Otto Rössler

1975-76

DES RÉACTIONS CHIMIQUES
À
LA TOPOLOGIE DU CHAOS

Réalisé par

Christophe Letellier

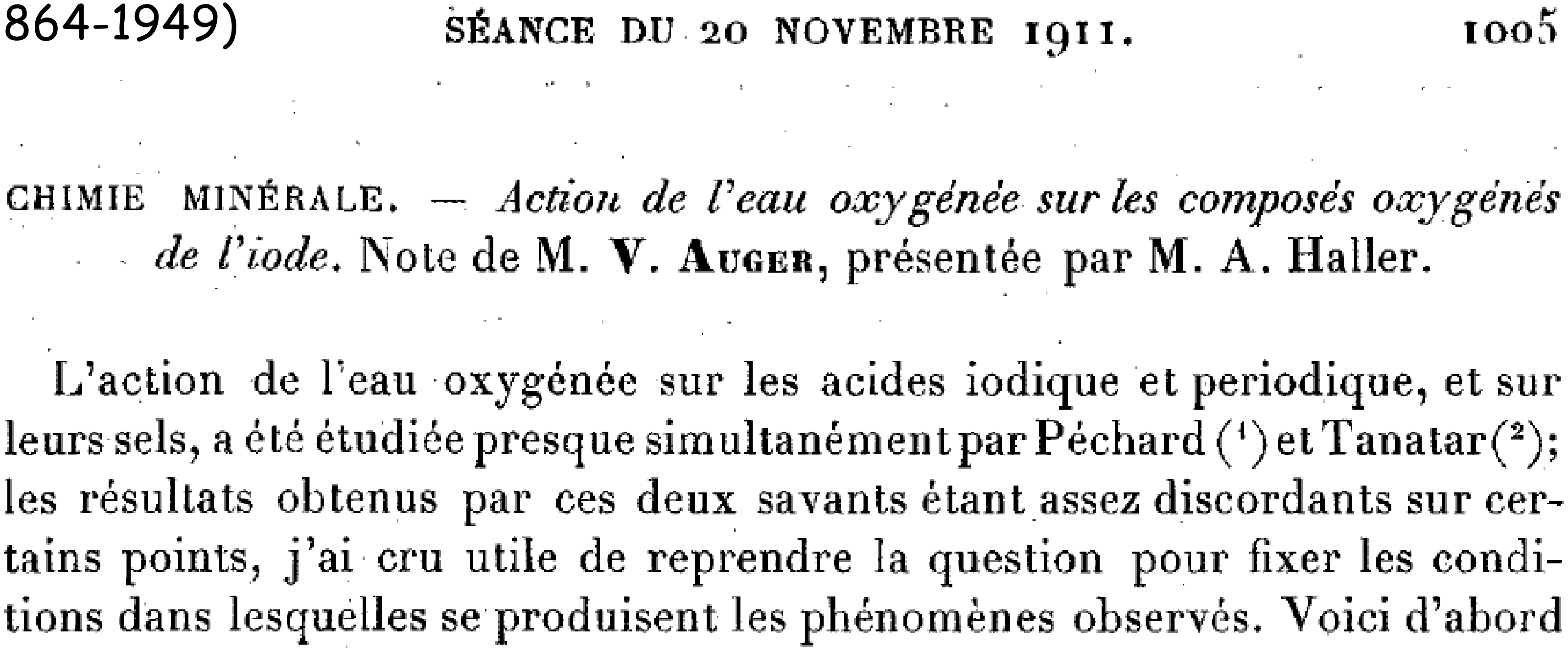
Normandie Université

1911

Première réaction chimique oscillante

Victor Auger
(1864-1949)

Action de l'eau oxygénée sur les composés iodés



SÉANCE DU 20 NOVEMBRE 1911. 1005

CHIMIE MINÉRALE. — *Action de l'eau oxygénée sur les composés oxygénés de l'iode.* Note de M. V. **Auger**, présentée par M. A. Haller.

L'action de l'eau oxygénée sur les acides iodique et periodique, et sur leurs sels, a été étudiée presque simultanément par Péchard (1) et Tanatar (2); les résultats obtenus par ces deux savants étant assez discordants sur certains points, j'ai cru utile de reprendre la question pour fixer les conditions dans lesquelles se produisent les phénomènes observés. Voici d'abord

1921

[Contribution from the Chemical Laboratory of the University of Californi

A PERIODIC REACTION IN HOMOGENEOUS SOLUTION AND ITS RELATION TO CATALYSIS.

By William C. Bray.

Received March 29, 1921.

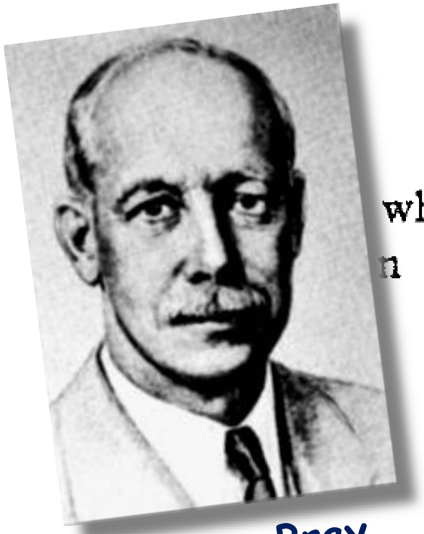
In 1916 A. L. Caulkins and the writer began an investigation of the dual rôle of hydrogen peroxide as an oxidizing agent and as a reducing agent. The reactions considered were the oxidation of iodine to iodic acid, and the reduction of iodic acid to iodine, *viz.*:

