Journées thématiques du GdR Automatique, groupe de travail

ALGÈBRES TROPICALES ...

et applications aux systèmes à événements discrets et à la commande optimale

(GdR "ALP" et "Automatique")

communes au Réseau EC-TMR

ALAPEDES

(The ALgebraic Approach to Performance Evaluation of Discrete Event Systems)

Du Lundi 29 au Mercredi 31 Mars 1999

- Salle Dussane, École Normale Supérieure, 45 rue d'Ulm, Paris (les lundi et mardi)
- Salle de Conférence, 46 rue d'Ulm, Paris (le mercredi).

http://amadeus.inria.fr/TROPICAL/
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Lundi 29 Mars. Exposés tropicaux.

9h00	Accueil des participants	
9h30	Claude Dellacherie	Théorie du potentiel élémentaire non linéaire
11h00	Pause	
11h30	Gérard Duchamp	Un critère de rationalité provenant de la géométrie non-commutative
12h30	Déjeuner	
14h30	Christian Commault	Commande des systèmes structurés : approche graphique.
16h00	Pause	
16h30	Stéphane Gaubert et Jean-Jacques Loiseau	Un bilan de trois ans d'activité tropicale
18h00	Table ronde	
19h00	Cocktail	

Mardi 30 Mars, Matin. Exposés tropicaux

9h00	Thomas Bonald	Effet du trafic transverse sur les performances de TCP
10h00	Marc Artzrouni	Distances projectives, coefficients d'ergodicité, et processus itératifs linéaires et "pseudo-linéaires" dans le cone positif de R^n
11h00	Pause	
11h30	Vassili Kolokoltsov	Deterministic approach to option pricing theory
12h30	Déjeuner	

Mardi 30 Mars, après midi. Exposés scientifiques du réseau ALAPEDES

14h00	Alain Jean-Marie	New bounds for Lyapounov exponents of stochastic (max,+) matrices
14h25	Bruno Gaujal	Analysis of preemptive periodic real time systems
14h50	Robert-Jan van Egmond	Railway capacity assessment, an algebraic approach
15h15	Jacob van der Woude	Conditions for the structural existence of an eigenvalue of a bipartite (min,max,+)-system
15h40	Jean-Pierre Quadrat	Max-Plus objects in Scilab and application to evaluation and optimisation of production systems.
16h00	Break	
16h25	Stephan Haar	Choosing a Semantics for Petri Nets
16h45	Sabrina Mantaci	Tree codes and the defect theorems for trees
17h10	James Martin	Fast Jackson Networks

Mercredi 31 Mars, matin. Exposés scientifiques du réseau ALAPEDES

9h00	Natacha Portier	The minimal realization problem in max-plus algebra is NP-hard
9h25	Sam Lifsches	Hybrid Systems and Monadic Logic
9h50	Eleni Katirtzoglou	Titre non parvenu
10h15	Bernd Heidergott	The Maclaurin Series for (Max,+) Lyapunov Exponents: The Bernoulli Case

Cette réunion scientifique ALAPEDES, ouverte à tous, sera suivie par une réunion d'évaluation, cette fois ci non-publique.

RÉSUMÉS / ABSTRACTS

Théorie du potentiel élémentaire non linéaire

Claude Dellacherie. Laboratoire AMS, Université de Rouen, 76821 Mont Saint Aignan Cédex.

Résume. Soit L l'espace des fonctions réelles bornées sur un ensemble E, muni de la norme uniforme, et P une contraction croissante de L dans L. Posant $A = I - P$, ou I est l'identité, on montrera qu'un système d'inegalités du type

$$v \geq u, A(v) \geq f \quad \text{pour}\, u, f \text{ donnés dans } L$$

admet, sous des conditions très générales, une plus petite solution, et que cela mène aux notions fondamentales de la théorie du potentiel : réduite, potentiel, principe du maximum, solution du problème de Dirichlet, etc. On insistera sur les deux propriétés essentielles de l'opérateur A :

1) (derivation) pour u, v dans L, si on a $u \leq v$ partout et $u(x) = v(x)$ en un point x, alors on a $A(u)(x) \geq A(v)(x)$

2) (production) pour tout u dans L et toute constante $c \geq 0$ on a $A(u + c) \geq Au$

et on terminera par quelques mots sur la théorie non élémentaire pour laquelle les deux propriétés précédentes sont les axiomes de départ.

Un critère de rationalité provenant de la géométrie non-commutative

Gérard Duchamp. LIFAR, Universite de Rouen, 76134 Mont-Saint-Aignan Cedex France.

