Amplification of broadband incoherent light in homogeneously broadened media in the presence of Kerr nonlinearity

J. Garnier and J. P. Fouque

Centre de Mathématiques Appliquées, Centre National de la Recherche Scientifique, Unité de Recherche Associée 756, Ecole Polytechnique, 91128 Palaiseau Cedex, and Commissariat à l'Energie Atomique, Centre d'Etudes de Limeil-Valenton, 94195 Villeneuve St. Georges Cedex, France

L. Videau and C. Gouédard

Commissariat à l'Energie Atomique, Centre d'Etudes de Limeil-Valenton, 94195 Villeneuve St. Georges Cedex, France

A. Migus

Laboratoire pour l'Utilisation des Lasers Intenses, Centre National de la Recherche Scientifique, Unité Mixte de Recherches 100, Ecole Polytechnique, 91128 Palaiseau Cedex, France

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We have developed a statistical nonlinear model to explain an anomalous intensity saturation observed in the amplification of intense broadband incoherent pulses on neodymium-doped glass power chains. The physics behind this model is basically self-phase modulation creating new wavelengths scattered in the tail of the gain profile. The theory shows qualitative agreement with the experimental results. © 1997 Optical Society of America [S0740-3224(97)02310-2]

1. INTRODUCTION

There are strong analogies between the nonlinear propagation of ultrashort coherent pulses and temporally incoherent light, implying that similar effects should be observed in both cases. Incoherent light with ultrashort coherence times is of interest for linear and nonlinear coherent spectroscopy[1,2] and for many other applications such as tomography in random media[3] and smoothing techniques for uniform irradiation in plasma physics.[4] Furthermore, pulses obtained by optical shaping methods for coded telecommunication purposes[5] are indeed deterministic but nevertheless may have similarities to incoherent fields. The present study was triggered by the initial observation of an anomalous intensity saturation effect in the amplification of intense incoherent pulses in a large Nd:glass power chain.[6] It is well known that on one hand gain narrowing effects limit the duration of ultrashort pulses in amplification devices and on the other hand nonlinear processes that appearing in the propagation, such as self-phase modulation (SPM), produce spectral broadening. Such delicate balances among dispersion, narrowing, and broadening effects help in the solitonlike generation of ultrashort pulses in cavities. This is so, for instance, in the case of a Nd-doped glass actively mode-locked glass oscillator, for which 100-fs pulses, an order of magnitude shorter than expected,[7] have been produced. However, the interplay between effects such as gain narrowing and SPM are difficult to observe because of other cumulative effects in a cavity. Such mechanisms are easier to study in the amplification of incoherent pulses. In the research reported in Ref. 6 the pulses at hand were time–space modulated, so it was difficult to distinguish among the phenomena that might be responsible for the observed saturation. We present data that correspond to a recent separate series of experiments in which pulses have time-incoherent modulations but no spatial fluctuations, so spatial effects such as self-focusing do not accumulate with the effects of temporal incoherence. Therefore we are able to analyze precisely the time mechanisms that give rise to intensity saturation. In what follows, we develop a closed-form theory of the combined effects of self-phase modulation and gain narrowing for incoherent pulses and discuss their applicability to the experimental results obtained on the Phebus high power laser chain with broadband pulses.

2. NONLINEAR STATISTICAL MODEL

We consider plane waves and assume that the initial field propagating along the z axis is $E_0(t)\exp - i(\omega_0 t - k_0 z)$, where ω_0 and k_0 are the carrier frequency and the corresponding wave vector, respectively. The envelope $E_0(t) = a_0(t)\exp i\phi_0(t)$ is assumed to follow stationary Gaussian statistics with a Gaussian correlation function of correlation time T_c:

$$\langle E_0(t) E_0{}^*(t + T)\rangle = I_0 \exp(-T^2/2T_c^2), \tag{1}$$

where $\langle \ \rangle$ stands for statistical averaging with respect to the distribution of E_0. In particular, (1) the amplitude $a_0(t)$ and the phase $\phi_0(t)$ are statistically independent, (2) the distribution of intensity $I(t) = |a_0(t)|^2$ admits of a density $p(I) = 1/I_0 \exp(-I/I_0)$, where I_0 the initial averaged intensity, and (3) $\phi_0(t)$ obeys a uniform distribution over $[0, 2\pi]$. The pulse duration τ_p is assumed to be many orders of magnitude larger than the coherence time, so the stationary condition is a reasonable assumption. As a consequence, the field E_0 is time ergodic, which implies that the time average coincides with the statistical average.