$$5H_2O_2 + I_2 = 2HIO_3 + 4H_2O \qquad (1)$$

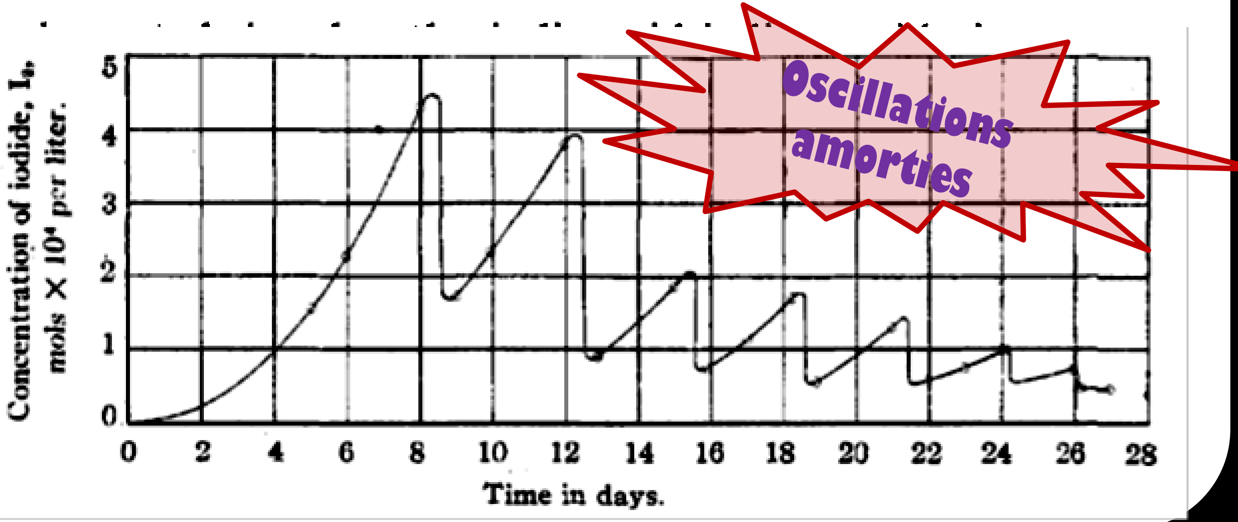
$$5H_2O_2 + 2HIO_3 = 5O_2 + I_2 + 6H_2O, \qquad (2)$$

which, it is

n "couple."



William Bray
(1879-1946)



Karl Bonhöffer
(1899-1957)

IE NATURWISSENSCHAFTEN

gang 40 Heft 11 (Erstes Juniheft) 1953

Modelle der Nervenerregung.

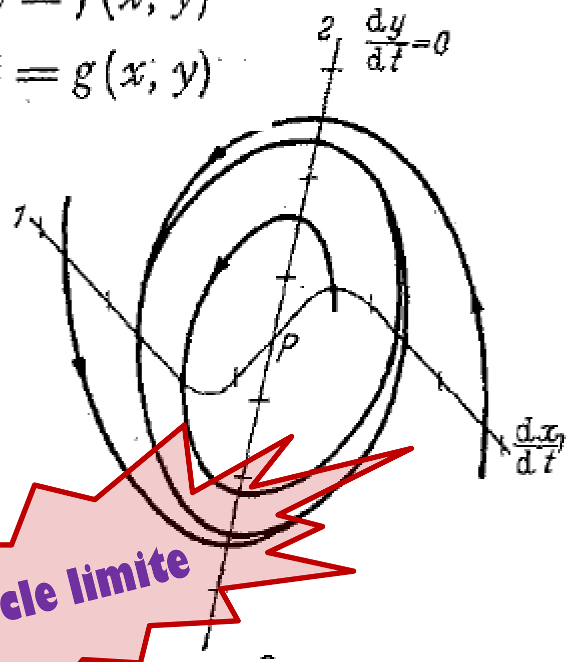
Von K. F. Bonhoeffer, Göttingen.

1948

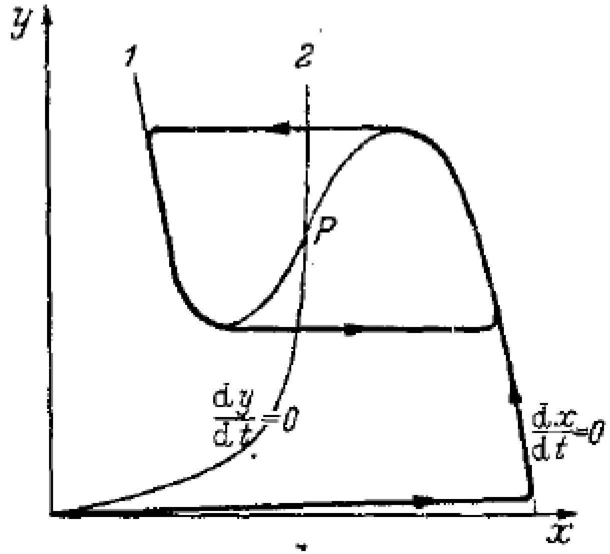
Un modèle 2D (incomplet!)