Résumé. Dans cet exposé, on résout une conjecture qu'Alain Connes a publié dans son livre "Non Commutative Geometry" en utilisant des techniques de théorie des automates à multiplicités (commutatives et noncommutatives). Il s'agit d'etablir la rationalité de certains opérateurs dans des espaces de Hilbert de fonctions L^2 sur le groupe libre. La machinerie peut aussi être mise en place sans norme et fournit alors un critère de rationalité pour les séries formelles à coefficients dans un corps quelconque, ce dernier sert à redémontrer le résultat de G. Cauchon (TCS ,98 - 1992-) sur l'identité entre les ensembles de séries de Malcev-Neumann rationnelles à monomes positifs et les séries reconnaissables par automate.

Abstract. In this talk, we solve a conjecture whhich was publisheed by Alain Connes in his bbook "Non Commutative Geometry", using techniques from the theory of automata with multiplicities (commutative and noncommutative). The aim is to show the rationality of some operators on Hilbert spaces of L^2 functions over the free group. The machinery can also be set without norm and then gives a rationality criterium for formal series with coefficients in any field which allows to show in another way the result of G.Cauchon (TCS ,98 - 1992-) on the equality of the set of rational series with "positive" monomials and automaton-recognizable series.

Commande des systèmes structurés : approche graphique.

Christian Commault. Laboratoire d'Automatique de Grenoble, ENSIEG, BP 46, 38402 Saint-Martin d'Heres.

Résume. Dans cet exposé on présente la définition des systèmes structurés, leur intérêt et le graphe qui leur est associé de manière naturelle. On montrera que de nombreuses propriétés génériques des systèmes structurés peuvent se caracteriser très simplement sur le graphe associé. En particulier les conditions de solubilité de nombreux problemes de commande (rejet de perturbations, découplage, ...) se ramènent à l'étude de plus courts chemins entrée-sortie à sommets disjoints sur ce graphe.

Effet du trafic transverse sur les performances de TCP

Thomas Bonald, INRIA - Projet Mistral, 2004 route des Lucioles - BP 93, 06902 Sophia-Antipolis Cedex.

Résume. Nous nous intéressons à l'effet du trafic transverse sur le mécanisme de contrôle de flux à fentre, utilisé par exemple par le protocole TCP dans l'Internet. L'approche consiste à étudier la stabilité d'un système à événements discrets, o chaque routeur traversé par les paquets d'une connexion TCP est représenté par une file d'attente FIFO, recevant par ailleurs un flux exogène stationnaire et ergodique. Nous montrons tout d'abord l'existence d'un débit maximum

autorisé par le mécanisme de contrôle de flux. Puis nous établissons des bornes sur la valeur de ce débit maximum. Ces bornes ne concident pas en général, et sont atteintes par des changements d'échelle en temps et en espace des flux exogènes. Ainsi, les performances du contrôle de flux à fentre ne dépendent pas uniquement de l'intensité de trafic des flux transverses, mais aussi de caractéristiques statistiques plus fines telles que la variabilité de ces flux. Ces résultats sont illustrés par plusieurs exemples, parmi lesquels celui d'une région de stabilité non monotone, non convexe et fractale.

Mots-clés : Contrôle de flux à fentre, TCP, stabilité, réseaux multi-classes, processus ponctuels stationnaires ergodiques, système (max,+)-linéaire.

Deterministic approach to option pricing theory

Vassili Kolokoltsov, Nothingham Trent University, UK.

Abstract. Unlike the original stochastic analysis method of deduction the famous Black-Sholes and Cox-Ross-Rubinstein formulas for option pricing, a deterministic approach, on the one hand, dwells to a more elementary foundation, and on the other hand, leads to a different type of generalisations of BS and CRR formulas characterised by more rough assumptions on the underlying common stocks evolution. This approach also provides a clear connection of option pricing theory with the theory of nonexpansive homogeneous maps, and thus with nonlinear idempotency.

Distances projectives, coefficients d'ergodicité, et processus itératifs linéaires et "pseudo-linéaires" dans le cone positif de R^n

Marc Artzrouni, Département de Mathématiques Appliquées, of Applied Mathematics
Universite de Pau et des Pays de l'Adour (IPRA), 64000 PAU.

Résume. On introduit la distance projective dans le cône positif de R^n. Cette distance est une pseudo-métrique car elle est nulle si et seulement deux vecteurs sont colinéaires. Cette distance permet de définir le coefficient d'ergodicité $\tau(M)$ d'une matrice positive M; $\tau(M)$ est un coefficient de contraction de M, considéré comme un opérateur linéaire dans le cône (pour la distance projective). Un coefficient d'ergodicité local est introduit qui est utile dans l'étude de la propriété contractante de M au voisinage d'un vecteur propre positif. Ces notions sont utilisées dans l'étude du processus linéaire nonhomogène $Y(k) = A(k)Y(k-1)$ dans le cône positif de R^n avec une suite de matrices $A(k)$ irréductibles. Plus les vecteurs propres de Perron $V(p)$ des matrices $A(p)$ ont varié lentement (pour la distance projective) dans le passé récent précédent k, plus $Y(k)$ sera proche de $V(k)$ (pour la distance projective). Dans un processus itératif pseudo-linéaire la matrice $A(k)$ est remplacée par une fonction matricielle $A[Y(k-1)]$ du vecteur Y(k-1) qui exprime un "feedback" de $Y(k-1)$ sur la matrice qui transformera $Y(k-1)$ en $Y(k)$. Lorsque cette fonction matricielle est homogène de degré 0 (cas de figure courant en dynamique des populations et en économie) un résultat nouveau de stabilité locale sera donné.