This incoherent pulse propagates along an amplifying medium that we assume to correspond to a homogeneously broadened two-level system embedded in a host medium. This host is characterized by a Kerr constant n_2, whereas the two-level system is characterized by a dephasing time T_2 and an excited-state lifetime that we neglect because it is much larger than all the characteristic time scales involved in the problem.

In what follows, we stay in the slowly varying envelope approximation for the field and the polarization, which is equivalent to $T_2 \ll T_c$. Assuming that the pulse is tuned to maximum amplification and neglecting the spatial transverse profile evolution, the coupled equations verified in the moving pulse time frame by the field E and the normalized polarization P are[8]

$$i \frac{\partial E}{\partial z} + \sigma \frac{\partial^2 E}{\partial t^2} + \frac{k_0 n_2}{2 n_0} |E|^2 E = \frac{1}{2} P, \tag{2}$$

$$T_2 \frac{\partial P}{\partial t} + P = i \gamma E, \tag{3}$$

where n_0 is the unperturbed value of the index of refraction and $n_2 > 0$ is the Kerr constant that characterizes the nonlinear correction of the index. The group-velocity dispersion (GVD) coefficient σ is related to the dispersion factor $\partial^2 \omega / \partial k^2|_{k_0}$ through the formula

$$\sigma = \frac{\omega''(k_0)}{2 \omega'(k_0)^3}, \tag{4}$$

where $\omega'(k_0)$ is the group velocity. Following the dispersive properties of the medium, the coefficient σ can be either positive in the case of a so-called anomalous-dispersion medium or negative in the opposite case of a normally dispersive medium. In this paper we consider mainly the latter case, because the value of σ for a Nd-doped glass laser at 1.05 μm is $\sigma \simeq -3.5 \times 10^{-4}$ $\mathrm{ps^2\,cm^{-1}}$. There is a further equation that expresses the evolution of γ,[8] the inverted population expressed in gain per length unit, but in this simplified analysis we focus on the case for which depletion of the stored energy can be neglected, so γ is kept constant.

3. INTERPLAY BETWEEN SELF-PHASE MODULATION AND GAIN

Let us denote by $\Delta\omega$ the bandwidth of the incident pulse, which is conversely proportional to the correlation time: $\Delta\omega = T_c^{-1}$. We assume in this section that, in the amplification length γ^{-1}, the time-delay dispersion of the different frequency components $\sigma \Delta\omega \gamma^{-1}$ is much smaller than the coherence time, which also reads as

$$\frac{\sigma}{\gamma T_c^2} \ll 1. \tag{5}$$

Time dispersion is then negligible, so each t cross section of the pulse evolves independently. As a consequence, we can neglect the time-dispersive effects and take $\sigma = 0$. The influence of the group-velocity dispersion is discussed in Section 5.

We first consider simultaneous SPM and gain but assume that the gain spectrum is flat ($T_2 = 0$), in which case $P = i\gamma E$. The pulse is then amplified with the intensity small-signal gain $g = \gamma z$ and acquires a nonlinear phase term, $\phi_{\mathrm{NL}}(t) = k_0 n_2 a_0^2(t)(e^g - 1)/(2 n_0 \gamma)$ that gives rise to spectral broadening. We can actually find a closed-form expression for the correlation function $C(T, z) = \langle E(t, z) E^*(t + T, z) \rangle$:

$$C(T, B)|_{T_2=0} = \frac{e^g I_0 \exp(-T^2/2T_c^2)}{\{1 + B^2[1 - \exp(-T^2/T_c^2)]\}^2}, \tag{6}$$

where $B = k_0 n_2 I_0 (e^g - 1)/(2 n_0 \gamma)$ is identified as the so-called B integral. In the limit $\gamma \to 0$ we then recover the expression of Manassah,[10] who considered the propagation of an incoherent field in a dispersionless Kerr medium without gain. In Figure 1 we plot the theoretical correlation function C given by Eq. (6) as a function of the B integral and the reduced time T, which puts into evidence a reduction of the correlation time of the pulse when the value of the B integral becomes of the order of 1. As a consequence we must take into account the finite dephasing time of the polarization even if the spectrum of the incident pulse is narrower than the gain spectrum.

