Modulational instability in optical fibers with polarization mode dispersion

F. Kh. Abdullaev
Physical-Technical Institute of the Uzbek Academy of Sciences,
G. Mavlyanov str. 2-b, 700084, Tashkent, Uzbekistan
J. Garnier
Centre de Mathématiques Appliquées, Ecole Polytechnique, 91128 Palaiseau Cedex, France
E. Seve and S. Wabnitz
Laboratoire de Physique de l'Université de Bourgogne,
avenue Alain Savary , B.P. 47 870, 21078 Dijon Cedex, France

Abstract

Random polarization mode dispersion leads to a substantial extension of the modulational instability domain in both the normal and anomalous dispersion regime of fibers.

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As is well known, the interplay between optical Kerr effect and chromatic dispersion leads to the phenomenon of modulational instability (MI) of light waves [1]. Such instability, also called Benjamin-Feir instability, occurs in different physical environments : plasmas, fluids, solid-state lattices, electrical circuits and nonlinear optics. MI leads to the breakup of a cw or quasi-cw beam into a train of ultrashort pulses and it can be used to generate a train of soliton-like pulses [2]. MI also sets a fundamental nonlinear limiting factor in the transmission of dense wavelength-division multiplexed signals in long-distance fiber links.

All these results were obtained by deterministic models. In realistic fiber transmission links, the chromatic dispersion, nonlinearity and birefringence are not constant but can fluctuate stochastically around a constant value. In this work we will study MI in fibers with random birefringence. In the model we adopted to treat random birefringence, the orientation of the principal axes is taken as constant but their magnitude varies randomly with distance. This simplified model has the advantage of leading to exact solutions for the linear stability analysis of plane waves. The evolution of the polarized fields in randomly birefringent fibers is ruled by a modified vector nonlinear Schrödinger system with a random group velocity mismatch or polarization mode dispersion (PMD) between the two modes [3-4]

$$iu_z + i\Delta(z)u_t + \beta u_{tt} + \left(|u|^2 + \alpha |v|^2\right) u = 0, \tag{1}$$

$$iv_z - i\Delta(z)v_t + \beta v_{tt} + \left(|v|^2 + \alpha |u|^2\right) v = 0. \tag{2}$$

Here standard dimensionless variables are used. The group velocity dispersion coefficient β is equal to 1 or -1, for the anomalous and normal dispersion regime, respectively. $\Delta(z)$ is the sum of a constant term Δ_0, and a white Gaussian-distributed noise

$$\langle \Delta(z) \rangle = \Delta_0, \quad \langle (\Delta(z) - \Delta_0)(\Delta(z') - \Delta_0) \rangle = 2\sigma^2 \delta(z - z'). \tag{3}$$

The standard linear stability analysis of MI consists in perturbing the stationary solution of the nonlinear Schrödinger equations [1]. In our notation, we write a perturbed plane wave as

$$u(z,t) = (A + u_1(z,t))e^{i\left(A^2+\alpha B^2\right)z}, v(z,t) = (B + v_1(z,t))e^{i\left(B^2+\alpha A^2\right)z}. \tag{4}$$

