammed M'rad*, "Progressive Utilities and Stochastic Flows" (advisor: N. El Karoui, one year funded by the chairs)

The recent financial crisis showed the need to develop appropriate tools to quantify financial risk, in particular long term risk. Over the last decade, new concepts as dynamic monetary risk measures or progressive forward utilities had been introduced in academic literature limiting the drawbacks induced by the classical concepts, in particular the strong dependence in maturity. Forward utilities, first introduced by M.Musiela and T. Zariphopoulou, as stochastic utilities $U(x,t)$, coherent with a liquid family of portfolios with given initial wealth x yield horizon-unbiased optimal portfolios. Such class of utilities is stable by change of numéraire, so we can still consider local martingale portfolio. Such processes may be characterised as solutions of nonlinear partial stochastic differential equation, or using the duality point of view. Conditions on the volatility are introduced to be sure that concavity and monotony are satisfied. Motivated by the fact that investors in general have a better idea of their optimal portfolio than their utility function, all progressive utilities having the same initial utility function and the same given optimal portfolio assumed to be increasing with respect to the initial wealth are characterized. The simplest is obtained by combining the initial utility and the inverse flow of the optimal wealth process. The general case is obtained in the same way , but now the derivative of the initial wealth is the initial condition of a family of parametric local martingales, linked to the optimal wealth by "martingale" conditions. Those utilities are, then, used to calculate certainty equivalents, indifference pricing, etc...

PhD defended in November 2009   *Mohamed Sbai*, "Dependence modeling and simulation of stochastic processes in finance" (advisor: B. Jourdain) In the first part of his thesis, Mohamed Sbai applies exact simulation techniques to the computation of the price of Asian options in the Black-Scholes model, proposes some high order discretization schemes for stochastic volatility models and obtains a uniform in time estimate of the Wasserstein distance between the marginals of an SDE and its Euler scheme. The second part addresses the modelling of dependence both in stock markets (joint model for the dynamics of an index and its composing stocks) and in credit risk (dynamic frailty model for the transitions of ratings in a large credit portfolio).

PhD in progress :15

- Advanced

  - *David Nicolay* is finishing his thesis with N. El Karoui. His research is focused on asymptotics of implied volatility in IRS derivatives.

  - *Gilles-Edouard Espinosa* is finishing his thesis with N. Touzi. His research is focused on the tracking of the maximum of a scalar diffusion and on portfolio management in a mean-field type model similar to the one suggested by Lasry and Lions, to take into account the imitative behavior of investors.

  - *Arash Fahim* (third year with N. Touzi; joint supervision with Teheran) His research focuses on one hand on the probabilistic numerical methods for non-linear PDE, for integro-differential equations, and Backward Stochastic Differential Equations, and on the other hand on the modelling of the carbon emission markets.

- *Isabelle Camilier* (third year with N. El Karoui): Numerical methods in credit risk, Long term interest rates modelling via forward utilities .

- *Harry Bensusan* (third year with N. El Karoui; funded by the chairs for one year): Asymptotic Wishart models for the interest rates and volatility. Modelisation of longevity evolution using dynamics of population models.

- *Laurent Duvernet* (last year with E. Bacry): Multifractal modelling in finance.

- *Emilie Fabre* (second year with N. Touzi, funded by the chairs) : Portfolio management. More precisely, she is interested in the optimal strategy to sell an illiquid asset in a market where other liquid assets are traded.

- *Carmine de Franco* (second year with P. Tankov, joint supervision with Paris VII): Portfolio insurance under risk measure constraint.

- *Trung-Lap Nguyen* (second year with N. El Karoui, funded by the chairs): Optimisation in finance and insurance: variable annuities, long term risk, longevity.

- *Abdelkoddousse Ahdida* (second year with B. Lapeyre and A. Alfonsi): Simulation and discretization schemes for Wishart processes, and application to finance.

- *Khalil Al Dayri* (second year with E. Bacry, joint supervision with ENSAE, funded by the Chair "Financial risks"): Modeling of order books.

- New PhD's

  - *Dylan Possamai* (first year with N. Touzi): Market illiquidity models.

  - *Xiaolu Tan* (first year with N. Touzi): On probabilistic numerical methods for non-linear degenerate PDE.

  - *Jose Infante Acevedo* (first year with T. Lelièvre and A. Alfonsi): Greedy algorithm for high-dimensional PDEs. Market microstructure modeling.

  - *Maxence Jeunesse* (first year with B. Jourdain and J.-P. Chancelier): Numerical methods for some financial issues.

  - *Chao Zhou, first year with A. Matoussi, funded by the chairs*: Stochastic control under model uncertainty.

1.3   Post-docs, Permanent visitors/associate researchers

Last year we strengthened our potential with permanent researchers. This year it is the turn of our long term collaborations, through invitation of post-docs, or by the introduction of the status of associated researcher.

- Post-docs

  - Gonçalo Dos Reiss (since January 2009, PhD in Humboldt University, Berlin; funded by the chairs): Quadratic BSDEs and Applications to weather derivatives.

  - Rudra Jena (since February 2009, PhD in theoretical physics in Tata Institute, India; funded by the Chair "Financial risks"): Hedging and Arbitrage in stochastic volatility models and modelling of systemic risk.

  - Christoph Frei (since October 2009, PhD in ETH Zurich): BSDE and Indifference Pricing.

- Mohamed M'rad (since October 2009, PhD CMAP): Stochastic forward utility; Liquidity problems in Credit Risk.
- Piergiacomo Sabino (January-June 2009, PhD university of Bari): Variance reduction in Quasi Monte Carlo methods, stratification.

- **Permanent Visitors**
  The presence of the Chairs has enabled us to create permanent visitor contracts under which a French or a foreign researcher spends a part of his / her time (usually, one day a week) at CMAP and contributes to the local research activities, or, alternatively, collaborates remotely on one of the specific topics of the chairs. For the year 2009, these permanent visitors were

  - Pauline Barrieu (LSE Londres)
  - Stéphane Loisel (Lyon)
  - Umut Çetin (LSE Londres)
  - Emmanuel Gobet (Grenoble)
  - Anis Matoussi (Le Mans)
  - Jean-François Muzy (Corsica University)

1.4 Short-term visitors

A number of invited researchers visited our team for more or less extended periods of time:

- Long periods

  - Michael Tehranchi is Senior lecturer at Cambridge University (invited by N. Touzi). He obtained his PhD thesis from Princeton University in 2004 with René Carmona. His research focuses on the modeling of volatility surfaces and the term structure of investor preferences.
  - Tom Hurd is Professor at the Department of Mathematics and Statistics of McMaster University. He started his carrier in mathematical physics, but since 1998 his research is focused on financial mathematics, where he made important contributions in particular in the fields of credit risk and portfolio risk management (invited by N. Touzi).
  - Jianfeng Zhang is Associate Professor at the University of Southern California. He obtained his doctorate from Purdue University in 2002 under Jin Ma. Jianfeng is an internationally recognized expert on Backward SDEs and their applications in finance. His current work with N. Touzi focuses on second order BSDEs.