New bounds for Lyapounov exponents of stochastic (max,+) matrices

Alain Jean-Marie, INRIA - Projet Mistral, 2004 route des Lucioles - BP 93, 06902 Sophia-Antipolis Cedex.

Résume. We consider the stochastic (max,+) recurrence $X(n) = A(n) \otimes X(n-1)$ in the case where the sequence $\{A(n)\}$ is i.i.d. and multinomial. Based on a decomposition of products of matrices in blocks of a certain form, we propose upper and lower bounds for $\mathrm{E}X(n)$ and the Lyapunov exponent of the system. The basic computational complexity of the computation is $\mathcal{O}(N^3)$ (N is the size of the matrix) but the accuracy can be improved at will at the expense of larger computation times.

Analysis of preemptive periodic real time systems

Bruno Gaujal, INRIA, technopole de Nancy-Brabois, 615 route du jardin botanique, BP 101, F-54606 Villers-les-Nancy.

Abstract. We present the modeling of on several tasks with preemption performed by a robot, using Marked Graphs. Then, with the help of the (max,+) representation, we derive simple tests to check real time constraints on those tasks.

Railway capacity assessment, an algebraic approach

Robert-Jan van Egmond, Delft University of Technology.

Abstract. The evaluation of the capacity of railway systems is of key importance for planning the future infrastructure. Railway capacity can be defined as the maximum frequency of a certain pattern of train movements for which delays do not propagate too much. A schedule of train movements can be modelled with the aid of a maxplus linear jobshop (also known as "heaps of pieces model"). The analysis of such a system leads to the minimum cycletime of the schedule and its critical part. Also the amount of delay propagation can obtained from this model. Next, one should add buffers to the schedule in order to get the amount of delay propagation below a predetermined upper bound. The cycletime of the schedule including the buffertimes is then a good measure for the capacity. To asses the value of this approach, a case study has been carried out on the station The Hague HS (which lies in the western part of the Netherlands).

Conditions for the structural existence of an eigenvalue of a bipartite $(\min, \max, +)$-system

Jacob van der Woude (joint work with Subiono). Delft University of Technology.

Abstract. In this talk we consider bipartite $(\min, \max, +)$-systems. We present conditions for the structural existence of an eigenvalue and corresponding eigenvector for such systems, where both the eigenvalue and eigenvector are supposed to be finite. The conditions are stated in terms of the system matrices that describe a bipartite $(\min, \max, +)$-system. Structural in the previous means that not so much the numerical values of the finite entries in the system matrices are important, rather than their locations within these matrices. The conditions presented in this talk can be seen as a version for bipartite $(\min, \max, +)$-systems of known conditions for the structural existence of an eigenvalue of a $(\max, +)$-system involving the (ir)reducibility of the associated system matrix.

References

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- G. J. Olsder, On structural properties of min-max systems, TWI report 93-95, Faculty of Technical Mathematics and Informmatices, 1993.

Max-Plus objects in Scilab and application to evaluation and optimisation of production systems.

Jean-Pierre Quadrat, INRIA, Domaine de Voluceau, BP105, 78153 Le Chesnay Cedex. (joint work with Guy Cohen and Stéphane Gaubert).

Abstract. We will discuss the state of the art of the implementation of the max-plus arithmetic and the max-plus linear system manipulations in Scilab. We will make an illustration of the power of the syntax and the graphic possibilities by presenting on a concrete example a very preliminary version of a future production system toolbox.

Choosing a Semantics for Petri Nets

Stefan Haar, ENSEM - INPL , Equipe TRIO, 2, Avenue de la fort de Haye, 54516 - Vandoeuvre les Nancy

Abstract. The semantics of a (Petri) net - or any concurrent system - is the space of its behaviors; differences arise according to the way that behaviour is understood and encoded. into two categories: unfolding oriented semantics (such as processes, Vogler's executions or Engelfriet's branching processes) represent *local* states and events, and path oriented semantics (comprising among others traces and event structures) that register transitions among global states. The theory for e.g. model checking logical properties on such structures is rich and growing; for quantitative results, as wanted for performance evaluation, more work needs to be done. Placiing the theories of heaps and of routing functions - both of which have been used for PN models of DEDS - into this context, we discuss several issues for DEDS: which semantic equivalences can be used for reductions, how can the path and unfolding viewpoints be combined, which semantics allow best to study performance properties for timed and stochastic systems, etc.

