Waves and imaging: Concepts, Theory and Applications

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General

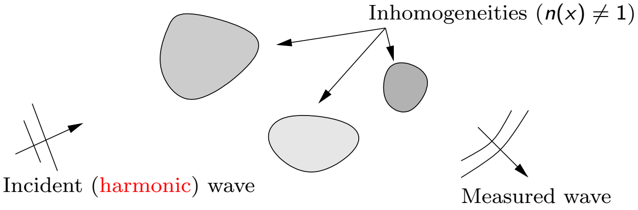
This course addresses some theoretical and numerical questions related to the resolution of inverse scattering problems...

We do it on a simple model problem: acoustic scattering from inhomogeneous medium.

Related (more complicated) models: Electromagnetic scattering (Maxwell's equations), Elastodynamic (Navier's equations),

Related applications: tomography, non destructive testing, medical imaging, radar and sonar, geophysics, etc...

Model problem for inverse (acoustic) scattering



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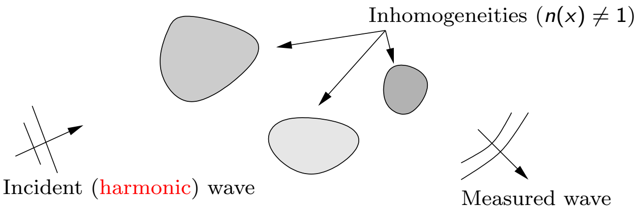

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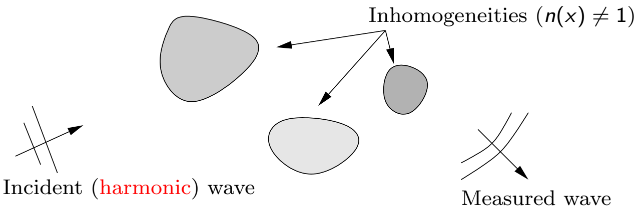

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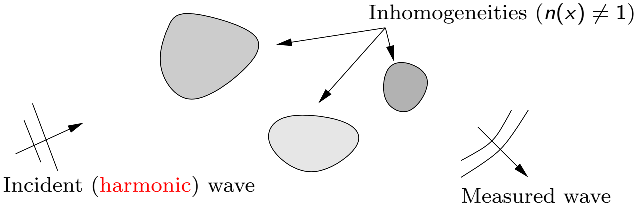

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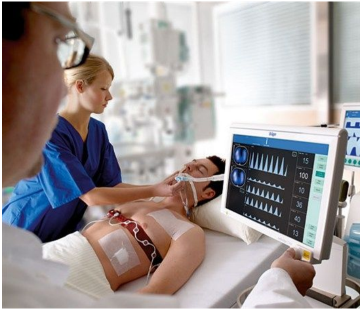
Both are non-linear and ill-posed problems...

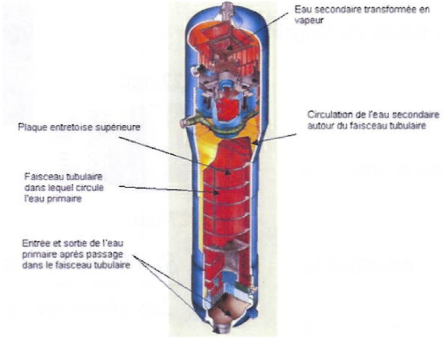
Example of applications: medical imaging

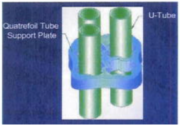
Possible applications include monitoring of lung function, detection of cancer in the skin and breast and location of epileptic foci.

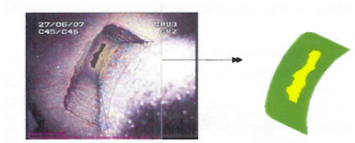


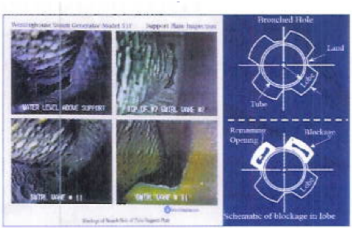
Source: `http://www.medicalexpo.com/`

Example of applications: Non destructive testing of nuclear fuel rods



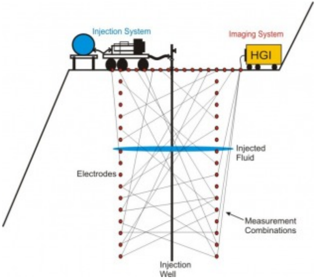


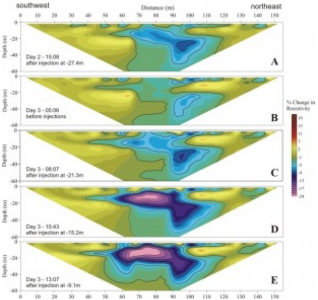




Example of applications: geophysics

Electrical resistivity tomography: use electrodes on the surface of the earth or in bore holes to locate resistivity anomalies.





Source: `http://dalerucker.com/heap-monitoring.html`

Program

1. Basic materials to study of scattering problems at fixed frequency (Radiation Condition, Fredholm theory, The Rellich lemma and unique continuation principle).
2. Discussion of some linearization approaches: Born approximation, Time reversal and Synthetic Aperture Radar principles

   Notion of ill-posed problems and some rudiments on regularization theory
3. Uniqueness in the framework of geometrical optic solutions.
4. Presentation and analysis of some nonlinear inversion algorithms (Landweber and Gauss-Newton).
5. Presentation and analysis of so-called sampling methods to solve the geometrical inverse problem (Linear Sampling Method, Factorization Method).

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Reference: *Inverse Scattering Theory and Transmission Eigenvalues*, F. Cakoni, D. Colton, and H. Haddar, SIAM publications, 88, 2016, CBMS Series.