Deterministic Weak-and-Marked Petri Net Languages are Regular

Stéphane Gaubert

INRIA, Roquencourt

BP 105, 78153 Le Chesnay Cédex, France, email: Stephane.Gaubert@inria.fr

Alessandro Giua

Dip. di Ingegneria Elettrica ed Elettronica, Università di Cagliari,

Piazza d'Armi, 09123 Cagliari, Italy

Phone: +39-070-675-5892 – Fax: +39-070-675-5900 – Email: giua@diee.unica.it

Abstract

The intersection of the class of deterministic weak and the class of deterministic marked Petri net languages is the class of regular languages. We prove this result using a lemma that characterizes regular deterministic Petri net languages.

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1 Introduction

The extension of the supervisory control theory to Discrete Event Systems modeled by Petri nets (PN) leads to non trivial decision problems for PN languages [2, 5]. For instance, checking the controllability of a PN specification with respect to a PN behavior requires more or less testing the inclusion of PN languages, a well known undecidable problem for general PN languages. This naturally leads to the investigation of appropriate subclasses of PN specifications and behaviors in which these basic problems become decidable. To this end, the subclass $\mathcal{L}_d$ of deterministic marked Petri net languages and the subclass $\mathcal{G}_d$ of deterministic weak Petri net (PN) languages were used in [2]. It is known that the set of regular languages $\mathcal{R}$ satisfies $\mathcal{R} \subset \mathcal{G}_d \cap \mathcal{L}_d$. Moreover, the class $\mathcal{L}_d$ is incomparable with $\mathcal{G}_d$. It was conjectured in [2] that $\mathcal{R} = \mathcal{L}_d \cap \mathcal{G}_d$. We prove here that this is the case. Thus, $\mathcal{L}_d$ and $\mathcal{G}_d$ provide proper and distinct extensions of regular languages.

Let us first recall some notation (the reader is refered to [2] for more details). Let Σ denote a finite alphabet. A Σ-labeled PN is a 4-uple $G = (N, \ell, M_0, F)$, where: N is a PN (whose sets of transitions and places are denoted respectively by T and P); ℓ is a labeling function $T \to \Sigma$; $M_0 \in \mathbb{N}^P$ denotes the initial marking; $F \subset \mathbb{N}^P$ is a finite set of final markings. The labelling function ℓ is extended to a morphism $T^* \to \Sigma^*$ in the canonical way. The *marked behavior* $L_m(G)$ is the ℓ-image of the set of firing sequences leading to a final marking, namely:

$$L_m(G) = \{\ell(\sigma) \mid \sigma \in T^*, M_0[\sigma\rangle M, \text{ with } M \in F\} \ .$$

The *weak behavior* $L_w(G)$ is defined by taking as accepting set the covering set C_F, i.e.:

$$L_w(G) = \{\ell(\sigma) \mid \sigma \in T^*, M_0[\sigma\rangle M, \text{ with } M \in C_F\}$$

where

$$C_F \stackrel{\text{def}}{=} \{M' \in \mathbb{N}^P \mid \exists M'' \in F, M' \geq M''\} \ . \tag{1}$$

In more general terms given a (possibly infinite) set of accepting markings $\mathcal{F}$, we set $L(G, M, \mathcal{F}) = \{\ell(\sigma) \mid \sigma \in T^*, M[\sigma\rangle M', \text{ with } M' \in \mathcal{F}\}$. We note that $L_m(G)$ is obtained from $L(G, M_0, \mathcal{F})$ by the specialization $\mathcal{F} = F$, while $L_w(G)$ is obtained from $L(G, M_0, \mathcal{F})$ by the specialization $\mathcal{F} = C_F$ as defined by (1). The set of reachable markings starting from a marking M will be denoted by $R(N, M)$. We say that G is *deterministic* if the marking reached after firing a sequence is uniquely defined from the sequence label, i.e., if $M_0[\sigma\rangle M$, $M_0[\sigma'\rangle M'$, and $\ell(\sigma) = \ell(\sigma')$ implies $M = M'$.

2 Characterization of regular Petri net languages

We begin with a lemma of general interest which characterizes all classes of regular deterministic PN languages. This extends a result of Ginzburg and Yoeli ([1], Theorem 1) for free-labeled closed PN languages. The regularity of Petri net languages has also been discussed by Valk and Vidal [6]. This lemma should be seen as the transcription in terms of reachable markings of the well known Myhill-Nerode characterization of a regular language by the finiteness of its set of residuals [3]. Given a language L and a string $w \in \Sigma^*$, the *residual* of L with respect to w is the language $w^{-1}L = \{z \mid \; wz \in L\}$. The language L is regular iff the set of its residuals as w ranges over Σ is finite, i.e., iff the set $\{w^{-1}L \mid \; w \in \Sigma^*\}$ is finite.