- Short periods

  - René Carmona (Princeton) is a Professor at the department of Operations Research and Financial Engineering of Princeton University. His research was initially focused on the applications of stochastic analysis in mathematical physics, making a significant contribution to the theory of stochastic PDE. He is interested in applications in statistics, signal and image processing and in finance. At Polytechnique, he is collaborating with N. Touzi on the modeling of CO2 emissions market.

- Mete Soner is Professor at ETH, Zurich. He spent an important part of his carrier in the US (Carnegie Mellon University, Princeton) and later in Istanbul (Coç, Sabanci). Mete is recognized as one of the foremost experts of stochastic control methods in finance. He has obtained important results on the theory of incomplete markets, hedging under constraints and markets with transaction costs. He is participating in a research project with N. Touzi on the second order BSDE with applications to probabilistic numerical methods and market illiquidity modeling.

- Jin Ma is a Professor at the University of Southern California. He specializes in Backward Stochastic Differential Equations (BSDE) whose development was stimulated by financial applications and had an important impact on the problems of hedging, portfolio management, risk measures and numerical methods. He is participating in a research project with Nizar Touzi, aiming to extend the large deviations results to Itô processes. This work is motivated by the modeling of implied volatility surfaces and in particular their short maturity asymptotics.

- Giulia Di Nunno is Associate Professor at the University of Oslo. Her research focuses on Malliavin calculus and the modeling of information asymmetry. Her current work with Jocelyne Bion-Nadal focuses on extension theorems for linear operators preserving "sandwich"-type conditions and applications to pricing in incomplete markets (invited by C. Hillairet and J. Bion-Nadal).

- Jim Gatheral : Bank of America Merrill Lynch and the Courant Institute, NYU. (4 days, invited by Emmanuel Bacry, speaker at the FIESTA seminar).

- Sasha Stoikov : Cornell University (USA) (3 days, ivited by Emmanuel Bacry, speaker at the FIESTA seminar)

- Fabrizio Lillo : University of Palermo (Italy) and Santa Fe Institute (USA) (3 days, invited by Emmanuel Bacry, speaker at the FIESTA seminar)

- Robert Almgren : Quantitative Brokers and New York University, USA (3 days, invited by Emmanuel Bacry, speaker at the FIESTA seminar).

- Robert Steltzer (TU Munich). Research topics : Stochastic volatility models, Lèvy processes, Time series analysis, Regime-switching models. (15 days, invited by Mathieu Rosenbaum)

- Mark Podolskij (ETH Zurich). Research topics : Semimartingale models of prices, jump models, volatility estimation. (9 jours, invited by Mathieu Rosenbaum)

- Anthony Reveillac: Humboldt University in Berlin (1 semaine, invited by Gonzalo Dos Reis).

- Jianing Zhang: Humboldt University in Berlin (1 semaine, invited by Gonzalo Dos Reis).

- Mats Broden (Lund University). Research topics: errors from discrete-time hedging; jump risk. Invited by P. Tankov.

- Martin Keller-Ressel (ETH Zurich). Research topics: volatility surfaces, affine processes. Invited by P. Tankov.

- Ekaterina Voltchkova (Toulouse I University). Research topics: integro-differential equations, numerical methods. Invited by P. Tankov.

- Martijn Pistorius (Imperial College London). Research topics: Lèvy processes, option pricing. Invited by P. Tankov

- Alexander Schied (Univ. Mannheim, formerly TU Munich). Alexander Schied is full professor at the university of Mannheim. His current research topics bring on risk measures and liquidity modelling. He is collaborating with A. Alfonsi and has been invited two days in June 2009.

- Jérôme Lelong (Grenoble) works mainly on Monte-Carlo methods and numerical methods in finance. He is a former phD student of Bernard Lapeyre and has strong links with the CERMICS. He has been invited in December by A. Alfonsi, and will collaborate regularly with the CERMICS in the future.

- Michel Vellekoop (University of Amsterdam) Research topics : modelling of dividends and pricing of American options in models with discrete dividens. His work [75] with B. Jourdain started during his one week visit in january 2009 focuses on the properties of the exercise boundary of the American Put options in the Black-Scholes model with discrete dividends.

- Reiichiro Kawai. Research topics : Monte Carlo methods, variance reduction, Lèvy processes. Invited one week by B. Lapeyre in April 2009.s

2 Research Activity

During the year 2009, the research activity of the chair was structured around three main topics: risk measurement and management, numerical methods, statistics of finance.

2.1 Liquidity risk

One of the two mini-courses of the European summer school in financial mathematics in August 2009 was dedicated to liquidity risk.

The general framework The financial mathematics literature contains various illiquidity models. in this paragraph, we do not discuss optimal liquidation problems à la Almgren and Chriss. Instead, our focus is on the problem of hedging derivative securities in financial market models close to the Black and Scholes model, leading to solutions which are implementable in reasonable time in order to be used in pratice. The following are two such examples which were introduced respectively by Soner-Touzi and Cetin-Jarrow-Protter:

- Hedging under Gamma constraints: find the minimal initial capital which allows to hedge a given derivative security by means of a portfolio strategy whose Gamma is restricted to some bounded interval. The bounds of the interval are fixed exogeneously and depend on the nature of the underlying asset. They represent thresholds above which the dynamic adjustment of the hedging strategy requires to go deep in the order book, and induce an immediate price impact supported by the hedger.

- Hedging under illiquidity costs: this model was suggested in the PhD thesis of Umut Cetin. The unit prices depend on the size of the trade, in agreement with real financial markets.

The continuous-time model leads to an expression of the portfolio value which differs from the classical one by a simple penalty proportional to the squared Gamma of the portfolio. The proportionality coefficient is given by the slope at the origine of the unit price process, and can be infered directly from real order book data.

Both problems have been studied in previous works:

1. The superhedging cost is characterized as the unique solution of a fully nonlinear partial differential equation.

2. The optimal hedging strategy is only known formally. It consists of a succession of Black-Scholes Delta hedging regime and static hedging regime of the *Buy-and-hold* type. A rigorous justification of the hedge is an open problem.

3. All existing results are specific to Vanilla options, and are not applicable to exotic options.

4. The numerical approximation of the solution can be performed by the finite differences method. This is another limitation as path dependent derivative securities are naturally suited with Monte Carlo based numerical methods.

Our contribution

- In order to handle the above questions 2 and 3, we have introduced recently the notion of second order backward stochastic differential equations. This is a suitable mathematical formulation which encompasses in a natural way the problem of hedging under Delta and Gamma constraints. This theory is developed in a series of four papers by Mete Soner, Nizar Touzi and Jianfeng Zhang, and leads to a non-trivial extension of the notion of backward stochastic differential equations, as introduced by Pardoux and Peng in 1991.

- As for the numerical approximation issues, we have developed two research directions:

- *small illiquidity asymptotic expansion techniques.* This allows to obtain an expansion of the hedging cost under illiquidity cost around the frictionless Black-Scholes model. We obtain natural expansions for Vanilla options defined by continuous payoff functions. For discontinuous payoff functions, as digital options, we find a phase transition effect in that the orders of the asymptotic expansion are not trivial. This problem is presently analyzed in details by Dylan Possamai, a first year PhD student at Ecole Polytechniue, supervised by Nizar Touzi.

