7^{th} **World Congresses of Structural and Multidisciplinary Optimization**
COEX Seoul, 21 May - 25 May 2007, Korea.

Optimal Design of Compliant Mechanisms by Level Set and Flexible Building Blocks Methods

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1 Abstract

This paper presents and compares two methods for optimal design of compliant mechanisms. The first one is the level set method, based on the classical shape derivative and the level set representation of the shapes. The shape derivative is computed by an adjoint method, and the level set representation leads to very efficient numerical algorithms. The second one is the flexible building blocks method, which has been developed at the French Atomic Energy Commission (CEA). This method has been implemented for planar mechanisms in a software called *FlexIn* (Flexible Innovation). It uses an evolutionary algorithm approach to optimize a truss-like structure made of an assembly of basic building blocks chosen in a given library. Both methods are briefly described. Then, they are illustrated by several examples of optimal synthesis of two-dimensional monolithic compliant mechanisms, like force inverters and micro-grippers. Finally, considering the respective advantages of both optimization methods, we propose a coupling strategy of these two techniques. After testing it on a force inverter example, we conclude on the feasibility of an improved synthesis method for the design of compliant mechanisms.

2 Keywords:

Shape and topology optimization; Compliant mechanisms; Level-set method; Genetic algorithm; Building blocks.

3 Introduction

Compliant mechanisms are single-body, elastic continua flexible structures that deliver the desired motion by undergoing elastic deformation as opposed to jointed rigid body motions of conventional mechanisms. The main advantages of compliant mechanisms are: simplified manufacturing, reduced assembly costs, no wear, no backlash, reduced kinematic noise and ability to accommodate unconventional actuation schemes. The compliant mechanisms have already been used in many applications including product design, MEMS, adaptive structures, surgical tools, etc (see e.g. [6] [7][16][17]). This paper compares two numerical methods for the design of compliant mechanisms.

Two approaches known in the literature for the systematic synthesis of compliant mechanisms are the *kinematics synthesis approach* and the *continuum synthesis approach*. The first approach, known as flexure-based synthesis approach, represents and synthesizes compliant mechanisms using a rigid-body kinematics with flexible joints, and uses pseudo-rigid-body model (Howell and Midha [16][17]). The *continuum synthesis approach*, based on the topology optimization method of continuum structures (Ananthasuresh *et al.* [6], Nishiwaki *et al.* [20], Sigmund [28]), focuses on the determination of the topology, shape and size. The methods based on this approach can be subdivided into, for example, the homogenization method and its variants [1][5] [8][9][20][28][15][26], the flexible building blocks method [10][12][19] and the level set method [2][3][4][29][30][31][32].

In a first part, the two numerical methods for optimal design of monolithic compliant mechanisms are briefly described. The first method is the level set approach, originally developed by Osher and Sethian for numerically tracking fronts and free boundaries [22]. Recently, it has been introduced in the field of shape optimization (Sethian and Osher [22][27], Allaire *et al.* [2][3][4], Wang *et al.* [29][30][32]), where it is based on a combination of the classical shape derivative and the Osher-Sethian level set algorithm for front propagation. In this method, the shape derivative is computed by an adjoint method. The second one is the flexible building blocks method, implemented in FlexIn, which has been developed by Bernardoni *et al.* (see e.g. [10][12]). It considers a compliant mechanism as an assembly of compliant building blocks, and a multi-objective genetic algorithm is used to optimize the blocks assembly. In order to validate this method, an experimental toolbox has been developed with MATLAB^{TM}, at the French Atomic Energy Commission (CEA).

This paper is organized as follows: First, we will briefly review the underlying idea of the level set method for topology optimization of compliant mechanisms. Secondly, we present the FlexIn methodology for the design of compliant mechanisms. In Section 4 we present some numerical examples of designs, to demonstrate the respective interests of both methods. Section 8 is devoted to some comparisons and concluding remarks.

