Amplification of incoherent light with wide spectrum

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Abstract

We consider a wide-frequency-bandwidth incoherent pulse and its propagation through an amplifier medium. We show how the intensity grows and how the correlation function behaves in the medium. We see how a small nonlinearity may greatly affect the amplification.

1 Problem

We aim at studying the propagation of a Gaussian incoherent field through a nonlinear amplifier medium. The medium, which draws power from a source other than the input signal, provides the pulse with energy. Our first task is to study this phenomenon called amplification. However, the medium is not perfectly linear, and we will have to consider that the index of refraction is slightly nonlinear. Then, because of self-phase modulation, we will see that it induces spectral broadening on the one hand, and that it affects the amplification on the other one.

The results of this paper seem to agree with with some experimental observations (see [2]) concerning the amplification and the spectral broadening of an incoherent laser beam. In particular a saturation of the output intensity is observed while the transverse effects and the variation in the population inversion are negligible.

1.1 Incoming field

We consider a plane wave incoming from the left $z < 0$, with the form $E_0(t)$ at $z = 0$.
We assume that E_0 has Gaussian statistics, that means that $E_0(t) = X_1(t) + iX_2(t)$, where X_1 and X_2 are independent stationary real Gaussian processes with the same correlation function $< X_i(0)X_i(t) >= \frac{A_0^2}{2} f(t)$, $f(0) = 1$. As a consequence E_0 is a complex Gaussian field with correlation function $K_0(t) =< E_0(0)E_0^*(t) >$ given by :

$$K_0(t) = A_0^2 f(t).$$

A_0 is the initial amplitude of the electric field (A_0^2 is the initial intensity). The angle brackets refer to the expectation with respect to the statistics of E_0. Besides we will denote by a_0 the modulus of the initial field and by ϕ_0 its phase, so that $E_0(t) = a_0(t)\exp(i\phi_0(t))$.

In the numerical simulations, we will assume moreover that the correlation function is Gaussian, $f(t) = \exp(-\frac{t^2}{2T_c^2})$, where T_c will be called the coherence time of the initial field. In that case, we can note that :

- $a_0(t)$ and $\phi_0(t)$ are independent at every time t,

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- the distribution of $a_0(t)$ admits a density which is equal to $p(a) = \frac{a}{A_0^2}\exp(-\frac{a^2}{2A_0^2})$,
- the distribution of $\phi_0(t)$ is uniform on $[0, 2\pi]$.

1.2 Amplification

The equation which governs the propagation is :

$$-2i\frac{\partial E}{\partial z} + \Delta_\perp E = P, \tag{1}$$

where P is the polarization of the amplifier medium.
Throughout the paper, the transverse aspect will be neglected. Moreover, the evolution of the polarization P is governed by :

$$\varepsilon\frac{\partial P}{\partial t} + P = -inE, \tag{2}$$

where n is the population inversion, which is the characteristic of the amplifier. What is important to notice is that the characteristic time of the evolution of the polarization is assumed to be much shorter than the coherence time of the initial field, i.e. that $\varepsilon \ll T_c$.

1.3 Self-phase modulation

During its propagation, the electric field is subject to a self-phase modulation, induced by a nonlinear index of refraction :

$$\left\{\begin{array}{l} E = E(z,T), \quad T = t - \frac{z}{v_g}, \\ -2i\frac{\partial E}{\partial z} + \gamma|E|^2E = P, \end{array}\right. \tag{3}$$

T is the reduced time at position z, and v_g is the group velocity.

1.4 First approximation

Let us first consider the very simple case where both the evolution of the polarization and the self-phase modulation are neglected. In the present approximation, the electric field E is, for any z, given by :

$$E_{\varepsilon=0,\gamma=0}(z,t) = e^{\frac{nz}{2}}E_0(t).$$

We define the average intensity as $I(z) =< |E(z,u)|^2 >$ and express it as :

$$I_{\varepsilon=0,\gamma=0}(z, A_0) = A_0^2 e^{nz}.$$

The correlation function $K(t,z) =< E(z,u)E^*(z,u+t) >$ is given by :

$$K_{\varepsilon=0,\gamma=0}(t,z,A_0) = A_0^2 e^{nz} f(t).$$

2 Evolution of the polarization

We will study here the effects induced by the evolution of the polarization on the electric field. We still neglect the nonlinearity and the self-phase modulation, i.e. we consider the limit case $\gamma = 0$. Moreover we consider that the population inversion n is constant.