- *Monte Carlo methods for fully nonlinear partial differential equations.* The mathematical analysis of these numerical probabillistic algorithms was initiated in the PhD thesis of Arash Fahim at Ecole Polytechnique, under the supervision of Nizar Touzi. The algorithm is a natural extension of those suggested by the discretization of backward stochastic differential equations. Such numerical techniques open the door for the treatment of higher dimensions than those of the classical finite differences methods. Extension to the integro-differential case are also available in Fahim's thesis, and allow to handle problems involving jump processes. Finally, Xialou Tan, a first year PhD student supervised by Nizar Touzi, is analyzing the extension to the degenerate case.

Performance concerned investors and financial bubles

Problem formulation Consider N investors on a financial market who seek to optimize their portfolio allocation. Their preferences account for the following two criteria:

- the absolute performance of the portfolio,

- the difference of their performance to the average performance of their competitors.

The latter performance concern criterion induces an interaction between the investors which has not been studied in the previous literature. Our objective is to justify and to understand the so-called financial bubles on some markets due to the presence of an important number of actors; the mimicking of the investors induced by their performance concern, leads to a self-fulfilling of their believes that the market is growing, and therefore to an over-investment on the market.

Our contributions Each investor optimizes the expected utility of a convex combination of his performance and the difference of his performance to the average performance of his competitors. The utility functions has constant absolute risk aversion, but our problem formulation allows for various sources of heterogeneity between the investors:

- the risk aversion coefficients are different,

- the performance concern coefficients are also different,

- the agents have different access to the financial market, and are subject to different portfolio constraints.

Our main interest is to prove the existence of a Nash equilibre, that is, a situation where each investor reaches an optimal strategy, given the portfolio strategies of the others.

This problem has been analyzed in the PhD thesis of Gille-Edouard Espinosa, a PhD student at Ecole Polytechnique, supervised by Nizar Touzi.

- Assuming that the price processes of the assets available for trading are defined by the multi-dimensional Black-Scholes model, we have proved existence and uniqueness of a Nash equilibrium, which we explicitly derive. In particular, we find an important evidence of the formation of financial bubles.

- If the coefficients of the prices processes dynamics are not constant, the question of existence and uniqueness of a Nash equilibrium reduces to a problem quadratic multi-dimensional backward stochastic differential equation. Unfortunately, we could not make any progress in this direction, and the problem of existence of a solution to the latter type of equation is still open...

- We next modify slightly the problem formulation by introducing a penalization by the risk induced by the strategies of the investors. This new formulation allows to reduce the problem of Nash equilibrium to the resolution of a multi-dimensional backward stochastic differential equation in the Lipschitz framework. We then can use classical results to prove existence and uniqueness od a Nash equilibrium.

2.2   Market microstructure, and Market imperfections

More and more problems related to the market microstructure are considered. This research direction is closely related to statistical finance discussed below in section 2.6

- **Limit order book** A. Alfonsi has continued his collaboration with A. Schied (Univ. Mannheim) on a simple limit order book model [62, 69, 70]. First, they have obtained optimal strategies for buying a large amount of shares with a given number of trades, when the market resilience is exponential. This problem is closely related to the absence of Price Manipulation Strategies. They have also considered the case of more general resilence kernels. In that case, the model is no longer Markovian and a new kind of manipulation strategies appears.

- **Microstructure Noise** A related problem is the study of the performance of the hedging strategy in the presence of market imperfections, addressed by P. Tankov for the case of discrete-time rebalancing [26, 36] and M. Rosenbaum for in the case of market microstructure noise [23]. One can also quote the work of E. Gobet and A. Makhlouf on the influence of pay-off regularity on the performance of Delta-Gamma hedging and the efficient choice of readjustment dates: [50].

2.3 Dynamic Risk measures and Forward utilities

The axiomatic study of risk measures has made an important progress since Basel II. This has been an important topic for the team for many years, essentially on the theoretical side. Also new directions are explored with the concept of the forward utilities.

- **Monetary Risk measures, reserves and ambiguity measurement**

  - How to calculate reserves when ambiguity also concerns the interest rate model is the motivation to study sublinear risk measures [31]
  - The research in this direction continued in 2009 [5, 12, 42].
  - It is also worth to mention the work by J. Bion-Nadal on dynamic pricing in the presence of transaction costs and liquidity risk [6].
  - The PhD thesis of C. de Franco studies the portfolio insurance under constraints given by a convex possibly dynamic risk measure.

- **Forward Utilities**

  - How to define a dynamic utility, with nice properties
  - Original contribution in the Phd Thesis of M'Rad, recently presented in several conferences (see section 1.2).

2.4 Market Risk

The market risk has remained one of the focal points of our research including the credit risk, the volatility risk/volatility smile

- **Market risk and stochastic volatility**

  H. Bensusan in his PhD thesis studies the stochastic volatility model in which the volatility of the asset follows the trace of a Wishart process. Contrary to a classic multifactor Heston model, the Wishart volatility model has additional degrees of freedom allowing to model stochastic correlation. Thanks to this flexibility, this

model produces a better fit of market data than the Heston model. Besides, the Wishart volatility model keeps a clear interpretation of its parameters and remains analytically tractable.

First, H. Bensusan presents a Monte Carlo simulation method for the evaluation of complex options. Secondly, the aim of his work is to provide an accurate approximation method to deal with asymptotic smiles and to apply this procedure to the Wishart volatility model in order to well understand the smile behavior and to make its calibration easier. Inspired by the singular perturbations method introduced by J.-P. Fouque, H. Bensusan suggests an efficient procedure of perturbation for affine models that provides an approximation of the asymptotic smile (for short maturities and for a two-scale volatility). Thanks to the affine properties of the Wishart volatility model, the perturbation of the Riccati equations provides the suitable approximations. The resulting approximations allow to study the influence of different model parameters and can also be used for calibration.

Smile behavior in interest rate models is analysed by asymptotic methods by D. Nicolay in his thesis.

- **Other Market Risks**

  - Credit risk (N. El Karoui [11, 43, 44] and A. Alfonsi [71, 72])
  - the jump risk (P. Tankov [25, 27])

- **Misspecified Volatility** An ongoing research project of P. Tankov and R. Jena on arbitrage opportunities in misspecified stochastic volatility models aims to provide a firm theoretical foundation for the market practices used in the field of volatility arbitrage. There is a vast empirical evidence of systematic mispricing in option markets: the implied volatility is higher than the historical one; the volatility of volatility and correlation estimated from the historical data are different from those implied from option prices. The objective of P. Tankov and R. Jena is to characterize the optimal investment strategies in the presence of such misspecification. In particular, new characterizations of well-known structures such as risk-reversals and butterflies is given in terms of their optimality for volatility arbitrage.
This research was presented during the joint seminar with Société Générale on December 7 and benefited from discussions with L. Bergomi and other representatives of Société Générale.

- **Correlation risk and dependence modeling.** B. Jourdain and M. Sbai [73] have developed a model coupling an index with its underlying stock components. These stocks are supposed to be influenced by the index which reflects the view of the investors on the state of the economy. More precisely, the return of each stock is decomposed into a systemic part driven by the index and an idiosyncratic part driven by the stock itself. In the limit of a large number of stocks, the index evolves according to a local volatility model and each stock according to a stochastic volatility model driven by the index. According to Dupire's formula, it is then possible to fit the smile of the index. Then the idiosyncratic part of the dynamics of each stock may be calibrated using a well chosen system of interacting stochastic differential equations. This research has benefited from numerous discussions with L. Bergomi and J. Guyon (Société Générale).