The interesting configuration is indeed the one in which the actions of SPM and gain narrowing accumulate, as described by Eqs. (2) and (3) with $\sigma = 0$. In what follows, we develop an asymptotic analysis in which we assume that the spectrum of the initial incoherent light is much narrower than the gain spectrum, which also reads as $\delta = T_2/T_c \ll 1$. Using a perturbation method (see

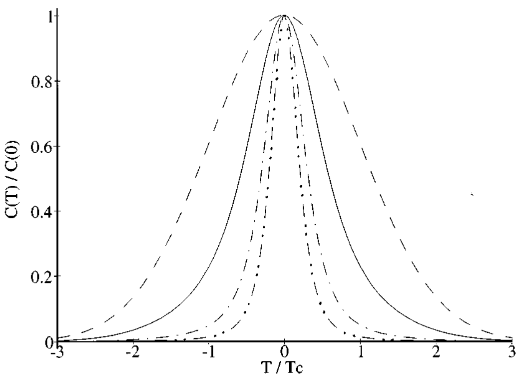

Fig. 1. Theoretical correlation function in the limit $T_2 = 0$ as a function of the reduced time T expressed in units of T_c for different values of the B integral. For each B value we have normalized the maximum of the function to 1. The dashed, solid, dotted–dashed, and double-dotted–dashed curves plot cases $B = 0, 1, 2, 3$, respectively.

Appendix A), we compute the asymptotic expansion of the electric field and find the average field intensity I at position z:

$$I(g, B) \simeq e^g I_0[1 - \delta^2 g - 2\delta^2 B^2 f_2(g) + O(\delta^3)]. \quad (7)$$

The term $f_2(g) = (1 - 4e^{-g} + 2ge^{-2g} + 3e^{-2g})(1 - e^{-g})^{-2}$ is equal to 1 in case of large gain. The second term in brackets in relation (7) is the usual gain-narrowing effect, and the third term is a new corrective term specific to the mutual action of SPM and a finite dephasing time. Both are always negative, so they lead to a decrease in the expected gain. The striking observation is that the third coupled term depends on the amplified intensity and not on the energy and that it may become even larger than the gain line-shape effect if B gets large enough ($B > \sqrt{g/2}$). However, we should also point out that the expansion in relation (7) is valid only if these two corrective terms are quite a bit smaller than 1.

The physics behind this intensity saturation is made more transparent when we estimate the evolution of the incoherent pulse spectrum. The method consists then of getting an asymptotic expansion of the correlation function $C(T, z)$ and Fourier transforming it. Actually this is a delicate computation that is sketched in Appendix A and yields

$$C(T, g, B) \simeq \frac{e^g I_0 \exp(-T^2/2T_c^2)}{\{1 + B^2[1 - \exp(-T^2/T_c^2)]\}^2} \times [1 + \delta^2 h_1(g, T/T_c) + 2\delta^2 B^2 h_2(g, T/T_c) + O(\delta^3)]. \quad (8)$$

For a high gain we get $h_1(g, \tau) = g(\tau^2 - 1)$ and $h_2(g, \tau) = g^2[1 - \tau^2 - \exp(-\tau^2)]$. The first ratio in relation (8) corresponds to the sole effect of SPM, and the first corrective term in the brackets originates from gain narrowing. The last corrective term, which can become quite important, is more interesting because it describes the coupled but competing mechanisms of the spectral narrowing that is due to the gain profile and the spectral broadening that results from SPM. The global result is that the correlation function may get narrower after propagation in the amplifier medium, corresponding to spectral broadening. This result provides a simple explanation of the intensity loss saturation that appears in relation (7): new wavelengths created by the SPM mechanisms are scattered in the wings of the gain profile and therefore are less amplified than expected or are not amplified at all. This result also implies that intensity saturation is automatically associated with spectral broadening, the important parameter being the B-integral value defined above.

4. COMPARISON WITH EXPERIMENTAL DATA

Here we develop numerical cases relevant for the experimental amplification of broadband pulses in Nd-doped glass amplifiers. In short, the experiments consist of creating a time-incoherent pulse with a spatially uniform profile by using a longitudinal multimode glass laser whose 14-ns output has the characteristics of a Gaussian stationary statistical process. The incoherent broadband pulse, typically of 1.2 nm FWHM bandwidth, corresponding to $T_c = 1.1$ ps, is sent into a pulse slicer to yield an output square shape of 1.3 ns. This pulse is then sent into a large Nd-doped phosphate glass able to deliver as much as 1.5 kJ of energy at 1.053 μm. Before going into the large disk amplifier the pulses go through high-gain preamplifiers at the output of which neither broadening nor saturation can be observed. We therefore consider only the section from the midchain to the final output. We can claim from previously known results[11] that two photon absorption is negligible in these experimental conditions.