$$d\,x/d\,t = f(x, y)$$

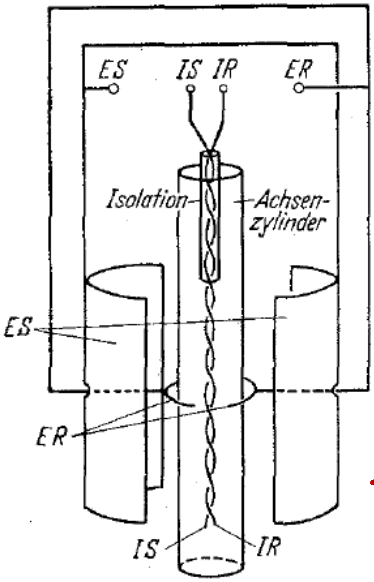
$$d\,y/d\,t = g(x; y)$$



Cycle limite



Système lent/rapide



Un modèle de la conduction nerveuse

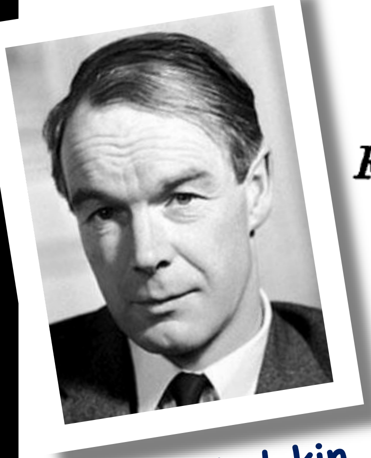
Cite B. van der Pol & J. van der Mark

A QUANTITATIVE DESCRIPTION OF MEMBRANE CURRENT AND ITS APPLICATION TO CONDUCTION AND EXCITATION IN NERVE

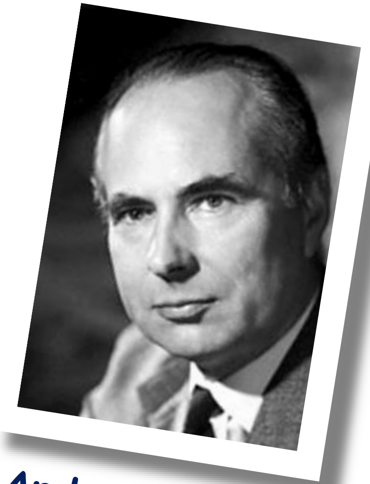
1952

By A. L. HODGKIN and A. F. HUXLEY

From the Physiological Laboratory, University of Cambridge



Alan Hodgkin
(1914-1998)



Andrew Huxley
(1917-)

Courant membranaire total I

$$I = C_M \frac{\mathrm{d}V}{\mathrm{d}t} + \bar{g}_{\mathrm{K}} n^4 (V - V_{\mathrm{K}}) + \bar{g}_{\mathrm{Na}} m^3 h (V - V_{\mathrm{Na}}) + \bar{g}_l (V - V_l),$$

where

$$\mathrm{d}n/\mathrm{d}t = \alpha_n(1-n) - \beta_n n,$$

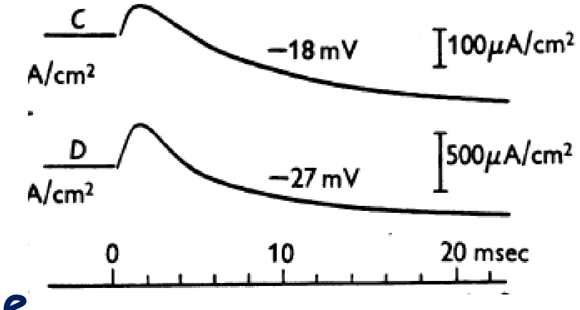
$$\mathrm{d}m/\mathrm{d}t = \alpha_m(1-m) - \beta_m m,$$

$$\mathrm{d}h/\mathrm{d}t = \alpha_h(1-h) - \beta_h h,$$

n conductance adimensionnelle du potassium

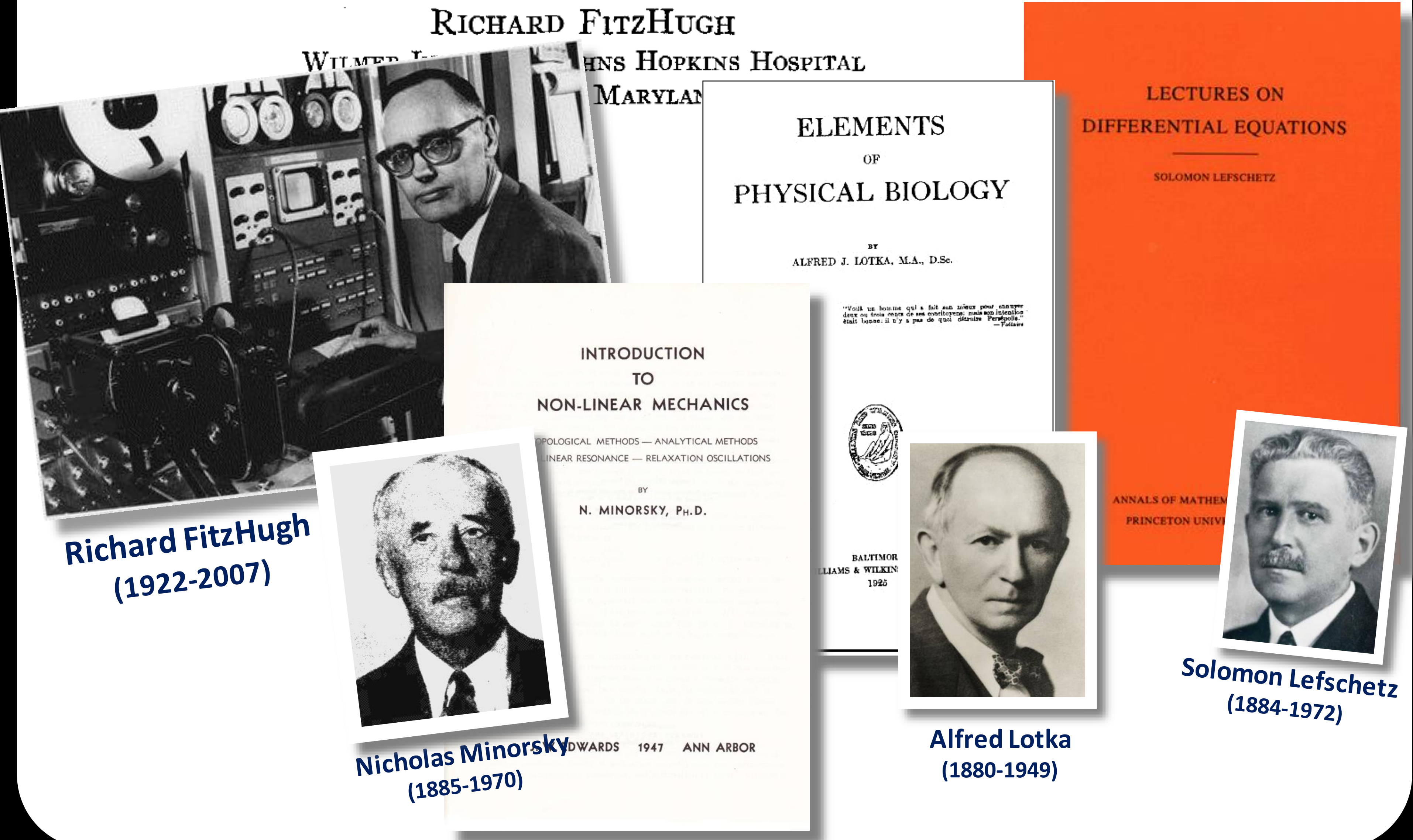
m fraction des molécules actives dans la membrane