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- **Approximate option pricing formulas (E.Gobet)** An important part of research of E. Gobet deals with the development of approximate pricing formulas for vanilla options in the models frequently used in finance. There exists an extensive literature in this domain, using regular or singular perturbation techniques, either from the analytic (PDE) point of view (as e.g., in the SABR or other stochastic volatility models) or using stochastic perturbation methods introduced by Watanabe. However, these approaches do not allow to treat the case of time-dependent coefficients, and the error estimates, when they exist, do not show the relative influence of different parameters.

  E. Gobet and hist students have introduced another stochastic expansion approach which resolves these difficulties using the notion of a proxy model (usually, Black-Scholes, Black or Merton). Using an ingenious parameterization of the distance from the model to its proxy, the option price can be written as the price in the proxy model plus a weighted sum of greeks in this proxy model. Using the closed formulas available in the proxy model, the option pricing is very fast, reducing the overall model calibration time by a factor of 100 or more, and achieving excellent precision on realistic examples.

  The models studied so fare include

    - Local volatility model with lognormal jumps: [4]. Second order expansion. This work was carried out before E. Gobet joined the chair.
    - Local volatility model: [3]. Expansion to arbitrary order. This work was carried out before E. Gobet joined the chair.
    - Time-dependent Heston model: [34]. Second order development in volatility of volatility. This work was carried out before E. Gobet joined the chair..
    - Local volatility model with stochastic interest rates: [33].
    - Basket options and Asian options in a local volatility model. Work in progress with M. Miri.
    - Vanilla options with underlying paying discrete dividends. Work in progress with P. Etore.

2.5 Numerical methods

2.5.1 Monte Carlo methods for non-linear PDE

For linear PDE, such as the Black-Scholes pricing equation, it is well known that the solution can be represented in the form of an expectation. This representation allows to construct probabilistic numerical methods, such as the Monte Carlo method, for the numerical approximation of the solution.

In a research project led by N. Touzi, it has been shown that this methodology can be extended to nonlinear PDE, and the convergence rates of the algorithm have been established [45]. Recently, these results have been extended to integro-differential equations by A. Fahim in his PhD thesis in Ecole Polytechnique.

The Monte Carlo methods are appreciated by the financial industry for their flexibility and the possibility to solve path-dependent problems. The extension of this methodology to nonlinear setting is also very important for the markets. For example, the robust pricing of a financial position in the case when volatility or correlation is uncertain, or when liquidity constraints are present, leads to a non-linear PDE.

Variance reduction for Monte Carlo methods Research aimed at developing robust variance reduction methods for Monte Carlo simulations is conducted at CERMICS under the direction of B. Lapeyre and B. Jourdain. The idea is to design algorithms in which the variance reduction parameters are tuned adaptively using information from the simulations already performed. It has already been applied successfully to importance sampling [65] and stratification [67, 68].

Approximation of stochastic differential equations Also in the field of numerical methods, one can quote the works of G. Dos Reis [15, 16] on the approximation of BSDE with quadratic growth (such BSDE arise for example in indifference pricing), by P. Tankov [52] on jump-adapted discretization schemes for Lèvy-driven SDEs (these schemes allow, for instance to construct fast simulation algorithms for the Libor market model with jumps) and by E. Gobet on adaptive Monte Carlo methods for Backward stochastic differential equations [48], the impact of fractional regularity of terminal condition [49] and the method of shifting the barrier for the simulation of stopped diffusion processes and for pricing barrier options [14][13].

Simulation of stochastic volatility models A. Ahdida and A. Alfonsi have worked on simulation methods for Wishart matrices. Wishart matrices process are increasingly used in finance, especially to model a multiname volatility. They have obtained an exact simulation procedure for Wishart processes, and also a general second-order scheme for general Affine diffusions on symmetric positive matrices.

B. Jourdain and M. Sbai [74] are interested in the simulation of stochastic volatility models, in which the process driving the volatility of the asset price evolves according to an autonomous one-dimensional stochastic differential equation with smooth coefficient. Using Itô's formula, they get rid, in the asset price dynamics, of the stochastic integral with respect to the Brownian motion driving this SDE. Taking advantage of this structure, they propose
-a scheme, based on the Milstein discretization of this SDE, with order one of weak trajectorial convergence for the asset price,
- a scheme, based on the Ninomiya-Victoir discretization of this SDE, with order two of weak convergence for the asset price.

2.6   Statistical finance

This research team, structured around the FIESTA seminar and study group, aims to explore the interactions between the financial mathematics and the statistics in the domains like market microstructure, modeling of order books, volatility estimation, hedging, etc. The works of M. Rosenbaum [20, 22, 51, 57, 58, 59] and E. Bacry [2, 29, 53] are part of this activity.

Microstructure modeling and estimation under microstructure noise Several statistical issues in the context of high frequency data have been investigated by the team. In [20], the impact of price discretness on volatility estimates is studied. In [59], Robert and Rosenbaum have developed a microstructure model for transaction prices and durations. This model called "model with uncertainty zones" accommodates the assumption of a continuous efficient price with the inherent properties of ultra high frequency transaction data (price discreteness, irregular temporal spacing, diurnal patterns...). In this model where microstructure noise and trading times are endogenous, the same authors have studied the problem of getting accurate

volatility and correlation estimates from high frequency data [58]. Indeed, these quantities are key elements for the risk manager in order to optimize intraday strategies. Finally, in [57], the question of the interest of high frequency hedging strategies is investigated through the model with uncertainty zones.

Estimation of relevant quantities In [51], Hoffmann, Rosenbaum and Yoshida study the problem of lead lag estimation from non synchronous data. Indeed, market participants usually agree that certain pairs of assets share a leadlag effect", in the sense that the lagger price process tends to partially reproduce the oscillations of the leader price process, with some temporal delay. Moreover, the lead-lag effect may have a lot of importance in practice when assessing the quality of risk management indicators for instance.

Statistical procedures for multifractal processes are investigated in a series of papers by Bacry, Muzy and other collaborators [2, 29, 53]. It is well known that multifractal processes reproduce well the main stylized facts of empirical data. However, classical estimation strategies are usually biased in the multifractal setting. In particular, estimation strategies in the MRW process introduced by Bacry and Muzy are considered in [2, 53].

2.7   Seminars and study groups

The research activities of the chair are structured in study groups ("groupes de travail") on the themes of the chair. This basic organization is the same as last year, but the themes of the groups undergo some evolution every year. During the second semester of 2008–2009, the two weekly study groups at Ecole Polytechnique were dedicated to simulation of rare events (organized by E. Gobet) and to portfolio insurance (organized by P. Tankov). The study group at ENPC is oriented towards numerical methods for finance and takes place every Friday afternoon.