Figure 2 shows the experimental energy output as a function of midchain energy input in the cases of a monochromatic pulse and of an incoherent pulse of bandwidth 1.2 nm. Some energy saturation, which is due to the energy depletion that the model does not take into account, can be seen in the case of the standard pulse. However, the striking observation is the stronger saturation in the case of the incoherent pulse. In a separate series of measurements we identified that this gain lowering is related to the pulse peak intensity, suggesting that it originates

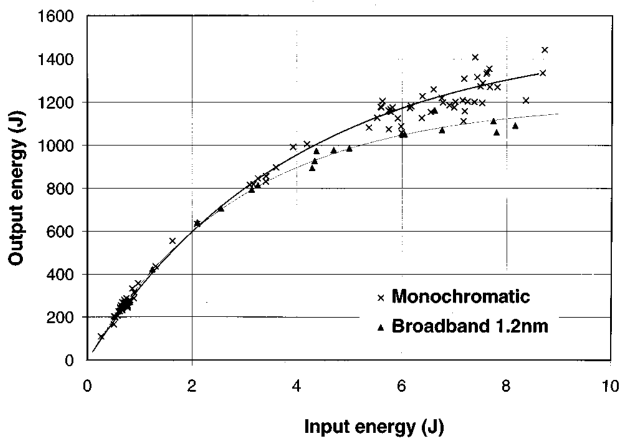

Fig. 2. Experimental energy output as a function of midchain energy input for a monochromatic pulse and for an incoherent pulse of bandwidth (FWHM) 1.2 nm.

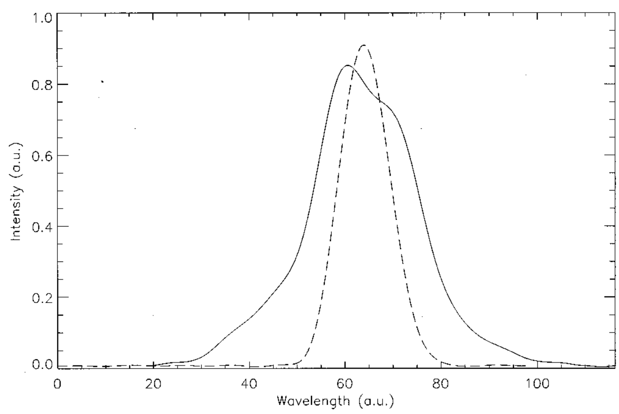

Fig. 3. Experimental chain output spectra. The preamplifier output spectrum (dashed curve) has a FWHM of 1.2 nm. The chain output spectrum (solid curve) has a FWHM of 2.4 nm and corresponds to an output energy of 1.1 kJ.

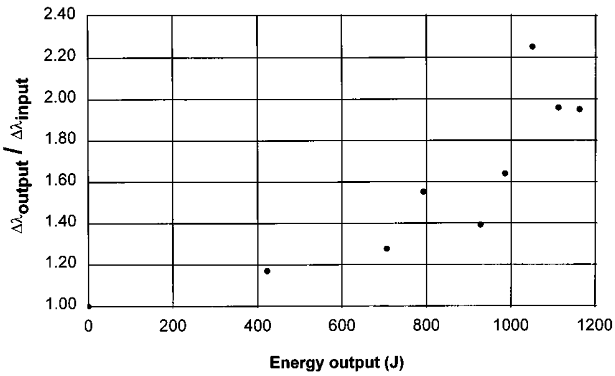

Fig. 4. Experimental spectral broadening as a function of energy output with an incoherent pulse of bandwidth (FWHM) 1.2 nm.