h fraction des molécules inactives dans la membrane



MATHEMATICAL MODELS OF THRESHOLD PHENOMENA IN THE NERVE MEMBRANE

1955



RICHARD FITZHUGH

WILMER I
HNS HOPKINS HOSPITAL
MARYLAN

ELEMENTS
OF
PHYSICAL BIOLOGY

BY
ALFRED J. LOTKA, M.A., D.Sc.

BALTIMOR
LLIAMS & WILKIN
1925

LECTURES ON
DIFFERENTIAL EQUATIONS

SOLOMON LEFSCHETZ

ANNALS OF MATHEM
PRINCETON UNIV

INTRODUCTION
TO
NON-LINEAR MECHANICS

OPOLOGICAL METHODS — ANALYTICAL METHODS
LINEAR RESONANCE — RELAXATION OSCILLATIONS

BY
N. MINORSKY, PH.D.

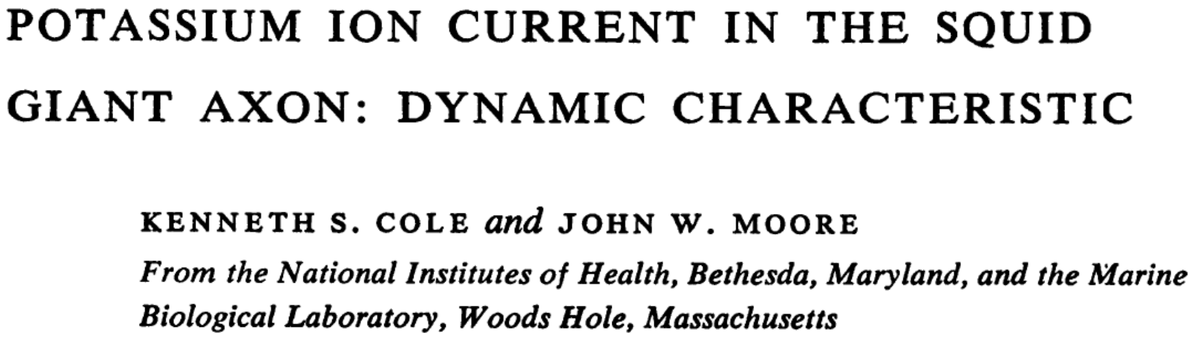
DWARDS 1947 ANN ARBOR

Richard FitzHugh
(1922-2007)

Nicholas Minorsky
(1885-1970)

Alfred Lotka
(1880-1949)

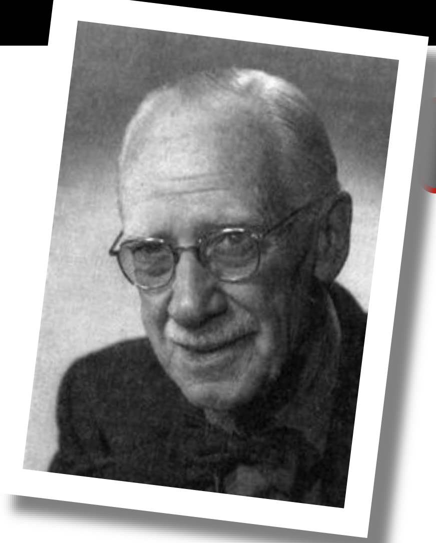
Solomon Lefschetz
(1884-1972)



POTASSIUM ION CURRENT IN THE SQUID GIANT AXON: DYNAMIC CHARACTERISTIC

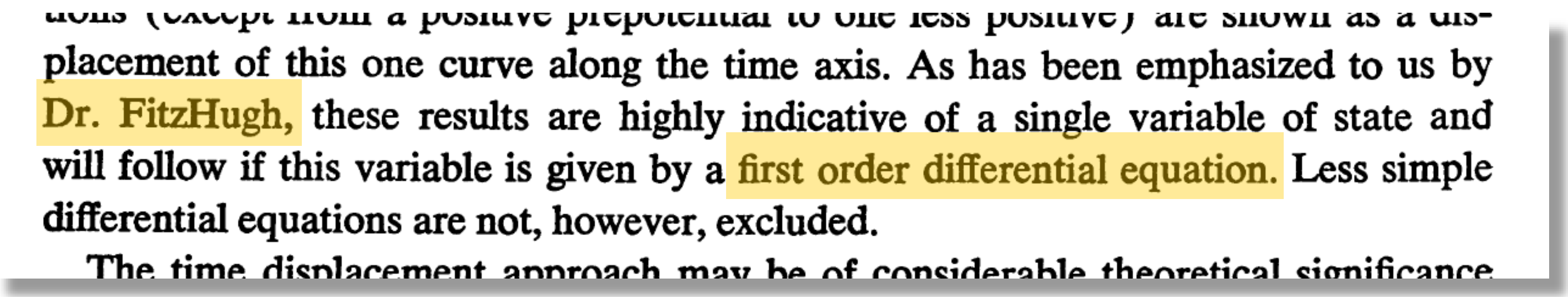
KENNETH S. COLE *and* JOHN W. MOORE

From the National Institutes of Health, Bethesda, Maryland, and the Marine Biological Laboratory, Woods Hole, Massachusetts