One obtains a linear system of equations for u_1 and v_1 ; using the complex representation, $u_1 = \bar{c}+i\bar{d}$, $v_1 = \bar{e} + i\bar{f}$ and performing the Fourier transform $c = \int \bar{c}e^{-i\omega t}d\omega$, we obtain a differential system $\frac{dq}{dz} = Q(z)q$ with $q = (c,d,e,f)^t$. This ODE system describes the evolution of the amplitude of the perturbation along the fiber. The eigenvalues of the associated matrix Q give the MI gain. Unlike the deterministic case, the perturbation matrix is no longer constant but varies randomly with distance. The study of the first moments of the perturbations is not sufficient to determine stability in the random case. Indeed, the MI gain for the average values of the perturbation coefficients is simply reduced, owing to the presence of random phases of the kind $\exp(\pm i\omega\sigma \int_0^z \Delta(y)dy)$ that multiply the coefficients c, d, e, and f, so that their expectation values decay exponentially along the fiber. It is therefore necessary to consider the moments of the moduli $|c|^2$, $|d|^2$, $|e|^2$ and $|c|^2$. By applying the Furutzu-Novikov formulae [5], one obtains a 16-dimensional differential system $\frac{dr}{dx} = Rr$ where r is a row vector whose elements are $\langle|c|^2\rangle$, $\langle|d|^2\rangle$, $\langle|e|^2\rangle$, $\langle|f|^2\rangle$, and the real and imaginary parts of $\langle c^*d\rangle$, $\langle c^*e\rangle$, $\langle c^*f\rangle$, $\langle d^*e\rangle$, $\langle d^*f\rangle$ and $\langle e^*f\rangle$, while R is a 16×16 dimensional matrix. Simple analytical formulae for the MI gain can be found for $\sigma \ll 1$ or $\sigma \gg 1$. In the general case, the MI gain can be evaluated by computing the eigenvalues of the stability matrix R through a numeric manipulation package. The left part of figure 1 shows MI gain curves for the anomalous dispersion ($\beta = 1$) case, different average PMDs and standard deviations σ. When the average PMD is zero, the MI region is enhanced so that all frequencies are unstable as soon as $\sigma^2 > 0$. Nevertheless, the MI peak gain is reduced with respect to the deterministic case. We find that the MI gain peak is equal to $2(1+\alpha)A^2$ when $\sigma = 0$, and it decays to $2A^2$ as σ goes to infinity.

Consider now the case $\Delta_0 > 0$. When $\Delta_0 < \alpha A^2$, MI is present for all sideband frequencies, but the peak gain is reduced and converges to $2|\omega|\sqrt{2A^2 - \omega^2}$ (which is the MI gain corresponding to the case $\alpha = 0$). Finally, if $\Delta_0 > \alpha A^2$ and $\sigma = 0$, the MI gain consists of a first peak at low frequencies and a second peak which lies close to Δ_0. When taking into account PMD fluctuations, the second peak is strongly reduced and ultimately it disappears for large σ, while the first peak converges to $2|\omega|\sqrt{2A^2 - \omega^2}$.

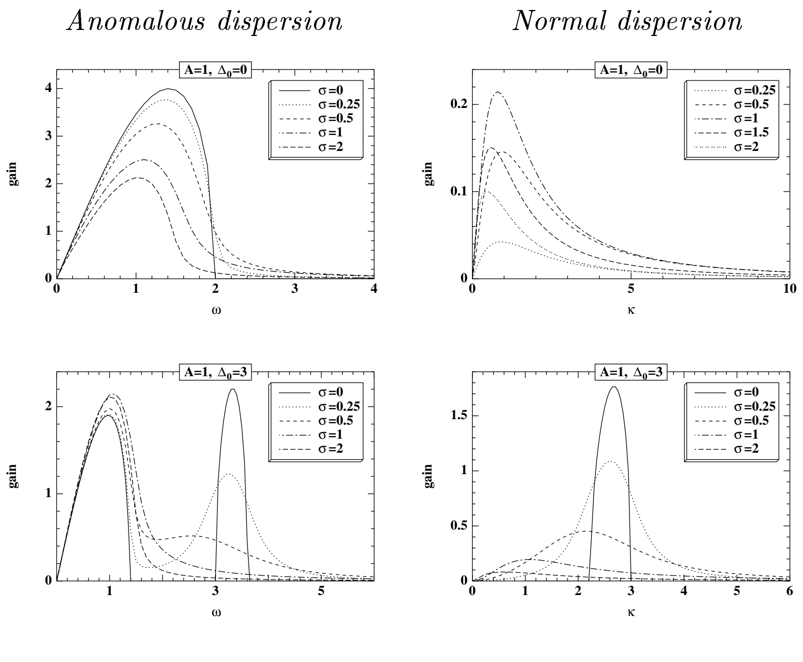

Figure 1: *MI gain vs frequency for anomalous ($\beta = 1$) and normal dispersion ($\beta = -1$), $\alpha = 1$ and different average PMD Δ_0 and standard deviation σ.*