Then we are able to find the new expressions for the field intensity and the correlation function for any z :

$$I_{\varepsilon,\gamma=0}(z,A_0) \simeq A_0^2 e^{nz}\left(1-\left(\frac{\varepsilon}{T_c}\right)^2(nz)+O\left(\frac{\varepsilon}{T_c}\right)^4\right),$$

$$K_{\varepsilon,\gamma=0}(t,z,A_0) \simeq A_0^2 e^{nz} f(t)\left(1+\left(\frac{\varepsilon}{T_c}\right)^2(nz)g(t)+O\left(\frac{\varepsilon}{T_c}\right)^4\right),$$

with $g(t)=\left(\frac{t}{T_c}\right)^2-1$ when $f(t)=\exp(-\frac{t^2}{2T_c^2})$. Note that the expansion of the correlation function is valid for times t such that $\varepsilon t \ll T_c^2$. According to the experimental data, where nz is of order 1 and $\frac{\varepsilon}{T_c}\ll 1$, the corrective term of order $\left(\frac{\varepsilon}{T_c}\right)^2 nz$ is negligible.

3 Self-phase modulation

In this section we neglect the evolution of the polarization, which will be taken to be equal to $-inE$. Thus the electric field is affected by the self-phase modulation according to the following equation :

$$\begin{cases} E=E(z,T), \quad T=t-\dfrac{z}{v_g}, \\ -2i\dfrac{\partial E}{\partial z}+\gamma|E|^2E=-inE, \end{cases}$$

The electric field is therefore given by :

$$E_{\varepsilon=0,\gamma}(z,T)=e^{\frac{nz}{2}}a_0(T)\exp i\left(\phi_0(T)-\frac{\gamma}{2n}\alpha(nz)a_0^2(T)\right), \tag{4}$$

where $\alpha(x)=e^x-1$.

On the one hand, since the self-phase modulation only affects the phase of the electric field, it does not induce any effect on the field intensity. We still have :

$$I_{\varepsilon=0,\gamma}(z,A_0)=e^{nz}A_0^2. \tag{5}$$

On the other hand, we can explicitly compute the correlation function (cf appendix), and find that it is greatly affected by the self-phase modulation :

$$K_{\varepsilon=0,\gamma}(T,z,A_0)=\frac{e^{nz}A_0^2 f(T)}{\left(1+\left(A_0^2\frac{\gamma}{2n}\right)^2\alpha(nz)^2(1-f(T)^2)\right)^2}. \tag{6}$$

Therefore, even if the nonlinear component of the index of refraction γ is small, we can deal with an important correction if the initial intensity A_0^2 and the size of the medium z are so large that $\frac{\gamma}{2n}\left(e^{nz}-1\right)A_0^2>1$.

In Fig 1, we compare different correlation functions plotted as functions of position z and reduced time T, for different values of A_0. For a fixed value of the initial intensity A_0^2, the correlation function $K(T,z)$ looks like $K_0(t)$ for small z. However for larger values of z, $T\mapsto K(T,z)$ has still the same shape as $K_0(T)$, but with a rescaling; we mean that $T\mapsto K(T,z)$ becomes more and more concentrated near $T=0$.

The higher the intensity is, the stronger the previously described trend is, i.e. the rescaling