4 Level set method for optimal synthesis of compliant mechanisms

In this section we present the mathematical formulation of the level set method for the problem of compliant mechanism synthesis. Our method has three main components: shape representation by level set, governing equations of the mechanism structure, and computation of the shape gradient (i.e. "derivative" of the objective-function with respect to shape variations). The results of this section were announced in Allaire, Jouve and Toader [2][3][4]. For a general and comprehensive discussion, we refer to [3].

4.1 Setting of the problem

Let $\Omega \subset \mathbb{R}^d$ ($d = 2$ or 3) be a bounded open set occupied by a linear isotropic elastic material with Hooke's law A. The boundary of Ω is made of two disjoint parts $\partial\Omega = \Gamma_N \cup \Gamma_D$, with Dirichlet boundary conditions on Γ_D, and Neumann boundary conditions on Γ_N. All admissible shapes Ω are required to be a subset of a working domain $D \subset \mathbb{R}^d$. We denote by f the vector-valued function of the volume forces and by g that of the surface loads. The displacement field u in Ω is the solution of the linearized elasticity system

$$\begin{cases} -\text{div }(Ae(u)) & = & f & \text{in } \Omega, \\ u & = & 0 & \text{on } \Gamma_D, \\ (Ae(u))n & = & g & \text{on } \Gamma_N, \end{cases} \tag{1}$$

where $e(u) = \frac{1}{2}(\nabla u + \nabla u^T)$ is the strain tensor.

Remark that the present study is restricted to the linearized elasticity case, while the level set method can easily be used in a nonlinear framework (e.g. large strains and large displacements) that could be valuable for compliant mechanism designs [3].

The shape optimization problem is formulated as a minimization problem

$$\inf_{\Omega \text{ admissible}} J(\Omega), \tag{2}$$

where $J(\Omega)$ is an objective function chosen to evaluate the performance of the mechanism. In the context of compliant mechanism optimization, many fitness functions are useful, that lead to different optimal designs. The most used in the literature are available in our numerical implementation of the level set method: mechanical advantage (MA), geometrical advantage (GA), work ratio (WR) (mechanical efficiency ME) and a least square error criterion (for details, see [3]).

Figure 1 illustrates a schematic monolithic compliant mechanism, with a fixed boundary subset Γ_d. At the input port ω_{in}, an input force F_{in} is applied, while the displacement magnitude u_{out} at the output port ω_{out} is computed and projected onto a given direction indicated by the vector l_{out}.

In this paper, we only consider the following objective function

$$J(\Omega) = -\frac{\displaystyle\int_\Omega \chi_{out}(x)(l_{out}(x), u(x))\ dx}{\left(\displaystyle\int_\Omega \chi_{in}(x)|u|^2(x)\ dx\right)^{1/2}} \tag{3}$$

where χ_{in} and χ_{out} are the characteristic functions respectively associated to the input and output ports. Formulation (3) is very similar to the Geometric Advantage (GA), maximized in [23][28], for example, and

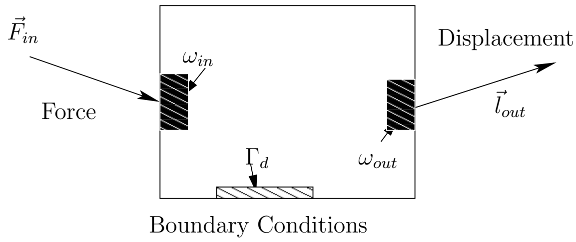

Figure 1: Example of problem specification for the design of a compliant mechanism.

defined by

$$\mathrm{GA} = \frac{(l_{out}, \mathbf{u}_{out})_{L^2(\Omega)}}{\| \mathbf{u}_{in} \|_{L^2(\Omega)}}$$