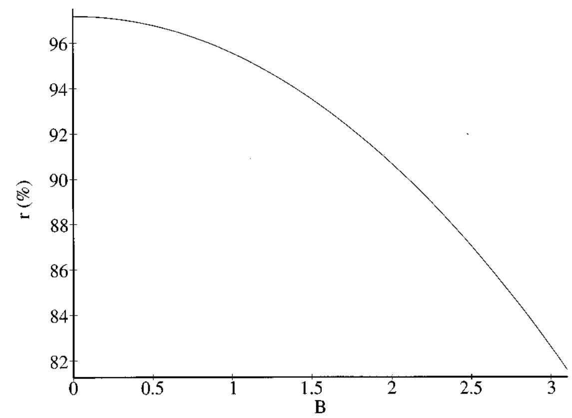

Fig. 5. Theoretical amplification efficiency, defined as the ratio r of the average output intensity given by relation (7) over the expected output intensity $I_0 e^g$. The efficiency is expressed as a percentage function of the B integral. The small signal-gain is taken to be $g = 3.5$ and the ratio is $\delta = 0.09$.

from nonlinear propagation effects as described in the model developed above. This conclusion is supported by the fact that the amplified pulse shows some important spectral broadening, typically by a factor of 2, relative to the injected spectrum. Figure 3 compares the preamplifier output spectrum with the chain output spectrum for one of the experiments, and Fig. 4 shows the experimental spectral broadenings observed for a series of experiments with different input and output energies.

To compare these data with our nonlinear statistical model, we assimilate the transition of Nd^{3+}-doped phosphate glass into a two-level homogeneously broadened system with a dephasing time $T_2 \sim 100$ fs. The small parameter is then $\delta = T_2/T_c \sim 0.09$. The measured gain corresponds to an amplification length unit of $\gamma^{-1} \sim 20$ cm, the small-signal gain being of the order of $g \sim 3.5$. Figure 5 represents the theoretical output as a function of the B integral for the given small-signal gain $g = 3.5$ when relation (7) is applied. It shows the same trend as the experimental data, consisting of saturation increasing with B, which itself is proportional to the input power. We get ~10% reduction of output for $B = 2$. At this stage it is instructive to use the correlation function of the output pulse given by relation (8) and, following the Wiener–Khintchine theorem,[12] to Fourier transform it to compute the output pulse spectrum. Figure 6 presents such a calculated output spectrum versus the input one for $B = 1.4$. These spectra depend only weakly on the value of the gain in the range $0 < g < 5$ but show a broadening that is due to self-phase modulation, which depends only on the B integral. This broadening seems roughly to follow the law $\sqrt{1 + 2B^2}$, which is an approximation that is valid for Gaussian pulses (Fig. 7). Figure 8 plots the calculated values of the B integrals of the experimental pulses whose output energies and spectral broadenings are shown in Fig. 4. For instance, spectral broadening by a factor of 2.2 theoretically corresponds to $B \sim 1.4$. We can also estimate the value of the B integral from energy measurements and the time profile of the pulse. This evaluation is difficult, however, because the temporal pulse shape is saturated and distorted by the complex amplifier system, which consists of many amplifiers with different diameters. We then use an efficient software called Mirò,[13] which can simulate propagation and amplification of laser beams in complex laser devices. Using the experimental time profile envelope as

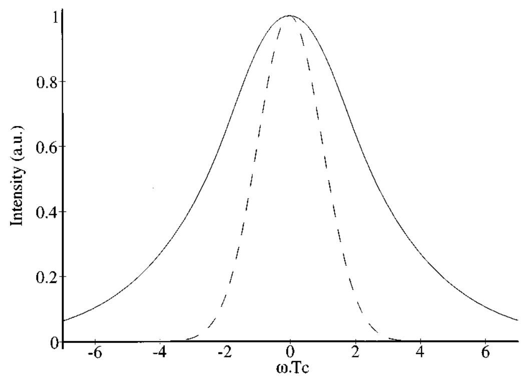

Fig. 6. Theoretical chain output spectrum versus the preamplifier spectrum (dashed curve) for $B = 1.4$ (solid curve). The small-signal gain is taken to be $g = 3.5$, and the ratio is $\delta = 0.09$.

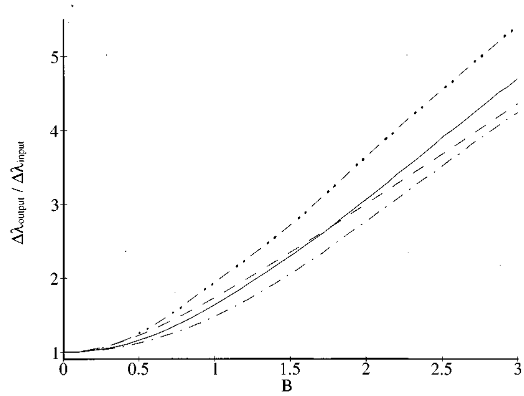

Fig. 7. Variation of the spectral broadening $\Delta\lambda_{output}/\Delta\lambda_{input}$ with the B integral. We compare the approximation that is valid for a coherent Gaussian pulse (dashed curve) with the theoretical chain output spectral broadening for incoherent light for $\delta = 0$ (dotted–dashed curve), for $g = 3.5$ and $\delta = 0.09$ (solid curve), and for $g = 6$ and $\delta = 0.09$ (double-dotted–dashed curve).