Kenneth Cole
(1900-1984)

1960

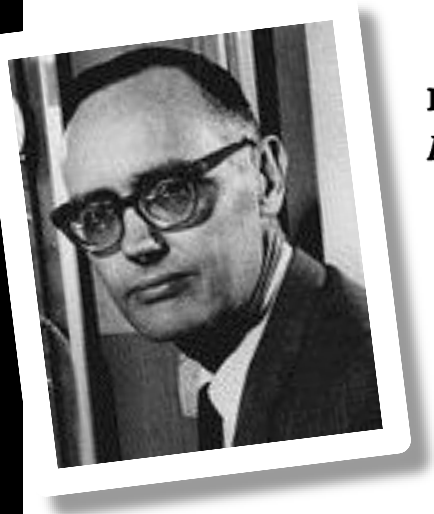


tions (except from a positive prepotential to one less positive) are shown as a displacement of this one curve along the time axis. As has been emphasized to us by Dr. FitzHugh, these results are highly indicative of a single variable of state and will follow if this variable is given by a first order differential equation. Less simple differential equations are not, however, excluded.

The time displacement approach may be of considerable theoretical significance

IMPULSES AND PHYSIOLOGICAL STATES IN THEORETICAL MODELS OF NERVE MEMBRANE

1961

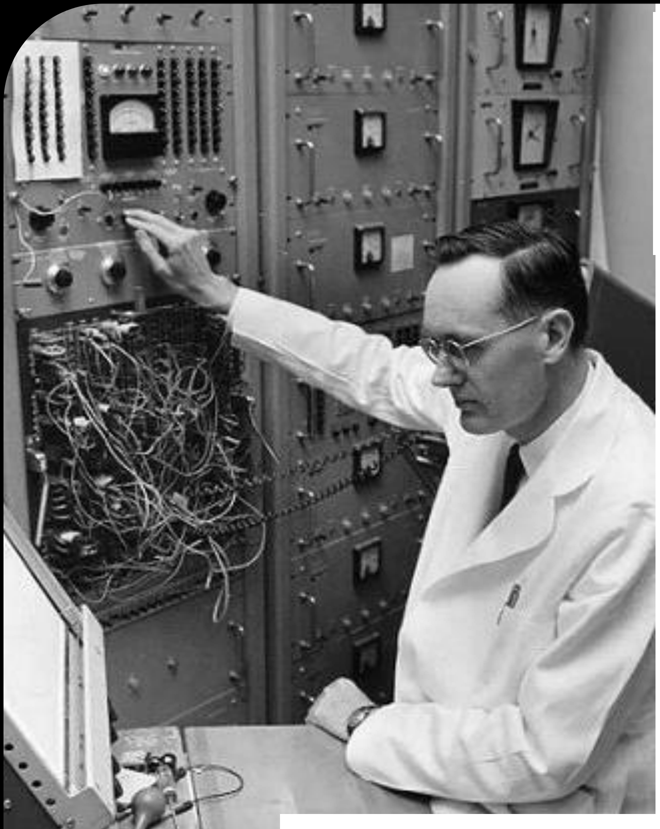


RICHARD FITZHUGH
From the National Institutes of Health, Bethesda

$$\ddot{x} + c(x^2 - 1)\dot{x} + x = 0$$

ABSTRACT Van der Pol's equation for a relaxation oscillator is generalized by the addition of terms to produce a pair of non-linear differential equations with either a stable singular point or a limit cycle. The resulting "BVP model" has two variables of state, representing excitability and refractoriness, and qualitatively resembles Bonhoeffer's theoretical model for the iron wire model of nerve. This BVP model serves as a simple representative of a class of excitable-oscillatory systems including the Hodgkin-Huxley (HH) model of the squid giant axon.

The possibility of representing excitable systems by a generalization of the van der Pol equation was suggested to the author by Dr. K. S. Cole.