4.2 Shape derivative

To apply a gradient method to the minimization problem (2) we use the classical notion of shape derivative of Murat and Simon (see e.g. [21][2][3]). The derivative of the objective function 3, with respect to normal variations of the shape, can be written as

$$\begin{aligned} J^{'}(\Omega)(\theta) &= \int_{\Gamma_N} \Big(\Big(\frac{(l_{out}, \chi_{out} u)}{C_1} - \frac{\chi_{in} |u|^2}{2C_2} \Big) J(\Omega) + Ae(p).e(u) - f.p - \frac{\partial (g.p)}{\partial n} - Hg.p \Big) \theta.n \ \mathrm{d}s \\ &+ \int_{\Gamma_D} \Big(\Big(\frac{(l_{out}, \chi_{out} u)}{C_1} - \frac{\chi_{in} |u|^2}{2C_2} \Big) J(\Omega) - Ae(p).e(u) \Big) \theta.n \ \mathrm{d}s \end{aligned} \quad (4)$$

where n is the unit normal to $\partial\Omega$ (boundary of Ω), θ is the normal elementary variation, H is the mean curvature of $\partial\Omega$, u is the solution of (1) in Ω, and p is the adjoint state in Ω defined as the solution of the adjoint problem:

$$\begin{cases} -\mathrm{div}\ (Ae(p)) &= \Big(\frac{\chi_{out} l_{out}}{C_1} - \frac{\chi_{in} u}{C_2} \Big) J(\Omega) & \text{in } \Omega \\ p &= 0 & \text{on } \Gamma_D \\ (Ae(p))n &= 0 & \text{on } \Gamma_N \end{cases} \quad (5)$$

where C_1 and C_2 are two constants given by

$$C_1 = \int_\Omega (l_{out}, \chi_{out} u)\ dx, \text{ and } C_2 = \int_\Omega \chi_{in} |u|^2\ dx.$$

For the details of the proof, see [18].

4.3 Shape representation by level set method

In the level set method, the boundary of a solid structure is implicitly represented as the zero level set of a scalar field $\psi(x)$, which is called level set function. If the solid region $\Omega \subset D$, then following the idea of Osher and Sethian [22][27], Ω and $\partial\Omega$ are implicitly described through the zero level set of ψ by

$$\begin{cases} \psi(x) = 0 & \Leftrightarrow & x \in \partial\Omega \cap D \\ \psi(x) < 0 & \Leftrightarrow & x \in \Omega \\ \psi(x) > 0 & \Leftrightarrow & x \in (D \backslash \bar{\Omega}) \end{cases} \quad (6)$$

The exterior normal n to $\partial\Omega$ is recovered as $\nabla\psi/|\nabla\psi|$ and the mean curvature H is given by $\mathrm{div}(\nabla\psi/|\nabla\psi|)$. During the optimization process, the shape $\Omega(t)$ is going to evolve according to a pseudo time parameter $t \in \mathbb{R}^+$, which corresponds to a down stepping parameter. Doing variations of the shape in the direction of the shape gradient amounts to solve the following Hamilton-Jacobi transport equation (cf. [2][3][22] for details),

$$\frac{\partial \psi}{\partial t} + V|\nabla\psi| = 0 \text{ in } D \quad (7)$$

where $V(x,t)$ is the normal velocity of the shape's boundary, computed using the shape derivative given by formula (4).

The main idea of the level set method is to avoid the characterization of the shape boundary and the meshing of the shape. The boundary is instead implicitly represented by the scalar function ψ on a fixed grid over D, and the shape variations are done through the resolution of the Hamilton-Jacobi equation (7) on the whole domain D.

4.4 Optimization algorithm

For the minimization problem (2) we use the level set algorithm, proposed in [2][3]. It is an iterative algorithm, structured as follows:

1. Initialization of the level set function ψ_0 corresponding to an initial guess Ω_0. Typically, Ω_0 is the full domain D perforated by a periodic distribution of circular holes.

2. Iteration until convergence, for $k \geq 0$:

   (a) Computation of the state u_k and the adjoint state p_k through two linear elasticity problems (1) posed in Ω_k.

   (b) Deformation of the shape by solving the transport Hamilton-Jacobi equation (7). The new shape Ω_{k+1} is characterized by the level set function ψ_{k+1} solution of (7) after a time step Δt_k starting from the initial condition $\psi_k(x)$ with velocity $V_k = -v_k$ computed in terms of u_k and p_k. The time step Δt_k is chosen such that $J(\Omega_{k+1}) \leq J(\Omega_k)$.