The right part of figure 1 displays the gains in the normal dispersion ($\beta = -1$) case. Whenever the PMD between the fields is identically zero (i.e. $\Delta_0 = 0, \sigma = 0$), there is no MI in the normal dispersion [1]. Nevertheless, if the group velocities are randomly mismatched (i.e., $\sigma > 0$), one finds that MI is present, and even that all frequencies are unstable! With increasing σ, the MI peak gain first grows larger, then it reaches a maximum for some particular σ_0 which depends on α, but not on the power A^2. For $\alpha = 1$, one finds that $\sigma_0 = 0.85$. For deviations larger than σ_0, the gain decays to zero. When $\Delta_0 \neq 0$, deterministic MI occurs for frequencies between $\sqrt{\Delta_0^2 - 2(1+\alpha)A^2}$ and $\sqrt{\Delta_0^2 - 2(1-\alpha)A^2}$. Whenever the PMD fluctuations increase, all frequencies are unstable but the peak gain decays to zero. For large σ, one gets the same behavior as for $\Delta_0 = 0$, whatever Δ_0.

Above mentioned results were obtained by means of the linear stability analysis of Eqs(1-2). To check the validity of the results we performed the numerical experiments by directly solving the system (1-2) with a randomly varying PMD $\Delta(z)$. The simulations were done using the split-step Fourier method. The discrete value of the deviation σ is $\sigma_{dis} = \sigma/\sqrt{dz}$, where dz is the z step. The initial condition was $u_1, v_1 = \varepsilon \exp(-i\omega t) + \varepsilon \exp(i\omega t)$, with $\varepsilon = 10^{-4}$. The number of points in the time domain was 256 and the z step is a small fraction of the propagation length ($dz = L/200$). The value of Δ was randomly changed after each step since we have considered a white Gaussian-distributed noise.

The results of the numerical simulations in the *anomalous* and *normal* dispersion regimes are presented in the left and right part of Fig. 2, respectively. In both cases, the analytic stability analysis (solid curves) predicts the extension of the spectral width of the MI gain, which leads to instability for all frequencies of modulations. These predictions are well confirmed by the numerical simulations in all cases (stars) : note the quantitative agreement with the numerically calculated MI gain values.

The PMD induced extension of MI to a broad range of wavelengths around the pumps both in the normal and anomalous dispersion regime, that we discovered in this work, may have interesting implications to the stability of fiber transmissions, in particular when operating close to the zero dispersion wavelength or whenever dispersion compensation techniques are employed.

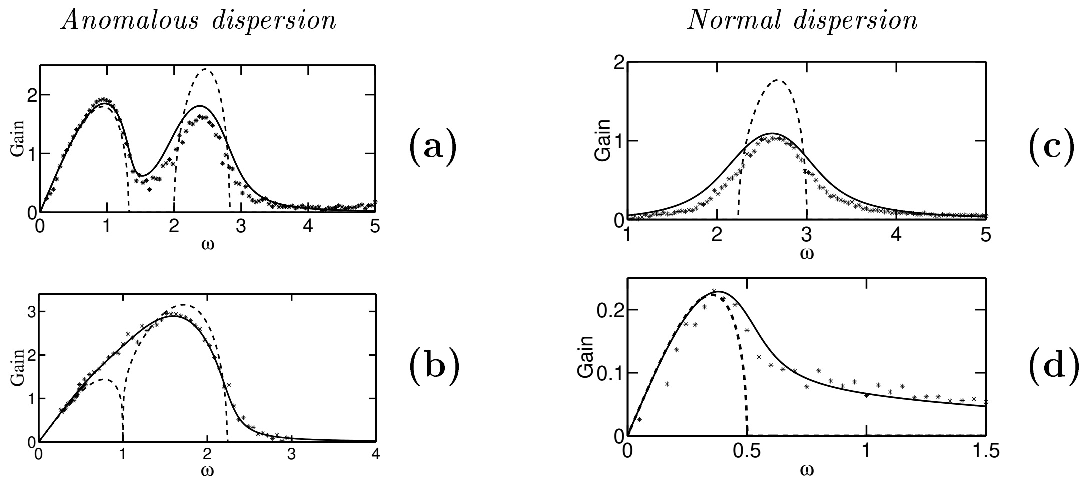

Figure 2: *Gain curves for $\beta = 1$ (a, b) and $\beta = -1$ (c, d). $A = 1$ and $\sigma = 0.25$ for both cases. Δ_0 2 (a), 1 (b), 3 (c) and 0.5 (d). The stars represents the MI gain obtained with the simulation of system (1-2), the solid curve is from linear stability analysis ; the dashed curve is the deterministic gain curve (i.e., with $\sigma = 0$).*

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