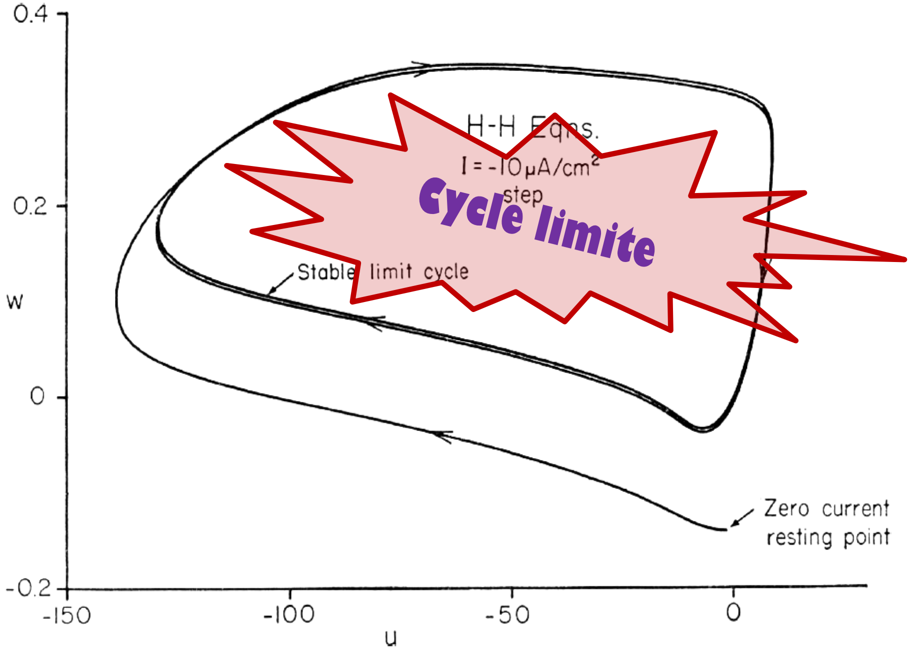
IMPULSES AND PHYSIOLOGICAL STATES IN THEORETICAL MODELS OF NERVE MEMBRANE

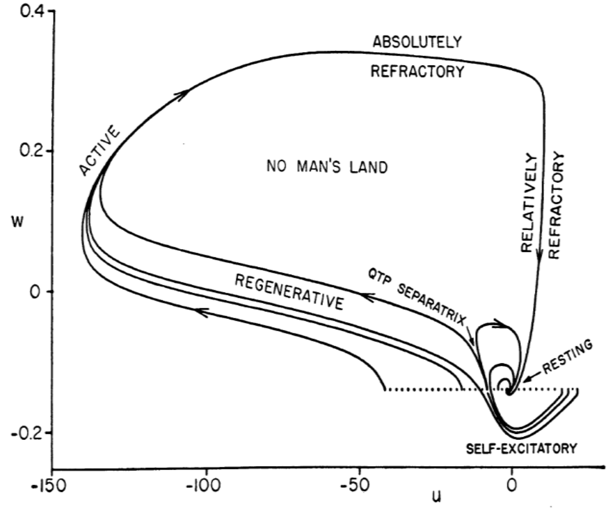
1961

RICHARD FITZHUGH
From the National Institutes of Health, Bethesda

Diagramme des états physiologiques obtenu par projection plane de l'espace des états du système de Hodgkin-Huxley

U=V=-36 m, w=$(n-h)/2$.





No. 4787 July 29, 1961 NATURE 457

1961

SUSTAINED OSCILLATIONS IN A CATALYTIC CHEMICAL SYSTEM

By R. A. SPANGLER* and F. M. SNELL
Department of Biophysics, University of Buffalo School of Medicine, Buffalo, New York

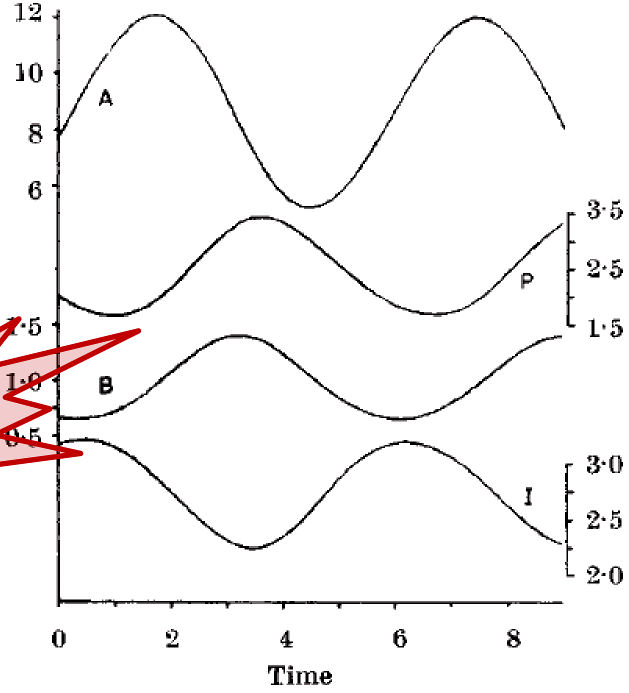
Etude d'un système catalytique intercouplé

$$\text{source } A \to A + E \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} B \xrightarrow{k_2} E + P \xrightarrow{k_3} \text{sink } P$$

$$\alpha P + E' \underset{k_{-4}'}{\overset{k_4'}{\rightleftarrows}} I' \qquad \alpha P' + E \underset{k_{-4}}{\overset{k_4}{\rightleftarrows}} I \qquad (1)$$

$$\text{source } A' \to A' + E' \underset{k_{-1}'}{\overset{k_1'}{\rightleftarrows}} B' \xrightarrow{k_2'} E' + P' \xrightarrow{k_3'} \text{sink } P'$$