Study group on the rare event simulation With the recent financial crisis, the interest towards the simulation of rare events has increased considerably. For the probabilities of order of 10^{-4} and less, specific methods are required to obtain reliable estimates. During the last years we have studied these advanced methods. We started with the works of P. Glasserman on the application of importance sampling to the computation of the Value at Risk. This allows to obtain first-order estimates for Gaussian-type risks. To go further, we then studied interacting particle metods developed among others by P. Del Moral et al. These methods usually consist in simulating interactig particles (scenarios), which multiply in case of favorable configuration or disappear otherwise. These algorithms are known as selection-mutation algorithms. A variant of this methodology was applied by Carmona et al. in the field of credit risk. We contunue researches in this field to develop more efficient algorithms for rare event simulation.

Owing to the presence of many prestigious visitors at CMAP, several sessions of the study groups were replaced by the talks of the visitors, in particular T. Hurd and J. Zhang gave a series of talks on their recent research. During the first semester of 2009–2010 academic year, the study group on the simulation of rare events has evolved towards other numerical methods in finance, in particular the asymptotic approximation methods, in preparation to the 3rd European Summer School in financial mathematics, which will include a course on this subject. The research initiated by the study group in portfolio insurance is now being pursued in the PhD thesis of Carmine de Franco. A new study group on second-order backward stochastic

differential equations and applications to finance, organized by A. Matoussi, has been created starting from September 2009.

A joint seminar with Société Générale with the aim to promote active research cooperation between the academic and professional participants of the Chair takes place every trimester. During this seminar, a talk by a practitioner from Société Générale is followed by a practitioner-oriented talk by one of the academinc members of the Chair and a meeting of the steering committee of the Chair. As most of the meetings of the Chair involving several institutions, it takes place at Institut Louis Bachelier. In 2009, the joint seminars took place on February 2 (with talks by O. Cohen and N. Touzi), September 21 (with talks by Julien Guyon and Benjamin Jourdain) and December 7 (with talks by Jérôme Brun and Peter Tankov).

Starting from September 2008 a new seminar on Finance and Statistics (FIESTA) was created by the team of high frequency data statistics, which was reinforced by the hiring of M. Rosenbaum. This seminar, which is organized by E. Bacry, M. Hoffman (ENSAE) and M. Rosenbaum, takes place about once a month. It is structured around a talk by an invited speaker followed by a detailled discussion of practical or numerical aspects of the specific problem of financial statistics. Since the beginning of the seminar, it was principally focused on the problems of order book modeling. The 2009 speakers were Aurelien Alfonsi (ENPC), Frédéric Abergel (ECP), Rama Cont (Paris VI and Columbia University), Robert Almgren (Quantitative Brokers and New York University), Roel Oomen (Deutsche Bank and University of Amsterdam), Sasha Stoikov (Columbia University) and Jim Gatheral (Bank of America Merrill Lynch and the Courant Institute, NYU).

The research seminar at ENPC is organized by A. Alfonsi and D. Goreac (UPEMLV) jointly with with the University Paris-Est Marne-la-vallée and the Mathfi project (INRIA). It takes place every week and mainly brings on numerical methods in finance (Monte Carlo methods and variance reduction techniques, simulation methods and discretization schemes for SDE and BSDEs, numerical Fourier inversion,...), and the related topics (financial modelling, Malliavin calculus,...).

The chair contributes to the organization of three regular seminars: sèminar of the team "Stochastic models in finance" of CMAP, Bachelier seminar at Institut Henri Poincarè and Financial Breakfast seminar at Institut Louis Bachelier.

3   Publications and Conferences

This section presents a global list of publications, preprints and conference talks by the members of our team. We have marked with the star (*) the items which are more closely related to the activities of the chair "Financial risks".

Published or accepted papers

Polytechnique

[1] R. Aid, L. Campi, A. Nguyen Huu et N. Touzi (2009). A Structural Risk Neutral Model of Electricity Prices, *International Journal of Theoretical and Applied Finance.*

[2] E.Bacry et J.F.Muzy (2009). Multifractal models for asset prices, To appear in *Encyclopedia of Finance 2009*

[3] E. Benhamou, E. Gobet, and M. Miri. Expansion formulas for European options in a local volatility model. *Forthcoming in International Journal of Theoretical and Applied Finance. Available at http://hal.archives-ouvertes.fr/hal-00325939/fr/*, 2008.

[4] E. Benhamou, E. Gobet, and M. Miri. Smart expansion and fast calibration for jump diffusion. *Finance and Stochastics*, 13(4):563–589, 2009.

[5] J. Bion-Nadal (2009)*. Time consistent dynamic risk processes, *Stochastic processes and their applications, vol 119, issue 2 p:633-654.*

[6] J. Bion-Nadal (2009)*. Bid-Ask dynamic pricing in financial markets with transaction costs and liquidity risk, To appear in *Journal of mathematical economics.*

[7] B. Bouchard, R. Elie et N. Touzi (2009). Discrete-Time Approximation of BSDEs and Probabilistic Schemes for Fully Nonlinear PDEs, *Radon Series on Computational and Applied Mathematics.*

[8] B. Bouchard, R. Elie et N. Touzi (2009). Stochastic Target Problems with Controlled Loss, *SIAM Journal on Control and Optimization.*

[9] A. Cretarola, F. Gozzi, H. Pham, and P. Tankov (2009). Optimal consumption policies in illiquid markets. To appear in *Finance and Stochastics.*

[10] R. Cont and P. Tankov (2009)*. Constant proportion portfolio insurance in the presence of jumps in asset prices, *Mathematical Finance, 19, pp. 379–401.*

[11] N. El Karoui and Y. Jiao (2009). Stein's method and zero bias transformation for CDO tranche pricing, *Finance and Stochastics (2009) 13:151-180*

[12] A. Galichon (2009)*. The VaR at Risk, *International Journal on Theoretic and Applied Finance, (special issue on the occasion of the 2008 X-Columbia conference in Paris)*

[13] E. Gobet and S. Menozzi. Stopped diffusion processes: Boundary corrections and overshoot. *Stochastic Processes and Their Applications*, 120:130–162, 2010.

[14] E. Gobet. *Handbook of Numerical Analysis, Vol. XV, Special Volume: Mathematical Modeling and Numerical Methods in Finance*, chapter Advanced Monte Carlo methods for barrier and related exotic options, pages 497–528. Elsevier, Netherlands: North-Holland, 2009.

[15] P. Imkeller, G. Dos Reis (2009). Path regularity and explicit convergence rate for BSDE with truncated quadratic growth, To appear in *Stochastic processes and their applications*

[16] P. Imkeller, G. Dos Reis and J. Zhang (2009)*. Results on numerics for FBSDE with drivers of quadratic growth, To appear in *"Special Springer volume in honour of Eckhard Platen's 60th birthday"*

[17] Anis Matoussi and Lucretiu Stoica (2009), The obstacle Problem for Quasilinear Stochastic PDE's, to appear in *Annals of Probabilty.*

[18] H. Pham and P. Tankov (2009)*. A coupled system of integrodifferential equations arising in liquidity risk model, *Applied Mathematics and Optimization, 59, pp. 147–173.*

[19] A.Porchet , N. Touzi and X. Warin (2009). Valuation of a powerplant under production constraints and markets incompleteness, *Mathematical Methods of Operations research, Volume 70, Issue 1, Page 47-75.*

[20] M. Rosenbaum (2009)*. Integrated volatility and round off error, *Bernoulli 15 (03) p 687-720.*

[21] M. Rosenbaum (2009). First order p-variation and Besov spaces, *Statistics and Probability Letters 79 (01) p 55-62*

[22] M. Rosenbaum (2009)*. A new microstructure noise index, to appear in *Quantitative Finance.*

[23] M. Rosenbaum, A. Tsykabov (2009). Sparse Recovery under Matrix Uncertainty, to appear in *Annals of Statistics*.