The Hamilton-Jacobi equation (7) is solved by an explicit upwind scheme on a Cartesian grid with a time stepping satisfying a CFL condition (see e.g. [27]).

5 FlexIn: A Compliant mechanisms stochastic design methodology

In this section, we briefly present the flexible building blocks method developped at CEA. This method has been implemented for in-plane mechanisms in a software called *FlexIn*, developed with MATLAB. It uses an evolutionary algorithm approach for the optimal design of compliant mechanisms made of an assembly of basic building blocks chosen in a given library (see Figure 2). A detailed description of the method can be found in [10][12].

5.1 Compliant building blocks

A library of compliant elements is proposed in FlexIn. These blocks are in limited number (the basis is composed of 36 elements, see Figure 2 and section 5.4). They are sufficient to represent a great variety of topologies, and it has been verified that they can describe many existing compliant structures of the literature. Moreover, the block feasibility related to fabrication process constraints can also be taken into account at this stage, which is not the case for classical beam-based approach.

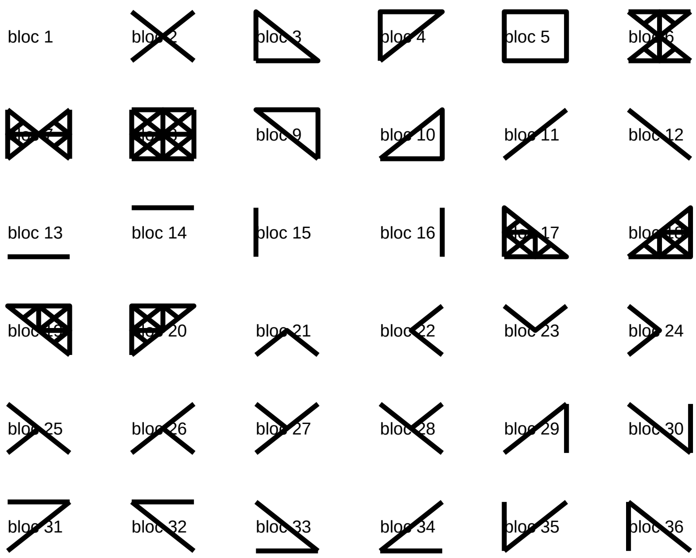

Figure 2: Compliant building blocks for two-dimensional compliant mechanisms synthesis using FlexIn.

5.2 Principle of the method

The purpose of FlexIn is to optimally design realistic compliant structures. The design method consists in searching for an optimal distribution of allowed building blocks, as well as the optimal set of structural parameters and material. Fixed node positions can also be considered as an optimization parameter.

The topology optimization method (see Figure 3, and section 5.3), inspired from Deb *et al* [14], uses a genetic algorithm approach, which allows true multicriteria optimization and the use of discrete variables (see **1** and **2** in Figure 3). The algorithm is structured as follows:

- Discrete variable parameterization of compliant mechanisms considering conception requirements (mesh size, topology, material and thickness, boundary conditions),
- Evaluation of individuals (computation of the design criteria),
- Stochastic operators for the optimization (modification of compliant mechanisms description).