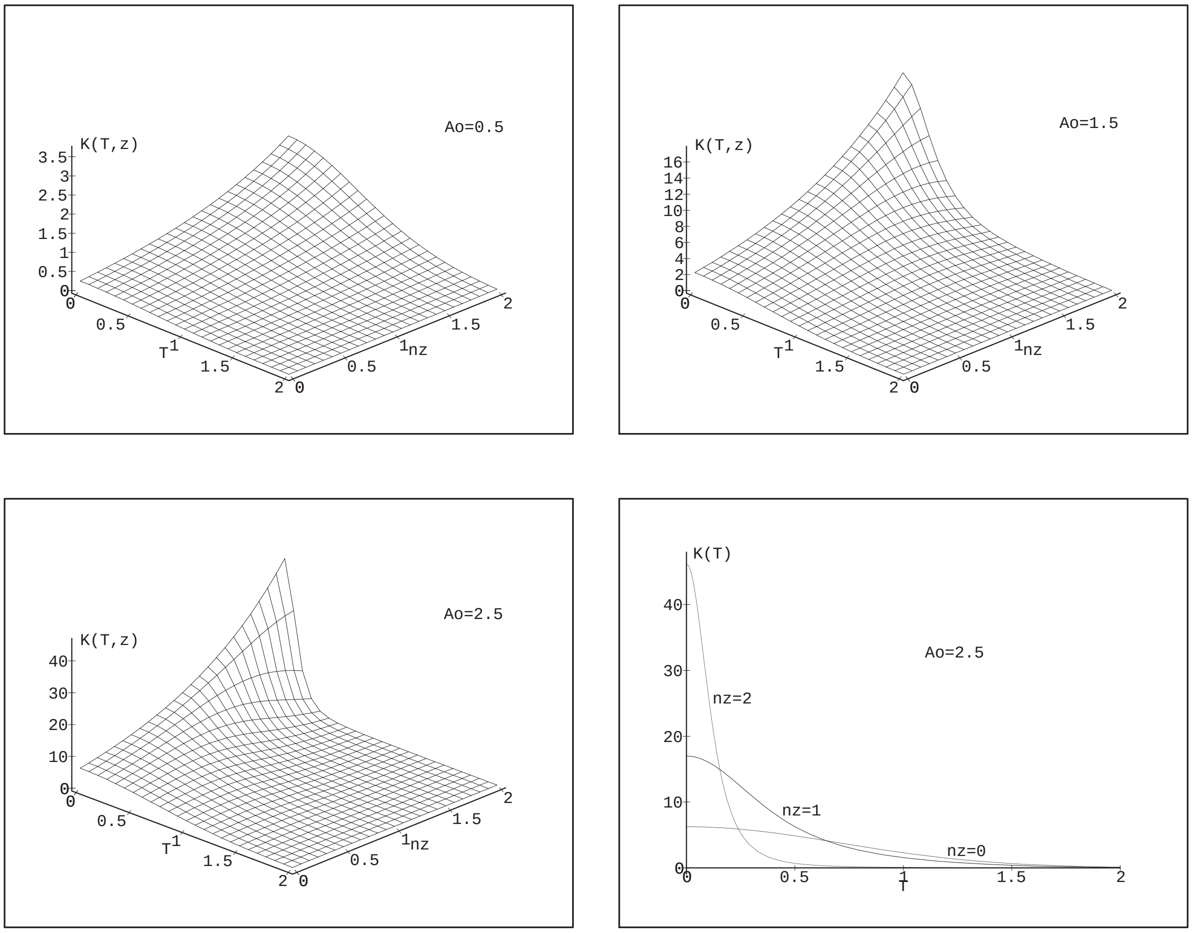

FIG. 1. *Correlation function* $K_{\varepsilon=0,\gamma}(T,z)$, $\dfrac{\gamma}{2n}=0.1$.

and the concentration of the correlation function near $T=0$ become more and more considerable.

In Fig 2, we compare different correlation functions plotted as functions of initial amplitude A_0 and reduced time T, for different values of position z. It is another way to underscore the concentration near $T=0$ of the correlation function when z and A_0 grow.

REMARK 3.1. *If we take the limit $n \to 0$ in (6), then we get back the result of Manassah [1], who has considered the propagation of an incoherent field in a dispersionless nonlinear Kerr medium.*

4 Joint action of self-phase modulation and polarization

Let us regard now how the evolution of the polarization and the self-phase modulation both affect the electric field and its statistics during its propagation.

4.1 Average intensity

First we consider the intensity of the electric field. We have already proved that the self-phase modulation does not affect the growth of the intensity if we do not take the evolution of the polarization into account. On the other hand we have shown that the evolution of