Lemma 1. *Let $\mathcal{F}$ denote an arbitary set of accepting markings. If $\{L(G, M, \mathcal{F}) \mid \; M \in R(N, M_0)\}$ is finite, then $L(G, M_0, \mathcal{F})$ is regular. The converse holds when G is deterministic.*

Proof. The proof is based on the following obvious observation:

$$\forall w \in \Sigma^*, \;\; w^{-1}L(G, M_0, \mathcal{F}) = \bigcup_{\sigma \in T^*, \;\; \ell(\sigma)=w, \;\; M_0[\sigma\rangle M} L(G, M, \mathcal{F}) \;\; . \tag{2}$$

If there are only finitely many $L(G, M, \mathcal{F})$ for $M \in R(N, M_0)$, we get readily from (2) that there are finitely many $w^{-1}L(G, M_0, \mathcal{F})$ for all $w \in \Sigma^*$ (since (2) writes $w^{-1}L(G, M_0, \mathcal{F})$ as a finite union of distinct subsets). Thus $L(G, M_0, \mathcal{F})$ is regular. Conversely, let $M \in R(N, M_0)$, with $M_0[\sigma\rangle M$. Obviously,

$$L(G, M, \mathcal{F}) \subset \ell(\sigma)^{-1}L(G, M_0, \mathcal{F}) \tag{3}$$

We prove that the converse inclusion holds for deterministic nets. Indeed, let $w \in \ell(\sigma)^{-1}L(G, M_0, \mathcal{F})$. Then, there exist $\sigma', \sigma'' \in T^*$ such that $\ell(\sigma') = \ell(\sigma)$, $\ell(\sigma'') = w$ and $M_0[\sigma'\rangle M'[\sigma''\rangle M'' \in \mathcal{F}$. Since G is deterministic, $M = M'$, hence $M[\sigma''\rangle M'' \in \mathcal{F}$, and thus $w = \ell(\sigma'') \in L(G, M, \mathcal{F})$. This shows the equality in (3), and implies that there are finitely many $L(G, M, \mathcal{F})$ as $M \in R(N, M_0)$. $\square$

We show how the converse of the lemma depends on the determinism of G with an example.

Example 1. Let G be the nondeterministic labeled net in Figure 1, with initial marking $M_0 = (0)$ and set of final markings $\mathcal{F} = \{(0)\}$. The set of reachable markings of this net is $R(N, M_0) = \mathbb{N}$. The language accepted starting from $M_i = (i)$ is $L(G, M_i, \mathcal{F}) = \{a^{i+2j} \mid \;\; j \geq 0\}$. Hence the set $\{L(G, M, \mathcal{F}) \mid \;\; M \in R(N, M_0)\}$ is infinite, while the language $L(G, M_0, \mathcal{F}) = (a^2)^*$ is regular.

3 Main result

We can then state the main result of this note.

Theorem 1. $\mathcal{R} = \mathcal{G}_d \cap \mathcal{L}_d$.

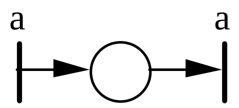

Figure 1: Non deterministic labeled net in Example 1.

Proof. The other inclusion being known [2], we prove that $\mathcal{L}_d \cap \mathcal{G}_d \subset \mathcal{R}$, by contradiction.

Let $G_1 = (N_1, \ell_1, M_{0,1}, F_1)$ and $G_2 = (N_2, \ell_2, M_{0,2}, F_2)$ be two deterministic labeled nets such that $L_w(G_1) = L_m(G_2)$ and assume that $L_w(G_1)$ is not regular. By Lemma 1, there must exist an infinite set of markings $\mathcal{M} \subset R(N_1, M_{0,1})$ such that for all $M, M' \in \mathcal{M}$, $M \neq M' \Rightarrow \; L(G_1, M, C_{F_1}) \neq L(G_1, M', C_{F_1})$. From this infinite set we can extract a (strictly) increasing infinite sequence $M_1, M_2, \ldots$ (see, e.g., [4, Theorem 2.5]). Hence $L(G_1, M_i, C_{F_1}) \subsetneq L(G_1, M_{i+1}, C_{F_1})$, for all i.

For each marking M_i, let σ_i be a firing sequence such that $M_{0,1}[\sigma_i\rangle M_i$, and let τ be a firing sequence such that $M_1[\tau\rangle M_f \in C_{F_1}$. Now, let us consider the net G_2. There exists a firing sequence σ'_i firable from $M_{0,2}$ and such that $\ell_2(\sigma'_i) = \ell_1(\sigma_i)$ for all i. Let $M'_i \in R(N_2, M_{0,2})$ be such that $M_{0,2}[\sigma'_i\rangle M'_i$. It follows from the equality in (3) for deterministic nets that $L(G_2, M'_i, F_2) = L(G_1, M_i, C_{F_1})$, hence $M'_i \neq M'_j$ for $i \neq j$. There must exist a firing sequence $\sigma'_{f,i}$ with $\ell_2(\sigma'_{f,i}) = \ell_1(\tau)$ such that $M'_i[\sigma'_{f,i}\rangle M'_{f,i}$ and $M'_{f,i} \in F_2$ for all i. Since ℓ_2 is non erasing, the length of $\sigma'_{f,i}$ is fixed. Thus, there are finitely many such $\sigma'_{f,i}$, and $M'_{f,i}$ differs from M'_i from a bounded quantity. Hence, being the set of all M'_i infinite, the set of all $M'_{f,i}$ must be infinite as well. This contradicts the hypothesis that the set of final markings F_2 of G_2 be finite. $\square$

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