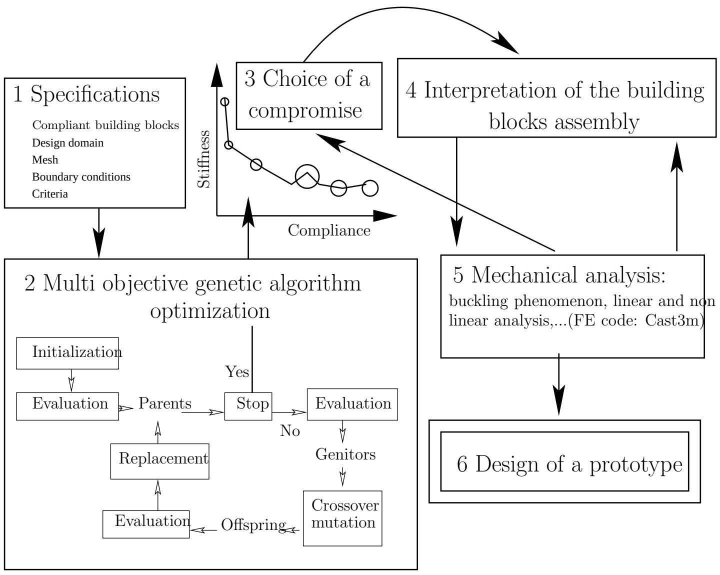

Figure 3: FlexIn compliant building blocks design method: Flowchart of the algorithm (example of multicriteria optimization with stiffness and compliance as objective functions).

5.3 A multi criteria genetic algorithm

Many fitness functions are available in FlexIn: displacement and force at the output port, strain energy (SE), mutual strain energy (MSE), maximal stress, geometric advantage (GA), mechanic advantage (MA), etc. The optimization algorithm generates a set of candidate solutions (see **3** in Figure 3), in the case of multicriteria optimization problem, and only one optimal solution for monocriterion optimization. The designer can choose, interpret and analyze the obtained structures that best suit his design problem (see **3** to **5** in Figure 3). Cast3m^{TM} can be used for subsequent FEA, to analyze the chosen design solution for criteria not considered during the optimization.

5.4 Mechanical model of the blocks

The specification of planar compliant mechanism problem considers specific boundary conditions: fixed frame location, input (actuators), contacts and output. In FlexIn, it is assumed that the compliant mechanisms are undergoing structural deformation, mainly due to the bending of the beams. Thus, the following assumptions have been made:

- Static state calculations,
- Small perturbations,
- Homogeneous and linear elastic model,

- Navier-Bernoulli beams with rectangular section.

The blocks are composed of beams modeled by Navier-Bernoulli beam-type finite elements. Structural parameters of each rectangular block are height, width and thickness. Material characteristics of each block are parameterized by Young's modulus, Poisson ratio and density. Firstly, the stiffness matrix of each block is calculated numerically, considering every combination of the discrete values allowed for the structural optimization variables. Then it is condensed, considering that non zero forces (i.e. inter-block connexion forces) act only on the four corners of the block. The computation of the reduced stiffness matrix of each valued-block is done only once, at the beginning of the optimal design problem, before running the genetic algorithm, thus saving running time. Even if the resulting model is not exact (for twelve blocks of the library), it has been found that it has few influence on the value of the objective functions for most of the compliant structures generated, due to the type of block assemblies that generally occur. The condensed model of each block induces smaller numerical problems for block assemblies, which is of great interest when using a genetic algorithm approach for multi-objective optimal design (here, numerous but simplified FE problems are being solved at each step). Let's note that Kim *et al.* [19] have proposed an original building blocks method that considers only four bars building blocks, characterized by their instant center based kinematics. But the chosen strategy limits this method to topological mono-objective optimization of GA, and needs, according to the authors, subsequent size and geometry optimization to refine the obtained design and consider other performance criteria.

6 Numerical results

The comparison between the two methods presented previously has been done considering a monocriterion optimization problem, with GA as the objective function. Many examples of compliant mechanisms, such as inverters and grippers, can be found in the literature. Here, some of them have been re-designed using the level set and FlexIn methods.

6.1 Force inverting mechanisms

We consider a force inverter. The design problem is sketched in Figure 4. The design domain is defined within a $L \times L$ square with $L = 1\ cm$. The purpose of the device is to generate the output displacement u_{out} in the opposite direction of an input force F_{in} at the input port. A force $F_{in} = 5.10^{-3}N$ is applied at the input port. The material Young's Modulus is assumed to be $192.10^5 MPa$ and the Poisson ratio 0.3. In level-set simulations, for void or holes are mimicked by an ersatz material with the same Poisson ratio and Young modulus 10^{-2}. In FlexIn simulations, we consider that the structures have a thickness $h = 10^{-3}mm$.

