Inverse problems: mathematical analysis and numerical algorithms

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This course adresses several mathematical and numerical questions related to the resolution of inverse problems arising in various fields: tomography, non destructive testing, imaging,...

The main feature of inverse problems is the loss of continuity of the solution with respect to the data. A first part of the course is then dedicated to the theory of regularization of (linear) ill-posed problems. We second specifically analyze the case of ill-posed Cauchy problems. In particular, the questions of unique continuation and stability will be treated. The remaining part of the course concerns non linear inverse problems, like for example the identification of a geometry or some physical properties (of a defect, an inclusion, a reservoir,...) from the knowledge of (surface) measurements. In particular, we analyze two families of numerical methods for solving such problems: non linear optimization methods and sampling methods (factorization method).

The course is divided into 12 lectures of 2 hours each, and covers:

- Inverse problems: prototypes and examples
- Regularization of linear ill-posed problems
- Ill-posed Cauchy problems: unique continuation, stability and algorithms
- Non linear inverse problems and optimization methods
- The factorization method and application to impedance tomography