A l'aide d'un ordinateur analogique



Oscillations entretenues

J. Theoret. Biol. (1967) **16**, 381–405

1967

Transfer Function Analysis of an Oscillatory Model Chemical System

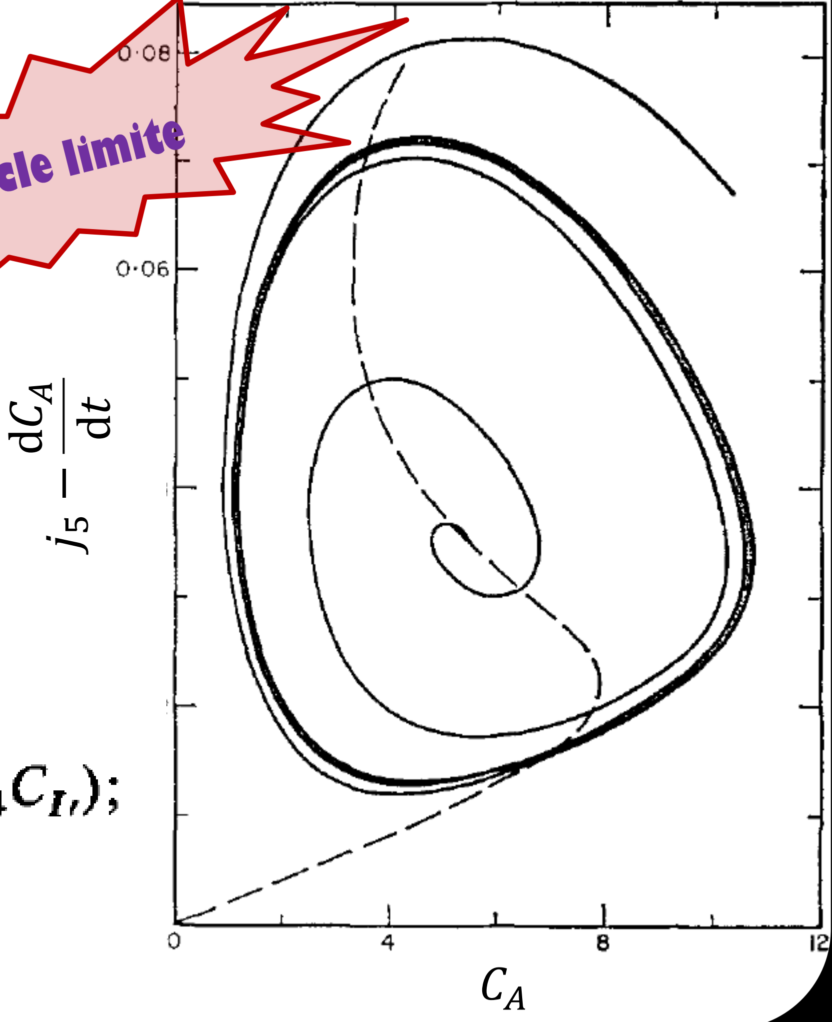
ROBERT A. SPANGLER

AND

FRED M. SNELL

Cycle limite tracé dans une projection de l'espace des états

Cycle limite



Système chimique 5D

$$\dot{C}_A \equiv \frac{dC_A}{dt} = j_5 - k_{-5}C_A - k_1C_AC_B + k_{-1}C_C.$$

$$\dot{C}_C = k_1C_AC_B - C_C(k_{-1}+k_2);$$

$$\dot{C}_P = k_2C_C - k_3C_P + j_{-3} - \eta_p(k_4C_P^{\eta_p}C_{B'} - k_{-4}C_{I'});$$

$$\dot{C}_{I'} = k_4C_P^{\eta_p}C_{B'} - k_{-4}C_{I'};$$

$$\dot{C}_B + \dot{C}_C + \dot{C}_I = 0;$$

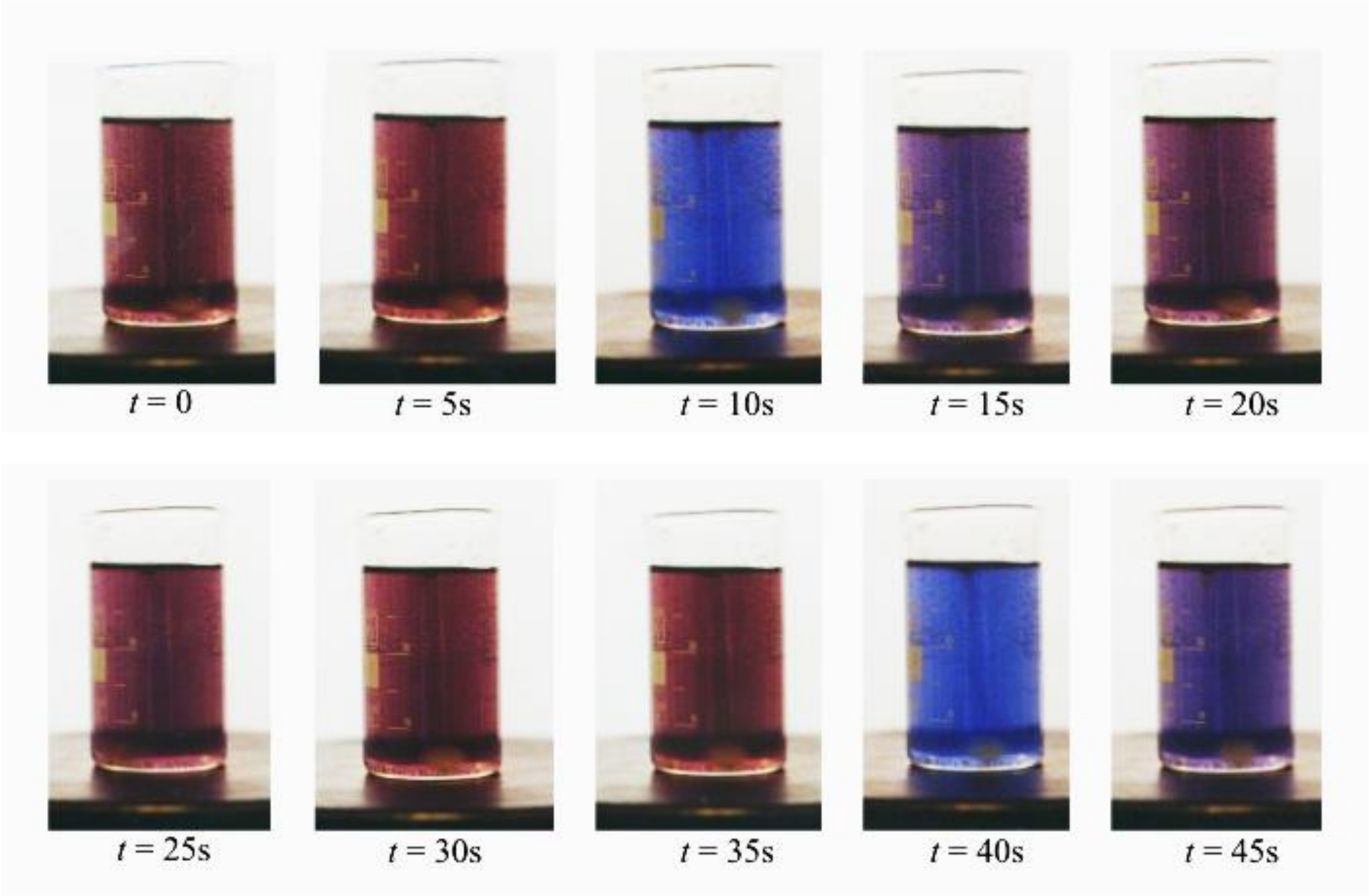
A periodic reaction and its mechanism

B. Belousov

1958

Boris Belousov
(1930-1998)