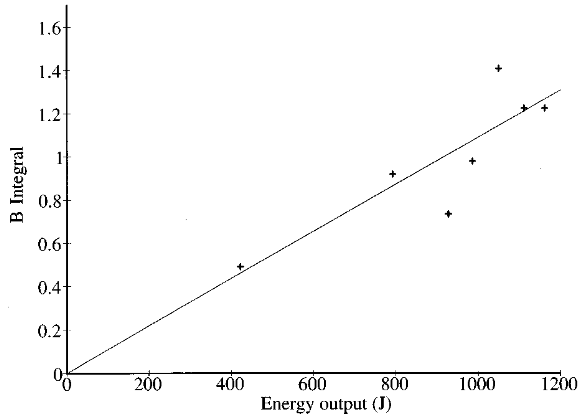

Fig. 8. Experimental B integral as a function of energy output with an incoherent pulse of bandwidth (FWHM) 1.2 nm. The values of the B integrals are calculated from the experimental values of the spectral broadenings given in Fig. 4 by the theoretical formula that connects the spectral broadening $\Delta\lambda_{\text{output}}/\Delta\lambda_{\text{input}}$ with the B integral and that is plotted in Fig. 7.

an entry for this software, we get the value $B_{\max} \simeq 1.7$ for the maximum of the B integral of the envelope. As the experimental time envelope has rather close to a flat profile, the actual value of the mean B integral is slightly less than 1.7, which is in reasonable agreement with the value 1.4 deduced from the spectral broadening. This B value corresponds to a calculated output energy loss of the order of 6%, which is close to the corresponding observed 10% loss of energy. Such a slight quantitative disagreement is not surprising in view of the uncertainty of the experiments and the simplifications of the model. First, we simulate the many amplifiers of the actual system by a single amplifier. Second, we do not consider energy saturation caused by the depletion of the inverted population. Therefore we believe that the nonlinear model of time incoherence developed here must capture the essential part of the phenomena responsible for the anomalous intensity saturation observed in a glass laser.

5. INFLUENCE OF GROUP-VELOCITY DISPERSION

We first discuss the effects of GVD in the case of a flat gain spectrum ($T_2 = 0$). The envelope of the field satisfies nonlinear Schrödinger equation (2) with small-signal gain γ, which reads as

$$i\,\frac{\partial E}{\partial z} + \sigma\,\frac{\partial^2 E}{\partial t^2} + \frac{k_0 n_2}{2n_0}\,|E|^2 E = \frac{i\gamma}{2}\,E. \tag{9}$$

The average intensity, i.e., the L^2 norm of the field, is then amplified with the exponential gain e^g. To put into evidence the effects of GVD we consider the contrast of the pulse, defined as the normalized variance of the intensity:

$$c^2 = \frac{\langle |E|^4\rangle - \langle |E|^2\rangle^2}{\langle |E|^2\rangle^2}, \tag{10}$$

which characterizes the relative fluctuations in intensity. The estimation of the L^4 norm of the field $\langle |E|^4\rangle$ requires the study of the L^2 norm of the time derivative of the field, which is simply related to the second time derivative of the autocorrelation function through the formula

$$\left\langle \left|\frac{\partial E}{\partial t}\right|^2 \right\rangle = -\mathrm{Re}\,\frac{\partial^2 C}{\partial T^2}\bigg|_{T=0}. \tag{11}$$

If we consider the GVD as a slight perturbation of Eq. (9), which holds true if

$$0 < \frac{\sigma}{\gamma T_c^2} \ll 1, \tag{12}$$

then we can obtain by a perturbation method an expansion of the norm of the time derivative of the field with respect to the small parameter $\sigma/(\gamma T_c^2)$, whose first corrective term is found to be (for large g)

$$\left\langle \left|\frac{\partial E}{\partial t}\right|^2 \right\rangle \simeq \frac{I_0 e^g}{T_c^2}\left[1 + 4B^2\left(1 + \frac{44B\sigma}{\gamma T_c^2}\right)\right]. \tag{13}$$