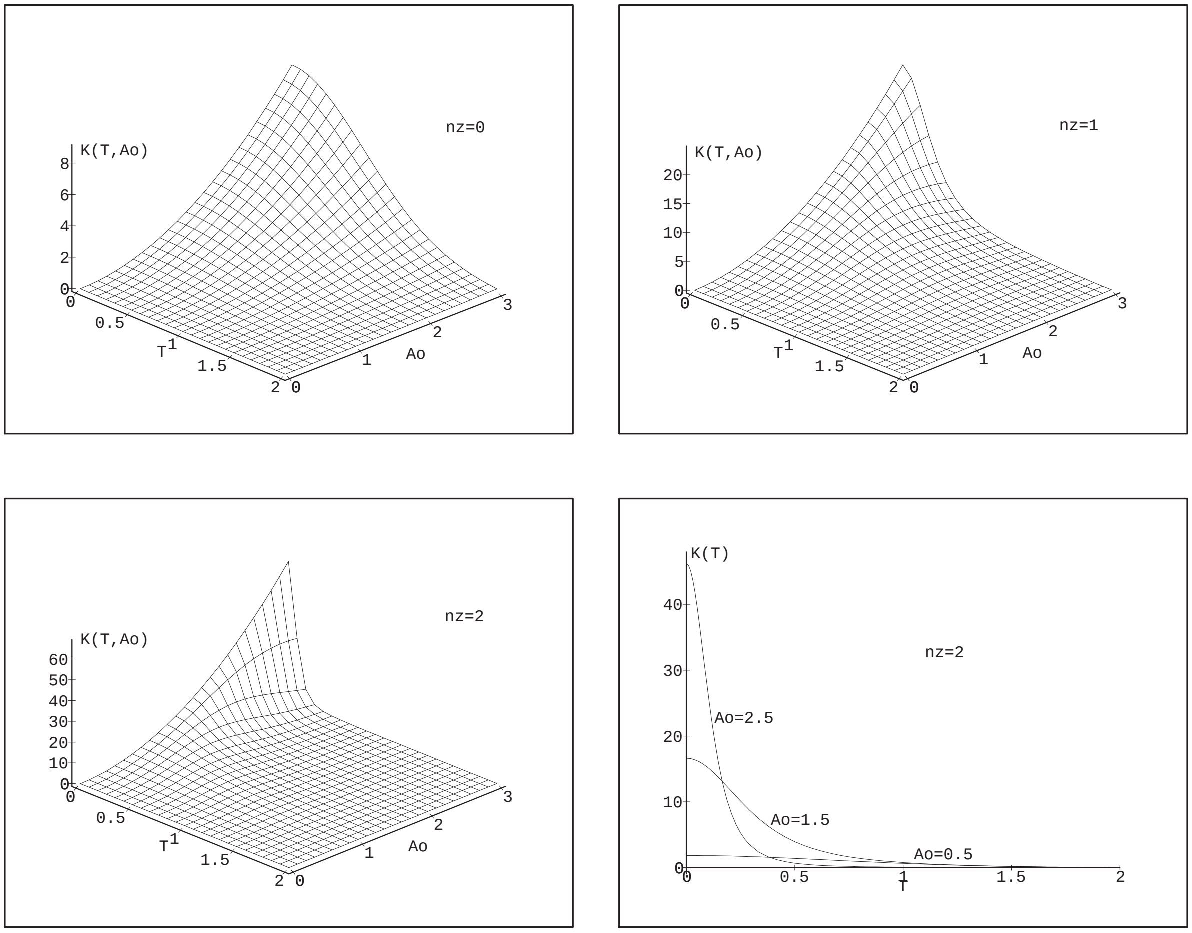

FIG. 2. *Correlation function* $K_{\varepsilon=0,\gamma}(T, A_0)$, $\dfrac{\gamma}{2n} = 0.1$.

the polarization has only a negligible influence on the intensity if we set aside the self-phase modulation.

We aim at showing here that if we take both the evolution of the polarization and the self-phase modulation into account, then it appears a new corrective term in the expression of the average intensity $I_{\varepsilon,\gamma}(z, A_0)$, which may be non-negligible.

$$(7)\qquad I_{\varepsilon,\gamma}(z, A_0) \simeq A_0^2 e^{nz} \times \left(1 - \left(\frac{\varepsilon}{T_c}\right)^2 nz - \left(\frac{\varepsilon}{T_c}\frac{\gamma}{2n}A_0^2 e^{nz}\right)^2 \beta(nz) + O\left(\left(\frac{\varepsilon}{T_c}\right)^3\right)\right),$$

where $\beta(x) = 2 - 8e^{-x} + 4xe^{-2x} + 6e^{-2x}$.

In the expression of $I_{\varepsilon,\gamma}$, we can recognize the term $\left(\dfrac{\varepsilon}{T_c}\right)^2 nz$ which corresponds to the direct effect of the evolution of the polarization (see the expression of $I_{\varepsilon,\gamma=0}$). Besides, we find the new corrective term $\left(\dfrac{\varepsilon}{T_c}\dfrac{\gamma}{2n}A_0^2 e^{nz}\right)^2 \beta(nz)$, which is due to the joint action of the evolution of the polarization and the self-phase modulation. This term depends on the initial intensity A_0^2 since it derives from a nonlinear effect. The higher the initial intensity is, the more important this corrective term is.

Moreover, it can be proved by a recursive argument (cf appendix), that the term

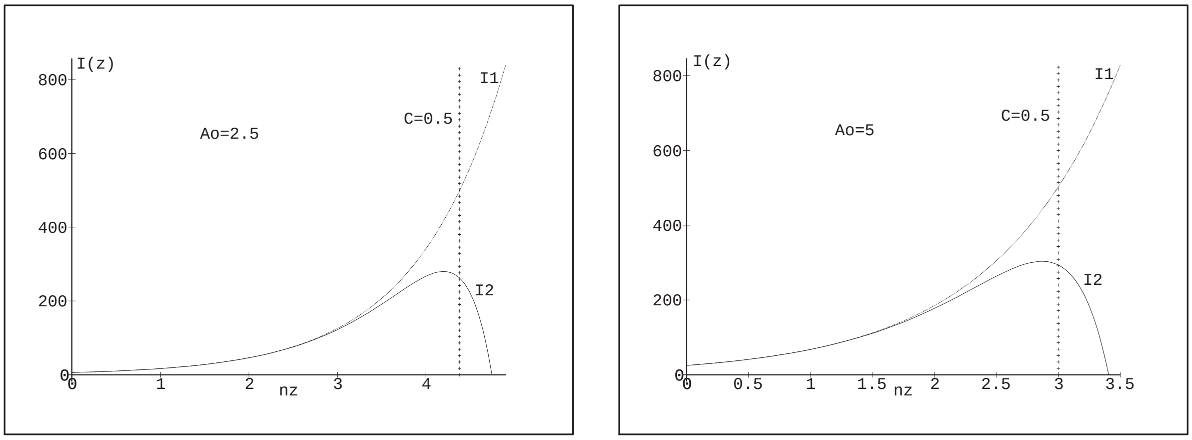

FIG. 3. *Amplification of the intensity* $I_{\varepsilon,\gamma}(z)$, $\dfrac{\gamma}{2n} = 0.1$, $\dfrac{\varepsilon}{T_c} = 0.01$.