Tree codes and the defect theorems for trees

Sabrina Mantaci, LIAFA.

Abstract. My research background at the time I joined ALAPEDES (January 1999) was mainly in the field of combinatorics on words and language theory. More in details the research I developed in order to relate my Ph.D thesis and in the following period, was devoted to the generalization to k-ary node-labeled trees of notions, properties and results of the combinatorial theory of words. In particular problems like extending the notion of "code", "equation", "periodicity", together with the connected theories, have been successfully studied. Under this point of view, many of the theorems of combinatorics of words can be seen like a special case of theorems in combinatorics on

trees, but, in the same time, some properties do not have a corresponding property for trees, due to their branching and asymmetric top-down/bottom-up structure. As a concrete example, here we consider the notion of tree code and the defect theorem, and their extention to the tree case. The defect theorem is one of the fundamental results of the combinatorial theory of words. It states that if a set X of words of cardinality n satisfies a nontrivial relation, then its elements can be expressed as a catenation of the words in another set Y having cardinality at most $n-1$. Actually there does not exist just one, but several results which formalize the above defect effect depending on the properties we require on Y (i.e., for instance, to be a code, a prefix/suffix code). We look at which defect theorems on words can be extended to trees. It turns out that in many cases this is possible. However, interestingly, the proofs are not just translations, but contain several different features compared to those for words. Moreover, due to the above differences, some results cannot be extended.

I just began to investigate some specific problems in the field of Discrete Event Systems. $(max, +)$-automata on words have been studied a lot, they are an important modelling tool for Discrete Event Systems. It seems interesting to study their counterpart for trees, that is $(max, +)$-tree automata.

Fast Jackson Networks

James Martin, DMI, ENS.

Abstract. We consider a generalisation of a traditional Jackson queueing network in which each node contains many servers, and in which a restricted "join the shorter queue" policy applies at each node. The model is of an open Markovian network, each of whose nodes consists of N identical channels, each with an infinite buffer and a single server. Upon arriving at a node, a customer selects m of the N channels at random and joins the one with the shortest queue. We analyse the behaviour of the system as N -¿ infinity, and show that the process representing the evolution of the system converges to a limiting trajectory which corresponds to the solution of a countably infinite system of ODEs. Under a standard non-overload condition, the system has a unique invariant distribution for each N, and these distributions converge to a limit corresponding to the unique fixed point of the system of ODEs. In the limit, the tail of the distribution of queue lengths decays super-exponentially, rather than exponentially as in the case of a standard Jackson network. If time permits, I might also mention some results concerning the point processes of arrivals and departures at a queue in such a system, concerning stochastic bounds for the queue lengths in the network in terms of systems of non-interacting nodes of a similar kind, and concerning, and concerning models in which the assumption of "complete exchangeability" of the servers is relaxed.

The minimal realization problem in max-plus algebra is NP-hard

Natacha Portier, Section de mathématique, Université de Liège. B37, B-4000 Liege, Belgique.

L. J. Meyer and A. R. Stockmeyer proved in 1973 that deciding if a rational expression over a unary alphabet is defining the all language is NP-hard. A consequence of this theorem is the NP-hardness of the following problems: Are there paths of all lengths between two given vertices of a finite directed graph? Are two one-letter rational series in a max-plus algebra different? Is a one-letter rational series in a max-plus algebra equivalent to a rational series of dimension 1? From this last result we infer that the minimal realization problem in max-plus algebra is NP-hard.

Hybrid Systems and Monadic Logic

Sami Lifsches, University of Groningen, Department of Mathematics and Computing Science.

Abstract. We will discuss the connections between Hybrid Systems, Tree Automata and Monadic second-order Logic. Our main result is that a Hybrid System is controllable by a tree automaton if and only if it is definable in the monadic theory of the binary tree. A system that is not controllable may become one if ”looking ahead” is allowed. In that case, the system can be approximated by a definable, controllable system.

The Maclaurin Series for (Max,+) Lyapunov Exponents: The Bernoulli Case

Bernd Heidergott, Delft University of Technology.

Abstract. We obtain the Maclaurin series for the Lyapunov exponent $\lambda(\theta)$ of a sequence of independently and identically distributed random matrices over the $(\max, +)$ semiring, generated via a Bernoulli θ scheme. The key assumption is that one of the possible outcomes of the random matrices has an eigenspace of dimension one. We apply the theory of weak differentiation to show that $\lambda(\theta)$ is analytical on $[0, 1)$. We show that the Maclaurin series converges to $\lambda(\theta)$ on $[0, p]$ for any $p < 1$. This result holds for the transient case as well. We will provide an intuitive explanation why we cannot expect the Maclaurin series to converge on the whole unit interval.