At this stage it is convenient to introduce a Hamiltonian, defined by

$$H := \sigma\left\langle \left|\frac{\partial E}{\partial t}\right|^2 \right\rangle - \frac{k_0 n_2}{4n_0}\,\langle |E|^4\rangle, \tag{14}$$

which is not preserved because of gain but obeys the differential equation

$$\frac{\partial H}{\partial z} = -\frac{\gamma k_0 n_2}{4n_0}\,\langle |E|^4\rangle + \gamma H. \tag{15}$$

Combining Eq. (15) with relation (13), we get that the L^4 norm of the field is not amplified according to the expected rate e^{2g}. Indeed, its expression can be written in the form of an expansion with respect to the parameter $\sigma/(\gamma T_c^2)$, whose first and second corrective terms can be derived from expressions (13)–(15):

$$\langle |E|^4\rangle \simeq \langle |E_0|^4\rangle e^{2g}\left(1 + \frac{16B\sigma}{\gamma T_c^2} + \frac{528B^2\sigma^2}{\gamma^2 T_c^4}\right). \tag{16}$$

In the case of normal dispersion ($\sigma < 0$), the interaction between the GVD and the SPM involves a spread of the pulse, whose spikes broaden and local maxima decrease. The expansion up to second order of the contrast of the pulse,

$$c^2 \simeq 1 + \frac{32B\sigma}{\gamma T_c^2} + \frac{1056B^2\sigma^2}{\gamma^2 T_c^4}, \tag{17}$$

exhibits a reduction of the relative fluctuations in intensity when $\sigma < 0$ and an enhancement when $\sigma > 0$.

Let us now analyze the effects of GVD in the asymptotic framework $0 < T_2 \ll T_c$. Taking into account the expression of the L^2 norm of the time derivative of the field given by relation (13), we can find the asymptotic expansion of the average field intensity I at position z for large gain, which generalizes relation (7):

$$I(g, B) \simeq e^g I_0\left[1 - \delta^2 g - 2\delta^2 B^2\left(1 + \frac{88B\sigma}{3\gamma T_c^2}\right) + O(\delta^3)\right]. \tag{18}$$

The derivation of the precise expansion valid for any g can be found in Appendix A. So it appears that anomalous dispersion induces an enhancement of the intensity saturation exhibited in Section 3, whereas normal dispersion succeeds in reducing the contrast as well as the intensity saturation. In the experimental conditions discussed in Section 4, the value of the corrective term $\sigma/(\gamma T_c^2)$ that is due to GVD is of the order of -8×10^{-3}; it was therefore fair enough to neglect that term in the analysis. However, the effect of GVD could be relevant for a much larger bandwidth or for a highly dissipative medium (we may think of fiber amplifiers doped with rare-earth ions). In such conditions it could be important to take into account the corrective terms in Eqs. 17 and 18. Moreover, if the absolute value of the parameter $\sigma/(\gamma T_c^2)$ becomes of the order of 1 or larger, then the above analysis based on a perturbation method is not valid because the time-dispersion effects cannot be considered a small perturbation anymore. The picture is actually much more complicated than the one presented here. The interplay between the SPM and the GVD in the anomalous regime will give rise to self-compression of the spikes toward the formation of solitons.[14] With normal dispersion the spikes tend to broaden and interact. Even dark solitons (i.e., solitonlike excitations with large but finite lifetimes) could appear between spikes; refer to Gredeskul and Kivshar[15] for a review of this topic.

APPENDIX A

We aim at sketching the calculations of the average intensity and the correlation function. A mathematical approach to this problem was developed elsewhere.[9] Using the perturbed function method, we can compute the asymptotic expansion of the electric field $E = ae^{i\phi}$ with respect to the small parameter $\delta = T_2/T_c$:

$$a(\tau, z) = \tilde{a}_0(\tau, z) + \delta\tilde{a}_1(\tau, z) + \delta^2\tilde{a}_2(\tau, z) + \dots,$$

$$\phi(\tau, z) = \tilde{\phi}_0(\tau, z) + \delta\tilde{\phi}_1(\tau, z) + \delta^2\tilde{\phi}_2(\tau, z) + \dots, \tag{A1}$$

where τ is the adimensional variable t/T_c. Within this framework the coherence time of the incident field is of the order of 1 with respect to δ, whereas the evolution equations of the field and the polarization can be written in the form