[24] M. Soner et N. Touzi (2009). The Dynamic Programming Equation for Second Order Stochastic Target Problems, *SIAM J. Control Optim. Volume 48, Issue 4, pp. 2344-2365.*

[25] P. Tankov (2009)*. Pricing and hedging gap risk, to appear in *Journal of Computational Finance.*

[26] P. Tankov and E. Voltchkova (2009)*. Asymptotic analysis of hedging errors in models with jumps, *Stochastic processes and their applications, 119, pp. 2004–2027.*

[27] P. Tankov and E. Voltchkova (2009)*. Jump-diffusion models: a practitioner's guide, *Banque et Marchés.*

Preprints

[28] R. Aid, O. Féron, N. Touzi et C. Vialas (2009). An arbitrage-free interest rate model consistent with economic constraints for long-term asset liability management. Submitted to *Bankers, Markets and Investors.*

[29] E.Bacry, A.Gloter, M.Hoffmann et J.F.Muzy (2009). Estimation of the multifractal spectrum in a mixed asymptotic framework, submitted to *Annals of Applied Probability*

[30] P. Barrieu, H. Bensusan, N. El Karoui, C. Hillairet, S. Loisel, C. Ravanelli, Y. Salhi (2009)*. Understanding, Modeling and Managing Longevity Risk : Key issues and Main Challenges, submitted to *Scandinavian Actuarial Journal*

[31] A.Benabid, H.Bensusan et N.El Karoui (2009)*. Wishart Stochastic Volatility: Asymptotic Smile and Numerical Framework. In revision for *Quantitative Finance.*

[32] A.Benabid, H.Bensusan et N.El Karoui (2009)*. Short term Smile In general Whishart Stochatic Volatility Model.

[33] E. Benhamou, E. Gobet, and M. Miri. Analytical formulas for local volatility model with stochastic rates. Rapport de recherche, LJK, France, October 2009. submitted.

[34] E. Benhamou, E. Gobet, and M. Miri. Time dependent Heston model. *In revision for SIAM Journal on Financial Mathematics, available at http://hal.archives-ouvertes.fr/hal-00370717/fr/,* 2009.

[35] B. Bouchard et N. Touzi (2009). Weak dynamic programming principle for viscosity solutions, submitted to *SIAM Journal on Control and Optimization.*

[36] M. Brodén, P. Tankov (2009)*. Errors from discrete hedging in exponential Lévy models: the L^2 approach, submitted to *Finance and Stochastics.*

[37] G. Carlier, A. Galichon, F.Santambrogio (2009). From Knothe's transport to Brenier's map and a continuation method for optimal transport, in revision for *SIAM Journal on Mathematical Analysis.*

[38] L.Carasso, N.El Karoui et Jan Obloj (2009). On Azèma-Yor processes, their optimal properties and the Bachelier Drawdown equation, submitted to *Annals of Probability.*

[39] G. Chemla, R. Aid, A. Porchet et N. Touzi (2009). Vertical integration and risk management in competitive markets of non-storable goods. In revision for *Management Science.*

[40] V. Chernozhukov, I. Fernandez-Val, A. Galichon (2009). Quantile and Probability Curves without Crossing, in revision for *Econometrica.*

[41] D. Crisan, K. Manolarakis et N. Touzi (2009). On the Monte Carlo simulation of BSDEs: an improvement on the Malliavin weights. In revision for *Stochastic Processes and their Applications.*

[42] I.Ekeland, A. Galichon, M. Henry (2009)*. Comonotonic measures of multivariate risks, in revision for *Mathematical Finance.*

[43] N. El Karoui, M. Jeanblanc and Y. Jiao (2009). The conditional density approach, in revision for *Stochastic Processes and their Applications.*

[44] N. El Karoui, M. Jeanblanc and Y. Jiao (2009). Modelling of succesive Default events, a density buttom-up approach.

[45] A. Fahim, N. Touzi, et X. Warin (2009). A Probabilistic Numerical Method for Fully Nonlinear Parabolic PDEs, in revision for *Annals of Applied Probability.*

[46] A. Galichon, M. Henry (2009). Inference in models with multiple equilibria , overhaul of the 2006 working paper "Inference in Incomplete Models," in revision for *Review of Economic Studies.*

[47] A. Galichon, M. Henry (2009). Dilation Bootstrap: A methodology for constructing confidence regions with partially identified models, in revision for *Journal of Econometrics.*

[48] E. Gobet and C. Labart. Solving BSDE with adaptive controle variate. *In revision at SIAM Numerical Analysis*, 2009.

[49] E. Gobet and A. Makhlouf. L2-time regularity of BSDEs with irregular terminal functions. *In revision for Stochastic Processes and their Applications, available at http://hal.archives-ouvertes.fr/hal-00291768/fr/*, 2009.

[50] E. Gobet and A. Makhlouf. The tracking error rate of the Delta-Gamma hedging strategy. *Preprint, available at http://hal.archives-ouvertes.fr/hal-00401182/fr/*, 2009.

[51] M. Hoffmann, M. Rosenbaum, N. Yoshida (2009)*. Estimation of the lead-lag parameter between two financial assets from non-synchronous data, submitted to *Finance and Stochastics.*

[52] A. Kohatsu-Higa and P. Tankov (2009)*. Jump-adapted discretization schemes for Lévy-driven SDEs, in revision for *Stochastic Processes and their Applications.*

[53] A.Kozhemyak, E.Bacry, J.F.Muzy (2009). Log-Normal continuous cascades: aggregation properties and estimation. Application to financial time-series, submitted to *Quantitative Finance.*

[54] Anis Matoussi and Hao Wang (2009), Probabilistic interpretation for Sobolev solution of semilinear parabolic partial integro-differential equations, with Hao Wang.

[55] Anis Matoussi, Wahid Faidi and Mohamed Mnif (2009), Maximization of Recursive Utilities: A Dynamic Maximum Principle Approach.