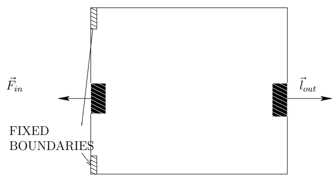

Figure 4: Design problem specification of a force inverter.

Figure 5 shows the initial design, the final optimal design obtained by the level set method and the deformed configuration of the force inverter. Figure 6 shows the convergence histories of the objective function and the volume fraction. On Figure 7 we present one typical solution given by FlexIn for the same problem. Table 1

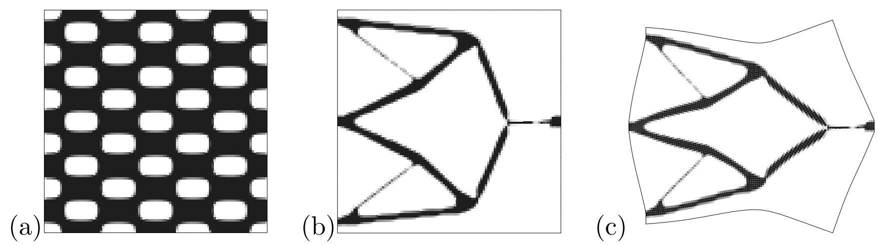

Figure 5: **Force inverter optimized by level set** (a): Initialization, (b): The final design, (c): The deformed configuration.

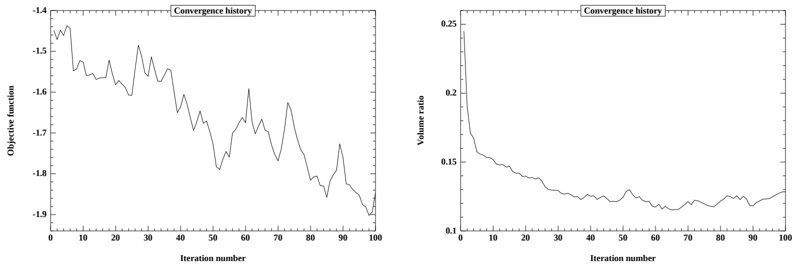

Figure 6: Convergence history of the objective function and the volume fraction for the level set method.

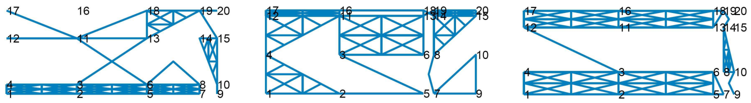

Figure 7: The final half design solution of a force inverter by FlexIn (horizontal symmetry axis).

exhibits the geometric advantage GA value and resulting input and output displacements for the force inverter with both methods.

force inverter	Level-set (with ersatz material)	Level-set with void)	FlexIn
GA ($=\frac{u_{out}}{u_{in}}$)	?	?	?
u_{out} (mm)	?	?	?
u_{in} (mm)	?	?	?

Table 1: Optimization of the geometric advantage by both methods: GA value and resulting output and input displacements for the force inverter.

6.2 Pull micro-gripper

In the third example, we consider the design of a pull micro-gripper. The problem is sketched in Figure **??**. The design domain is defined within a $L \times L$ square with $L = 1mm$, where the mechanism is supported at part of the left side and is subjected to an horizontal squeezing load in the middle of the left side. The output port is shown, where an output displacement is desired. Figures 9 and 10 show the optimal solutions obtained by the level set and FlexIn methods respectively. Table 2 exhibits the geometric advantage GA value and resulting input and output displacements for the pull micro-gripper with both methods.

7 Coupling strategy of two methods

In this section, we propose a coupling strategy of these two techniques, structured as follows:

1. One optimal solution obtained by *FlexIn* is selected (here discripted by the file of 2D maillage utilisabled by level-set code.
2. Computation of optimal solution by level-set method, with the initialisation by selected solution solution *FlexIn*.