$$i\frac{\partial E}{\partial z} + \frac{\sigma}{T_c^2}\frac{\partial^2 E}{\partial \tau^2} + \frac{k_0 n_2}{2n_0}|E|^2 E = \frac{P}{2}, \tag{A2}$$

$$\delta\frac{\partial P}{\partial \tau} + P = i\gamma E. \tag{A3}$$

Substituting the expressions given by Eqs. (A1) into Eqs. (A2) and (A3) and equating to zero the terms in front of δ^j, we obtain a set of differential equations for the functions $\tilde{a}_j$ and $\tilde{\phi}_j$, and finally we can find the asymptotic expansion of the average output intensity $I(z)$ at position z in the case $\sigma = 0$:

$$I(g) \simeq e^g\langle a_0^2\rangle - \delta^2(B/I_0)^2 f_2(g)e^g\langle a_0^4 a_0'^2\rangle - \delta^2 g e^g(\langle a_0'^2\rangle + \langle a_0^2\phi_0'^2\rangle) + O(\delta^3), \tag{A4}$$

where the prime stands for the derivative with respect to τ. In the case $\sigma \neq 0$, the term $\delta^2\sigma/(\gamma T_c^2)[k_0 n_2 e^g/(2n_0\gamma)]^3 e^g A(g, I_0)$ has to be added to the sum given by relation (A4); the coefficient A is found to be equal to

$$A(g, I_0) = f_3(g)\langle a_0'^4 a_0^4\rangle + f_4(g)\langle a_0''^2 a_0^6\rangle, \tag{A5}$$

where $f_3(g) \simeq 56/9$ and $f_4(g) \simeq -8/3$ in the case of large gain. Then we decompose $E_0 = a_0 e^{i\phi_0}$ into the sum $X_1 + iX_2$. Because the processes X_1 and X_2 are statistically independent and identically distributed, we can factor the expectations $\langle F(X_1)G(X_2)\rangle$ as $\langle F(X)\rangle\langle G(X)\rangle$, where X obeys the same distribution as X_j. Finally, after some calculations, we can verify that relation (7) holds in the limit when $\sigma = 0$, and also when $A(g, I_0) = I_0^4\bar{A}(g)$, where

$$\bar{A}(g) = -\frac{176}{3} + 176ge^{-g} - 352ge^{-2g} + 528e^{-2g} - 24g^2e^{-3g} - 176ge^{-3g} - \frac{1408}{3}e^{-3g}, \tag{A6}$$

which is equal to $-176/3$ for large gain. We can perform the same calculations for the correlation function. However, the computation is much more complicated, because many terms of order δ^2 are found. The prevailing one for large gain g is

$$\Delta C(\tau, g) \simeq \delta^2 e^g g^2\left[\frac{k_0 n_2(e^g - 1)}{2n_0\gamma}\right]^2\langle a_0^2 a_0'(0)a_0^2 a_0'(\tau) \times \exp\{i[\phi_0(0) + \phi_{\mathrm{NL}}(0) - \phi_0(\tau) - \phi_{\mathrm{NL}}(\tau)]\}\rangle. \tag{A7}$$

However, it is necessary to estimate all the terms to find the prevailing one. Decomposing the initial field E_0 into the sum $X_1 + iX_2$, developing and factorizing the expectations by the independence of the processes X_1 and X_2, we find that ΔC is equal to

$$\Delta C(\tau, g) \simeq 2\delta^2 e^g g^2 B^2 I_0 \times \frac{\exp(-\tau^2/2)[1 - \tau^2 - \exp(-\tau^2)]}{\{1 + B^2[1 - \exp(-\tau^2)]\}^2} + \text{negligible terms}, \tag{A8}$$

where the negligible terms are a sum of terms of the type

$$\delta^2\frac{I_0\exp(-\tau^2/2)h(g, B, \tau)}{\{1 + B^2[1 - \exp(-\tau^2)]\}^j},$$

with $j \geqslant 3$.

About the further terms of order $O(\delta^k)$, $k \geqslant 3$, by a recursive argument we can state that the asymptotic expansion of the average intensity can be written as

$$I(g, B) \simeq e^g I_0\left[1 - \delta^2 g - 2\delta^2 B^2 f_2(g) + \sum_{l=3}^{k} f_l(g, B)\delta^l + O(\delta^{k+1})\right], \tag{A9}$$

where the lth corrective term $f_l(g, B)$ is at most of the type B^l.

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