[56] C.Y. Robert, M. Rosenbaum (2009). On the limiting spectral distribution of the covariance matrices of time-lagged processes, submitted to *Journal of Multivariate Analysis.*

[57] C.Y. Robert. M. Rosenbaum (2009)*. On the microstructural hedging error, submitted to *SIAM SIFIN.*

[58] C.Y. Robert, M. Rosenbaum (2009)*. Volatility and covariation estimation when microstructure noise and trading times are endogenous, submitted to *Mathematical Finance.*

[59] C.Y. Robert, M. Rosenbaum (2009)*. A new approach for the dynamics of ultra high frequency data: the model with uncertainty zones, submitted to *Journal of Financial Econometrics.*

[60] P. Tankov (2009)*. Pricing and hedging in exponential Lévy models: review of recent results submitted to *Paris-Princeton lecture notes on Mathematical Finance.*

Ecole des Ponts
Published or accepted papers

[61] A. Alfonsi and B. Jourdain, (2009)*. Exact volatility calibration based on a Dupire-type Call-Put duality for perpetual American options,*Nonlinear Differential Equations and Applications, Vol. 16, No. 4, pp. 523-554.*

[62] A. Alfonsi, A. Fruth, A. Schied, (2010)*. Optimal execution strategies in limit order books with general shape functions. *Quantitative Finance DOI:10.1080/14697680802595700.*

[63] A. Alfonsi, (2009)*. High-order discretization scheme for the CIR process: application to the Heston model. *Mathematics of Computation, Vol. 79, No. 269, pp. 209-237.*

[64] A. Alfonsi, (2010)*. Cox-Ingersoll-Ross (CIR) model, *Encyclopedia in Finance, to appear in 2010.*

[65] B. Jourdain, J. Lelong, (2009)*. Robust Adaptive Importance Sampling for Normal Random Vectors, *Ann. Appl. Probab. 19(5), pp 1687-1718.*

[66] B. Jourdain, (2009)*. Adaptive variance reduction techniques in finance, *Advanced Financial Modelling, Radon Series Comp. Appl. Math 8, Ed. by H. Albrecher, W. Runggaldier and W. Schachermayer, de Gruyter, pp. 205-222.*

[67] P. Etorè, B. Jourdain*. Adaptive optimal allocation in stratified sampling methods, *to appear in Methodology and Computing in Applied Probability.*

[68] P. Etorè, G. Fort, B. Jourdain, E. Moulines*. On Adaptive Stratification, *to appear in Annals of operations research.*

Preprints

[69] A. Alfonsi, A. Schied, (2009)*. Optimal execution and absence of price manipulations in limit order book models, June 2009, *submitted.*

[70] A. Alfonsi, A. Schied, A. Slynko, (2009)*. Order Book Resilience, Price Manipulation, and the Positive Portfolio Problem, October 2009, *submitted.*

[71] A. Alfonsi, J. Lelong, (2009)*. A closed-form extension to the Black-Cox model, September 2009.

[72] A. Alfonsi, (2009)*. An introduction to the multiname modelling in credit risk (Survey that will be part of a book edited by T. Bielecki, D. Brigo and F. Patras).

[73] B. Jourdain, M. Sbai, (2008)*. Coupling Index and Stocks, December 2008, *submitted.*

[74] B. Jourdain, M. Sbai, (2009)*. High order discretization schemes for stochastic volatility models, August 2009

[75] B. Jourdain, M. Vellekoop, (2009)*. Regularity of the Exercise Boundary for American Put Options on Assets with Discrete Dividends, November 2009, *submitted*

Scientific conferences

- *Stochastic Analysis from and to finance*, Kyoto (Japan), August 3–7, 2009, N.Touzi.

- *Istanbul Workshop in Mathematical Finance*, Istanbul (Turkey), May 18–21, 2009, N.El Karoui, N.Touzi.

- *Workshop on Stochastic Analysis and Finance*, City University, Hong Kong, June 29–July 3, 2009, N.Touzi.

- *Signal and Finance, Workshop*, September 29, 2009, Télécom ParisTech, E. Bacry.

- *Euro Conference 2009*, Bonn (Allemagne) July 5–9, 2009, J. Bion-Nadal.

- *Fudan and Jinan Seminars in China*, March 2009, N.El Karoui

- *Colloquium Madrid 2009*, N.El Karoui

- *Sociètè Mathèmatique Marocaine, Rabat Avril 2009*, N.El Karoui

- *Natixis-Paris VII Conference*, Paris, September 2009, N.El Karoui

- *Frontiers in Financial Econometrics*, Princeton University, 25/09/2009, M. Rosenbaum.

- *33rd Conference on Stochastic Processes and Their Applications,*, TU Berlin, 27-31 July 2009, Gonzalo Dos Reis, M. Rosenbaum, P. Tankov.

- *Congrès des actuaires*, Paris, 29/06/2009, M. Rosenbaum.

- *QASS conference*, Queen Mary University London, 17/06/2009, M. Rosenbaum.

- *Journée "dépendance"*, ENGREF Paris, 05/06/2009, M. Rosenbaum.

- *41èmes Journées de Statistique de la SFDS*, Bordeaux, 15/05/2009, M. Rosenbaum.

- *Financial Econometrics Conference*, Toulouse School of Economics, 15/05/2009, M. Rosenbaum.

- *Statistique Asymptotique des Processus Stochastiques VII* , Université du Mans, 17/03/2009, M. Rosenbaum.

- *2do Simposio en Actuaria*, Bogota, septembre 2009, S. Loisel.

- *Workshop in honour of Hans Gerber*, Lausanne, juin 2009, S. Loisel.

- *Conférence IME (Insurance: Mathematics and Economics)*, Istanbul, mai 2009, S. Loisel.

- *ERM Symposium*, Chicago, avril 2009, S. Loisel.

- *Fourth General Conference on Advances Mathematical Methods in Finance*, Aalesund (Norway), 4-10 May, 2009, G. Dos Reis.

- *Third Conference on Numerical Methods in Finance*, Paris (France), 15-17 April, 2009, G. Dos Reis.

- *4ème Biennale Française des Mathématiques Appliquées et Industrielles, SMAI 2009*, La Colle sur Loup, France, mai 2009 E. Gobet.

- *Workshop on Mathematical Finance and Related Topics in Economics and Engineering*, Kyoto, Japan, August 2009, P. Tankov.

- *Croatian Quants Day*, Zagreb, Croatia, April 2009, P. Tankov.

- *International Conference on Stochastic Analysis*, Hammamet, 12-17 octobre 2009. N. El Karoui, C.Hillairet, A. Matoussi, N. Touzi.

- *Summer School on Financial Mathematics*, HEC, Jouy-en-Josas, France, 24-29 aout 2009. Mini-course on Backward Stochastic Differential Equations, E. Gobet.

- *Spring School in Finance*, Bologna, Italy, May 2009, minicourse on Lèvy processes and applications in finance, P. Tankov.

- *Recent Advancements in the Theory and Practice of Credit Derivatives*, Nice, 28-30 September 2009, A. Alfonsi.

- *workshop computational Finance*, RIMS Kyoto university, 10-12 August, B. Jourdain.

- *workshop on Optimal stopping and singular stochastic control problems in finance*, National University of Singapore, 9-15 december, B. Jourdain.

Conferences and articles for professionals/general public

- Tribune "Le capitalisme est mort, vive le capitalisme!" A. Galichon, P. Tibi, Les Temps Modernes, oct 2009.

- La tribune Novembre 2009: Les femmes face à la crise (N.El Karoui)

- Un moment dans l'expèrience probabiliste. Cahiers du centre Cournot pour l'Economie (Novembre 2009)

- *Confèrence au Boston Consulting Group* Janvier 2009, N.El Karoui

- *Confèrence Sciences à coeur, Univ Paris VI* Mars 2009, N.El Karoui

- *Confèrence Calyon* Mai 2009, N.El Karoui

- *Sèminaire Encre, CNRS, La Rochelle* Octobre 2009, N.El Karoui.