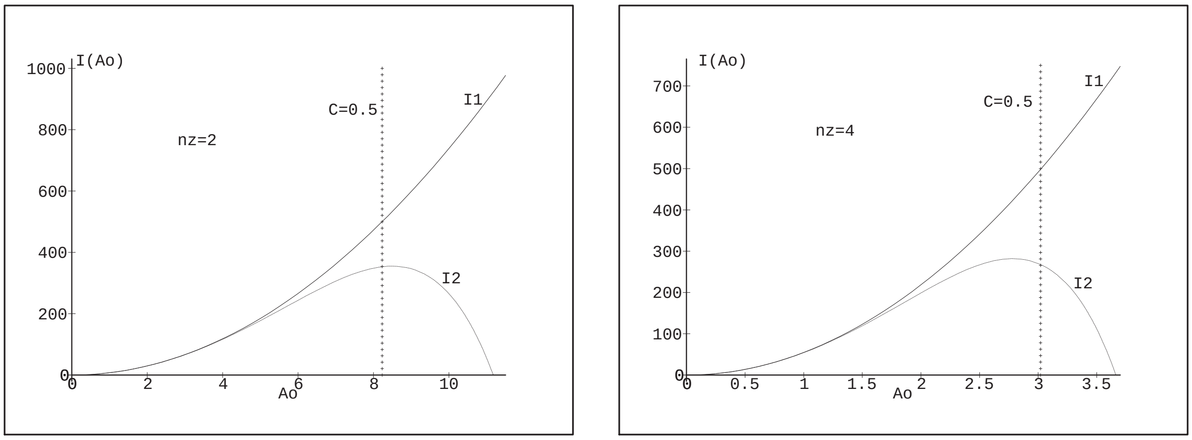

FIG. 4. *Amplification of the intensity* $I_{\varepsilon,\gamma}(A_0)$, $\dfrac{\gamma}{2n} = 0.1$, $\dfrac{\varepsilon}{T_c} = 0.01$.
$I1$ denotes $I_{\varepsilon=0,\gamma=0}$,
$I2$ denotes $I_{\varepsilon,\gamma}$ *except the term in* $O\left(\dfrac{\varepsilon}{T_c}\right)^3$,
C denotes the coefficient $\dfrac{\varepsilon}{T_c} A_0^2 \dfrac{\gamma}{2n} e^{nz}$.

$O\left(\dfrac{\varepsilon}{T_c}\right)^3$ is a sum of terms of the type $\left(\dfrac{\varepsilon}{T_c}\dfrac{\gamma}{2n}A_0^2 e^{nz}\right)^j$, $j \geq 3$, and some others which are negligible in front of these, like $\left(\dfrac{\varepsilon}{T_c} nz\right)^j$, $j \geq 3$.

Therefore anyone of these two curves are exact. Fig 3 and 4 must be understood as follows :

- When $\dfrac{\varepsilon}{T_c}\dfrac{\gamma}{2n}A_0^2 e^{nz} \ll 1$, then $I1 = I_{\varepsilon=0,\gamma=0}$ is a good approximation of the average intensity.
- When $\dfrac{\varepsilon}{T_c}\dfrac{\gamma}{2n}A_0^2 e^{nz}$ reaches values of order 1, then the average intensity does not follow the curve $I1$ anymore, but the curve $I2$: it means that the amplification is stopped.
- When $\dfrac{\varepsilon}{T_c}\dfrac{\gamma}{2n}A_0^2 e^{nz} > 1$, then we should take into account all the terms which are

contained in $O\left(\frac{\varepsilon}{T_c}\frac{\gamma}{2n}A_0^2e^{nz}\right)^3$ in (7). We do not know the behaviour of the average intensity in this region.

To sum up : We can affirm that the growth of the intensity is stopped when the initial amplitude A_0 is so high that $\frac{\varepsilon}{T_c}\frac{\gamma}{2n}A_0^2e^{nz}$ is not small compared to 1. Moreover the maximal output intensity that we can expect is independent of the input intensity and is of order $O\left(\frac{\varepsilon}{T_c}\frac{\gamma}{2n}\right)^{-1}$.

4.2 Correlation function

Let us now regard the joint action of the evolution of the polarization and the self-phase modulation on the correlation function of the electric field.

We find that the correlation function roughly behaves as in the limit $\varepsilon = 0$. However the joint action induces a corrective term which is of the same order as in the intensity, so that $K_{\varepsilon,\gamma}(T,z,A_0)$, is, when $\varepsilon T \ll T_c^2$, equal to :

$$(8)\quad K_{\varepsilon=0,\gamma}(T,z,A_0)\times\left(1+\left(\frac{\varepsilon}{T_c}\right)^2 nzg(T)+\left(\frac{\varepsilon}{T_c}\frac{\gamma}{2n}A_0^2e^{nz}\right)^2\lambda(nz)h(T)+O\left(\frac{\varepsilon}{T_c}\right)^3\right),$$

where $\lambda(x) = 2x^2\left(1+\underset{x\to\infty}{o}(1)\right)$.