We present the numerical results, obtained by this coupling strategy, in the inverter force case. We observe the modifications obtained after shape-optimization by level set method, are not verry important.

inverter	Optimization by *FlexIn*	Optimization by *level set*
GA ($=\frac{u_{out}}{u_{in}}$)	48.472	111.5

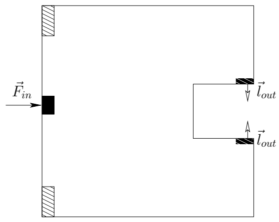

Figure 8: Design problem specification of a pull-gripper.

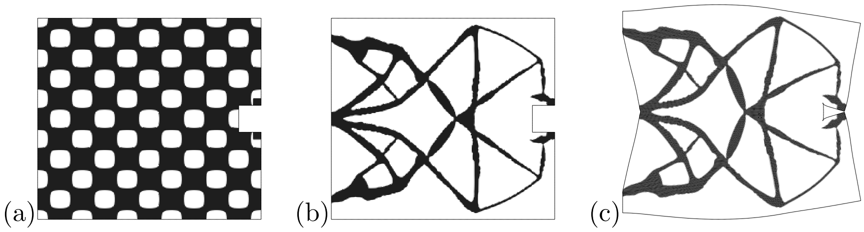

Figure 9: **Pull micro-gripper optimized by level set** (a): Initialization, (b): The final design, (c): Deformed configuration

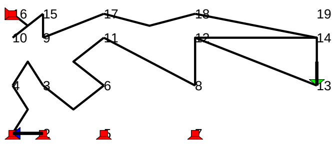

Figure 10: The final half design solution of a pull micro-gripper by FlexIn (horizontal symmetry axis).

Pull micro-gripper	Level-set (with ersatz material)	Level-set with void)	FlexIn
GA $(=\frac{u_{out}}{u_{in}})$	0.725727476	2.606832877	?
u_{out} (mm)	3.15204187E-06	8.28426919E-05	?
u_{in} (mm)	4.34328584E-06	3.17790575E-05	?

Table 2: Optimization of the geometric advantage by both methods: GA value and resulting output and input displacements for the pull micro-gripper.

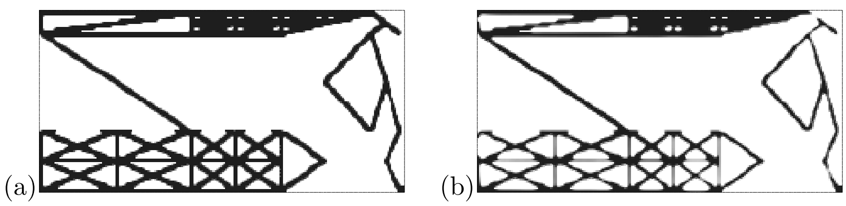

Figure 11: (a): Inverter optimized by $FlexIn$ (Initialization of level-set), (b): Inverter optimized by Level-Set.

8 Conclusions

In this paper, we have presented and compared two numerical methods suitable for topology and shape synthesis of compliant mechanisms. The first one is the level set method. It has a number of promising advantages, including its capability to handle topology changes, the moderate computational complexity on an Eulerian mesh, and its versatility in taking into account any type of objective functions or mechanical models.

The second one is a conceptual multiobjective design method of compliant mechanisms, although it has been used in the present study in a mono-objective context to allow the comparison with the level set method. It considers a compliant mechanism as a basic assembly of compliant building blocks. The user can choose a design among a set of pseudo-optimal solutions that may have various topologies. But there is a limitation due to the use of a beam model with linear elastic material. The user may have to investigate it further using classical Finite Element Analysis software. Indeed, geometrical and material non linearities, and dynamics are not implemented in FlexIn.

This research is still ongoing. One of the perspectives of this work is the refinement of both methods, and their coupling. Indeed, the genetic approach has two main advantages over the level set method and other deterministic methods: it can handle multiobjective criteria and can also optimize the positions of boundary conditions and actuators. It could be thus used as a pre-computation step, followed by a more precise design optimization by the level set method.

Acknowledgment. This work has been financially supported by the French Atomic Energy Commission (CEA) under the grants CEA-4000154343 and CEA-4000237437.

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