- *Confèrence ENST, Enseignement des mathèmatiques en grande Ecole* Novembre 2009, N.El Karoui.

- *Sèminaire Commission bancaire, Paris* Novembre 2009, N.El Karoui.

- *Confèrence Maths à venir, Paris* Dècembre 2009, N.El Karoui.

- *Confèrence Centre des Cordeliers, Paris* Dècembre 2009, N.El Karoui.

- *Confèrence Cercle des Economistes, Paris* Dècembre 2009, N.El Karoui.

4 Organization of conferences and seminars

4.1 Conferences in 2009

Third Conference on Numerical Methods in Finance, ENPC, 15-17 April 2009
This conference was organized by A. Alfonsi, J.-F. Delmas, B. Jourdain and B. Lapeyre with the help of the scientific committee headed by D. Talay and composed of H. Berestycki, J. Cvitanic, P. Garampon, G. Pagès and C. Rogers. Over 140 participants were welcomed at École des Ponts during the 3 days of the conference. The plenary talks were given by

- Yacine Aït Sahalia, Princeton University : The Fine Characteristics of Jumps and Volatility in High Frequency Financial Data,

- Damiano Brigo, Fitch Solutions : Credit Derivatives pre- and in-crisis: Dynamical models implying armageddon scenarios and extreme losses,

- Rama Cont, CNRS & Columbia University : Too interconnected to fail: contagion and systemic risk in financial networks,

- Min Dai, National University of Singapore : Numerical Methods for Portfolio Selection with Transaction Costs,

- Pierre Henry-Labordère, Sociètè Gènèrale : Numerical Methods for Non-Linear Problems in Quantitative Finance,

- Arturo Kohatsu-Higa, University of Osaka : Approximations for SDE's driven by Lèvy processes.

Moreover, 46 talks were given in parallel sessions dedicated to Monte Carlo methods, variance reduction techniques, discretization of partial differential equations, stochastic differential equations and backward stochastic differential equations, calibration of models, optimal stopping and American options, tree methods, quantization,... The conference dinner organized at the Musèe d'Orsay after a visit of the collections was appreciated by the participants.

Second SMAI European Summer School in Financial Mathematics, Paris, August 24–29, 2009 The second edition of the European summer school in financial mathematics took place at the CRC Castle on HEC Campus (Jouy-en-Josas). All the information about the summer school is available at the web site

<div align="center">

`http://www.cmap.polytechnique.fr/~euroschoolmathfi09/`
</div>

This second edition of the summer school attracted 80 participants among which 60 PhD students.

The summer school was structured around two minicourses followed by short presentations given by PhD participants. One afternoon was dedicated to interaction with financial institutions, with presentations by J. Lebuchoux (Reech AiM) and M. Simmonds (Nomura). The minicourses were

- Financial modeling under illiquidity by P. Bank (Berlin) and A. Schied (Mannheim).

- Backward stochastic differential equations with financial applications by E. Gobet (Grenoble) and J. Ma (Los Angeles).

The contributed talks by PhD participants were focused on a variety of topics including pricing, approximations of stochastic processes, mathematical theory of finance, fixed income models, credit risk, illiquidity models, optimal investment, statistical methods. The Third edition of the summer school will take place in August 2010 at the same location.

Finally, Alfred Galichon, member of the chair, was among the organizers of two conferences last year:

- Summer school "Stats in the Chateau", HEC (Paris), September 2009 (with P. Alquier, V. Czellar, E. Gautier, G. Stoltz). The main topics of this summer school were large dimensional statistics and inverse problems with applications to structural econom itrics and finance:

  `https://studies2.hec.fr/jahia/Jahia/statsinthechateau`

- Workshop "Optimization, Transportation and Equilibrium in Economics", Ecole des Mines de Paris, July 6-8th 2009 (with G. Carlier, F. Santambrogio, T. Tomala). This workshop was dedicated to optimal transportation and various economic applications (including finance with the talks on Mean-field games by J.-M. Lasry, P.-L. Lions and O. Guéant)

  http://sites.google.com/site/alfredgalichon/workshopotae2009

4.2 Future conferences

"Modeling and measuring financial risks", Paris, January 2011 To draw some conclusions from the first 4 years of the chair "Financial risks" we are planning to organize a conference "Modeling and managing financial risks" in January 2011. This conference will last 3 days and cover a range of topics including liquidity, market microstructure, risk measures, numerical methods, scenario simulation, calibration and model risk, counterparty risk, credit risk, hedging and regulatory aspects. We aim to attract academic and professionals speakers for both contributed and invited talks, and a total of about 150–200 participants. This will also be an occasion to celebrate the 20 years of the master program "Probability and Finance" of Paris VI and Ecole Polytechnique, and a special afternoon session for master students in finance will be organized.

4.3   Contribution to seminars

The Chair contributes to the funding (especially, the speaker invitations) of two important financial mathematics seminars of the Paris region: the Financial Breakfast seminar (`www.frontiers-in-finance.co`) and the Bachelier seminar (`www.bachelier-paris.com`). The Financial Breakfast seminar is organized by R. Cont, Y. Braouezec and P. Tankov in Institut Louis Bachelier with the aim to promote interaction between university researchers and finance practitioners. The Bachelier mathematical finance seminar is co-organized in Institut Henry Poincarè by Ecole Polytechnique, Paris Dauphine University, ENSAE, Universities of Paris VI and VII, Evry University and, Marne La Vallèe University.

5   Master-level courses

Risk measures in finance   This second-year master course, common for the master program "Probability and Finance" of Paris VI/Polytechnique and "Mathematics and Applications, finance track" of Marne-la-Vallèe University comprises a theoretical part by J.-F. Delmas (ENPC) and P. Tankov (Polytechnique) and a professional part by Lorenzo Bergomi, Pierre Garampon, Jèrôme Brun and Olivier Cohen (Socièté Gènèrale).

The goal of this course is to present in a theoretical part the risk measurement tools for trading and portfolio management on a short time scale (1 to 10 days). The main topics of the theoretical part (6 3-hours courses plus exam) are: monetary measures of risk, extreme value theory, and multidimensional representation of risks via copulas. In the second professionally-oriented part, the practitioners from Socièté Gènèrale present the methods used by different departments of their bank to evaluate financial risks.

The web page of the course is available at the following address:

$$\text{http://cermics.enpc.fr/~delmas/Enseig/mrf.html}$$

"Financial mathematics, risk management and Monte Carlo methods" in the joint master program ParisTech–Fudan University   Since almost 4 years now, a joint master program in mathematical finance between ParisTech (mostly Ecole Polytechnique) and Fudan University in Shanghai allows to promote the French school of financial mathematics in China and to attract the brightest students from Fudan University to the Paris VI master program. To boost this joint program, in 2009 we organized a course on financial mathematics, risk management and Monte Carlo methods in Fudan University taught by two professors of Ecole Polytechnique (M. Rosenbaum and P. Tankov), with the travel costs covered partially by the Chair (the local expenses and the professors' salaries are covered by Fudan University). A total of 30 students followed the course, 20 of which signed up for the final exam, and the feedback from the students and the Chinese colleagues was very positive.