When $f(t)=\exp(-\frac{t^2}{2T_c^2})$, we have $g(T)=\left(\frac{T}{T_c}\right)^2-1$ and $h(T)=1-\left(\frac{T}{T_c}\right)^2-f^2(T)$.

We do not write the negligible terms of the type $\frac{A_0^2e^{nz}\ldots}{\left(1+(A_0^2\frac{\gamma}{2n})^2\alpha(nz)^2(1-f^2(T))\right)^j}$, $j\geq 3$.

There are two corrective terms of order $O\left(\frac{\varepsilon}{T_c}\right)^2$:

The first one, $\left(\frac{\varepsilon}{T_c}\right)^2 nzg(T)$, which is the direct effect of the evolution of the polarization, is independent of the initial intensity and is always negligible.

The second one, $\left(\frac{\varepsilon}{T_c}\frac{\gamma}{2n}A_0^2e^{nz}\right)^2\lambda(nz)h(T)$, which is due to the joint nonlinear action of the evolution of the polarization and the self-phase modulation, depends on the initial intensity A_0^2. As in the case of the growth of the intensity, if the initial intensity A_0^2 becomes too high, then the second corrective term may become non-negligible.

5 Spectral broadening

5.1 Limit case $\varepsilon = 0$

We aim at exhibiting a relation between the spectral width and the output intensity. This is worth studying, since both of them are observable quantities.

First we consider the case $\varepsilon = 0$. In that case, the correlation function $K_{\varepsilon=0,\gamma}$ is given by (6). We define the coherence time DT as the width of the correlation function (the spectral width is then the inverse of the coherence time). More exactly we say that $DT_{\varepsilon=0,\gamma}$ is the unique time such that

$$\int_{-DT_{\varepsilon=0,\gamma}}^{+DT_{\varepsilon=0,\gamma}} K_{\varepsilon=0,\gamma}(T,z,A_0)dT = \frac{1}{2}\int_{-\infty}^{+\infty} K_{\varepsilon=0,\gamma}(T,z,A_0)dT.$$

Let us denote by Is the output intensity. We know that $Is_{\varepsilon=0,\gamma} = e^{nz}A_0^2$. Thus, according to the explicit form of the correlation function, we can write $K_{\varepsilon=0,\gamma}$ as a function of Is, T and of the parameter $\frac{\gamma}{2n}$, if we assume that $e^{nz} \gg 1$. We can write $DT_{\varepsilon=0,\gamma}$ as a function of Is and $\frac{\gamma}{2n}$.

THEOREM 5.1. *Let us assume that f admits an asymptotic expansion near $t=0$ of the type $f(t) = 1 - \lambda t^\alpha$.*

- *If $\alpha > \frac{1}{2}$, then $DT_{\varepsilon=0,\gamma} \times \left(\frac{\gamma}{2n} Is\right)^{\frac{2}{\alpha}} \underset{Is\to\infty}{\longrightarrow} \beta$,*
  where β is such that $\int_{-\beta}^{+\beta} \frac{1}{(1+2\lambda t^\alpha)^2} dt = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{1}{(1+2\lambda t^\alpha)^2} dt$.

- *If $\alpha < \frac{1}{2}$, then $DT_{\varepsilon=0,\gamma} \underset{Is\to\infty}{\longrightarrow} \beta$,*
  where β is such that $\int_{-\beta}^{+\beta} \frac{f(t)}{(1-f(t)^2)^2} dt = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{f(t)}{(1-f(t)^2)^2} dt$.

In fact, these behaviours hold as soon as $\frac{\gamma}{2n} Is \gg 1$.

Example. If $f(t) = \exp(-\frac{t^2}{2T_c^2})$, then $\frac{DT_{\varepsilon=0,\gamma}}{T_c} \times Is \times \frac{\gamma}{2n} \to a_1$, $a_1 \simeq 2.26$.

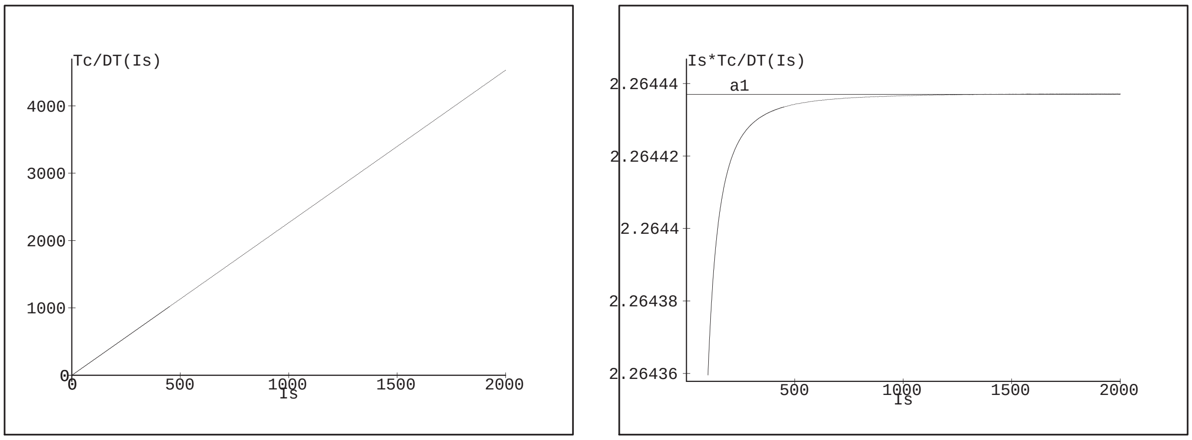

FIG. 5. *Coherence time $DT_{\varepsilon=0,\gamma}$, $\frac{\gamma}{2n} = 1$.*

5.2 Asymptotic case $\varepsilon \ll T_c$

We assume here that $\varepsilon \neq 0$ but $\varepsilon \ll T_c$, that means that the characteristic time of evolution of the polarization is much shorter than the coherence time of the incoming field.

According to the previous results, we know that the output intensity $Is_{\varepsilon,\gamma}$ is almost equal to $A_0^2 e^{nz}$ and the correlation function is given by (8) when $\frac{\varepsilon}{T_c}\frac{\gamma}{2n}A_0^2 e^{nz} \ll 1$. Therefore the coherence time behaves according to the formulas of theorem (5.1) when $\frac{\varepsilon}{T_c}\frac{\gamma}{2n} Is \ll 1$.

When the intensity is such that the quantity $\frac{\varepsilon}{T_c}\frac{\gamma}{2n}A_0^2 e^{nz}$ reaches values of order 1, then we have shown on the one hand that the amplification of the output intensity was stopped, i.e. Is does not grow anymore when $A_0^2 e^{nz}$ increases. On the other hand it is easy to check from (8) that the coherence time still behaves according to the formulas of theorem (5.1), if we replace Is by $A_0^2 e^{nz}$. Combining these facts, we can affirm that $\frac{\partial DT_{\varepsilon,\gamma}}{\partial Is}$ goes to infinity when Is reaches values such that $\frac{\varepsilon}{T_c}\frac{\gamma}{2n}Is$ becomes of order 1. However, as in the previous sections, we are not able to describe the behaviour of the coherence time when the quantity $\frac{\varepsilon}{T_c}\frac{\gamma}{2n}A_0^2 e^{nz}$ is larger than 1.

Example. In the case $f(t) = \exp(-\frac{t^2}{2T_c^2})$,

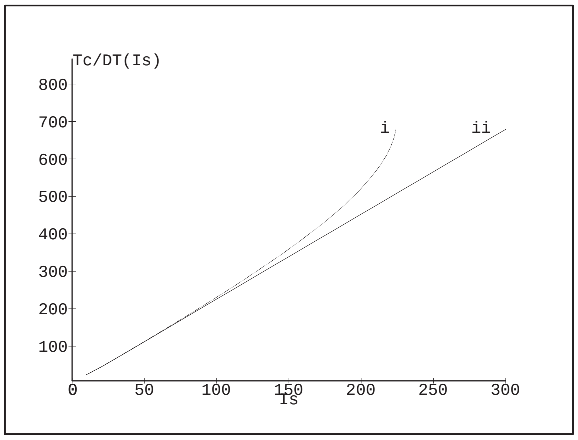

FIG. 6. *Coherence time* $DT_{\varepsilon,\gamma}$.
Curve i : $\frac{\gamma}{2n} = 0.1$, $\frac{\varepsilon}{T_c} = 0.01$, *Curve ii :* $\frac{\gamma}{2n} = 0.1$, $\frac{\varepsilon}{T_c} = 0$.

6 Appendix

We aim at sketching the calculations of the average intensity and the correlation function.

First consider the case $\varepsilon = 0$. By the expression (4) of the electric field $E_{\varepsilon=0,\gamma}$, the correlation function $K_{\varepsilon=0,\gamma}$ is :

$$K_{\varepsilon=0,\gamma}(T,z,A_0) = e^{nz} \times < E_0(u)E_0(u+T)^* \exp i\frac{\gamma}{2n}\alpha(nz)\left(|E_0(T+u)|^2 - |E_0(u)|^2\right) > .$$

Because of the stationarity of the process E_0 on the one hand and the decomposition of E_0 into the sum $X_1 + iX_2$ on the other hand, $K_{\varepsilon=0,\gamma}(T,z,A_0)$ can be rewritten as :

$$e^{nz} \times < (X_1(0)+iX_2(0))(X_1(T)-iX_2(T))\, e^{i\alpha(nz)\frac{\gamma}{2n}\left(X_1(T)^2+X_2(T)^2-X_1(0)^2-X_2(0)^2\right)} > .$$

However the processes X_1 and X_2 are independent and identically distributed, so we can factorize the expectation $< F(X_1) \times G(X_2) >=< F(X) > \times < G(X) >$, where X has the same distribution as X_i. As a consequence,

$$\begin{aligned} K_{\varepsilon=0,\gamma}(T,z,A_0) = 2e^{nz} \quad & \times < X(0)X(T)\exp i\frac{\gamma}{2n}\alpha(nz)\left(X(T)^2 - X(0)^2\right) > \\ & \times < \exp i\frac{\gamma}{2n}\alpha(nz)\left(X(T)^2 - X(0)^2\right) > . \end{aligned}$$

Calculating the brackets gives (6).

Consider from now on the case $\varepsilon \neq 0$, $\varepsilon \ll T_c$. Using the perturbed function method, we can compute the asymptotic expansion of the electric field E. We obtain that the average output intensity $I_{\varepsilon,\gamma}$ at position z is :

$$e^{nz} < a_0^2 > -\varepsilon^2 \left(\frac{\gamma}{2n}\right)^2 \beta(nz) e^{3nz} < a_0^4 {a_0'}^2 > -\varepsilon^2 nz e^{nz} \left(< {a_0'}^2 > + < a_0^2 {\phi_0'}^2 >\right) + O\left(\varepsilon^3\right).$$

After some calculations, you can check that (7) holds. We can do the same for the correlation function. However the computation is much more delicate. We find a lot of terms of order ε^2. The one which prevails is :

$$\varepsilon^2 e^{nz} (nz)^2 \alpha(nz)^2 \left(\frac{\gamma}{2n}\right)^2 B(T, z, A_0),$$

$$B(T, z, A_0) = < a_0(0)^2 a_0'(0) a_0(T)^2 a_0'(T) e^{i\left(\alpha(nz)\frac{\gamma}{2n}\left(a_0(T)^2 - a_0(0)^2\right) + \phi_0(0) - \phi_0(T)\right)} > .$$

(However it is necessary to compute all the terms in order to find the prevailing one). Decomposing the initial field $E_0(t) = a_0(t) e^{i\phi_0(t)}$ into the sum $X_1 + iX_2$, developing and factorizing the expectation by the independence of the processes X_1 and X_2, we have found that :

$$B(T, z, A_0) = \frac{2}{T_c^2} \times \frac{A_0^2 f(T) h(T)}{\left(1 + \left(A_0^2 \frac{\gamma}{2n}\right)^2 \alpha(nz)^2 \left(1 - f^2(T)\right)\right)^2} + \text{negligible terms},$$

where the 'negligible terms' represent a sum of terms of the type $\dfrac{A_0^2 \ldots}{\left(1 + \left(A_0^2 \frac{\gamma}{2n}\right)^2 \alpha(nz)^2 (1 - f^2(T))\right)^j}$, with $j \geq 3$.

About the further terms of order $O\left(\varepsilon^k\right)$, $k \geq 3$, by a recursive argument we can affirm that :

- The asymptotic expansion of the average intensity $I_{\varepsilon,\gamma}(z, A_0)$ can be written as :

$$e^{nz} A_0^2 \times \left(1 - \left(\frac{\varepsilon}{T_c}\right)^2 nz - \left(\frac{\varepsilon}{T_c} \frac{\gamma}{2n} A_0^2 e^{nz}\right)^2 \beta(nz) + \sum_{l=3}^{k} C_l(z, A_0) \left(\frac{\varepsilon}{T_c}\right)^l + O\left(\frac{\varepsilon}{T_c}\right)^{k+1}\right),$$

where the l-th corrective term $C_l(z, A_0)$ is at most of the type $\left(\frac{\gamma}{2n} A_0^2 e^{nz}\right)^l$.

- The correlation function can still be written as (except the 'negligible terms'):

$$K_{\varepsilon,\gamma}(T, z, A_0) \simeq \frac{e^{nz} A_0^2 f(T)}{\left(1 + \left(A_0^2 \frac{\gamma}{2n}\right)^2 \alpha(nz)^2 (1 - f^2(T))\right)^2} \times$$

$$\left(1 + \left(\frac{\varepsilon}{T_c}\right)^2 nz g(T) + \left(\frac{\varepsilon}{T_c} \frac{\gamma}{2n} A_0^2 e^{nz}\right)^2 \lambda(nz) h(T) + \sum_{l=3}^{k} D_l(T, z, A_0) \left(\frac{\varepsilon}{T_c}\right)^l + O\left(\frac{\varepsilon}{T_c}\right)^{k+1}\right)$$

where $D_l(T, z, A_0)$ is at most of the type $\left(\frac{\gamma}{2n} A_0^2 e^{nz}\right)^l \times \left(1 + (nz)^l\right) \times h_l(T)$.

Acknowledgement. This paper constitutes a part of a joint work in progress with Claude Gouédard (CEA Limeil) [2].

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