In Collection of short papers on radiation medicine for 1958
Med. Publ. (Moscow) 1959

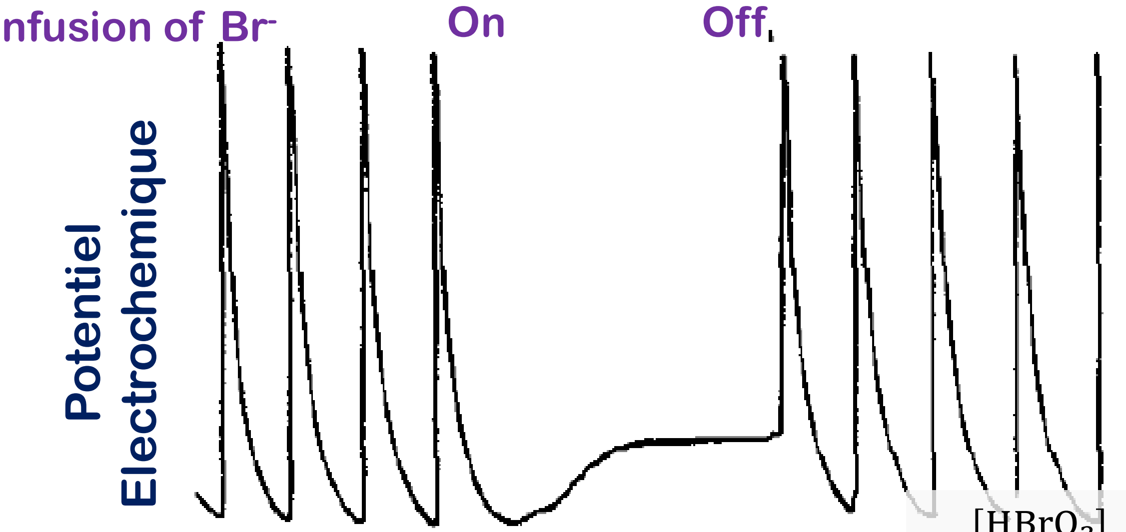


V. A. Vaivilin, A. M. Zhabotinsky & A. N. Zaikin

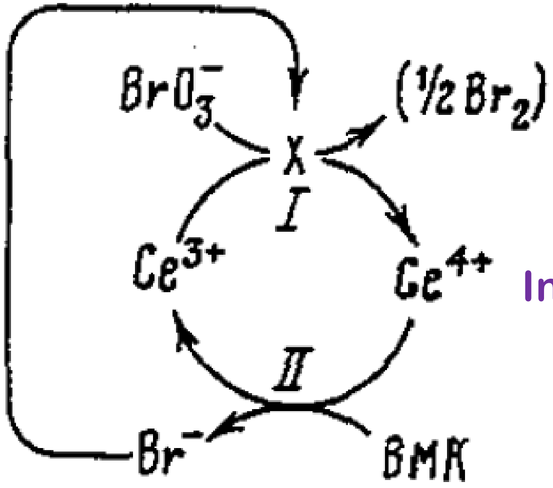
Russian Journal of Physics & Chemistry, **42**, 3091, 1968

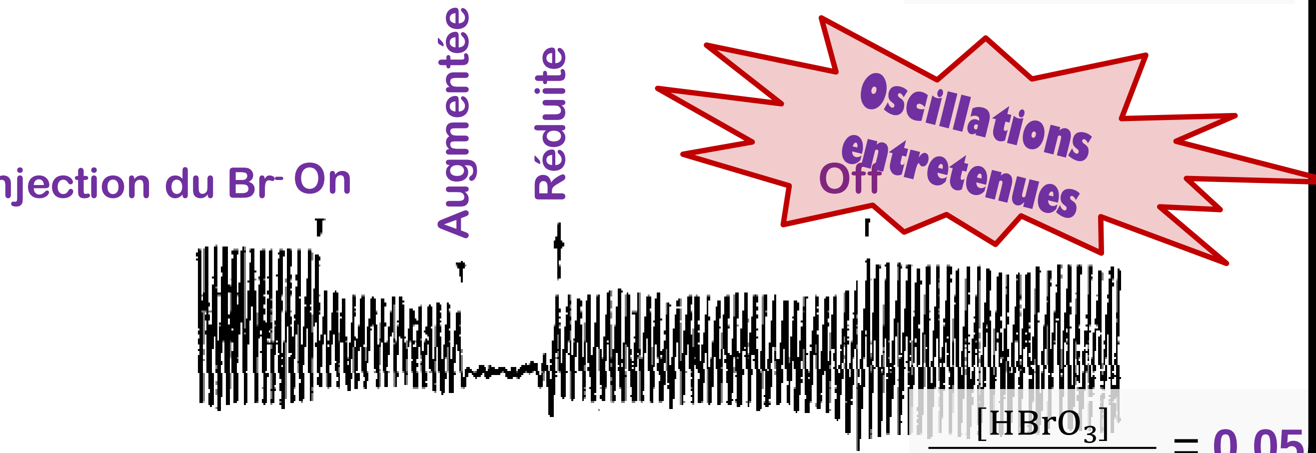
1964

Anatol Zhabotinsky
(1938-2008)



$$\frac{[\mathrm{HBrO_3}]}{[\mathrm{CH_2(COOH)_2}]} = 1.0$$





$$\frac{[\mathrm{HBrO_3}]}{[\mathrm{CH_2(COOH)_2}]} = 0.05$$

NATURE VOL. 225 FEBRUARY 7 1970 535

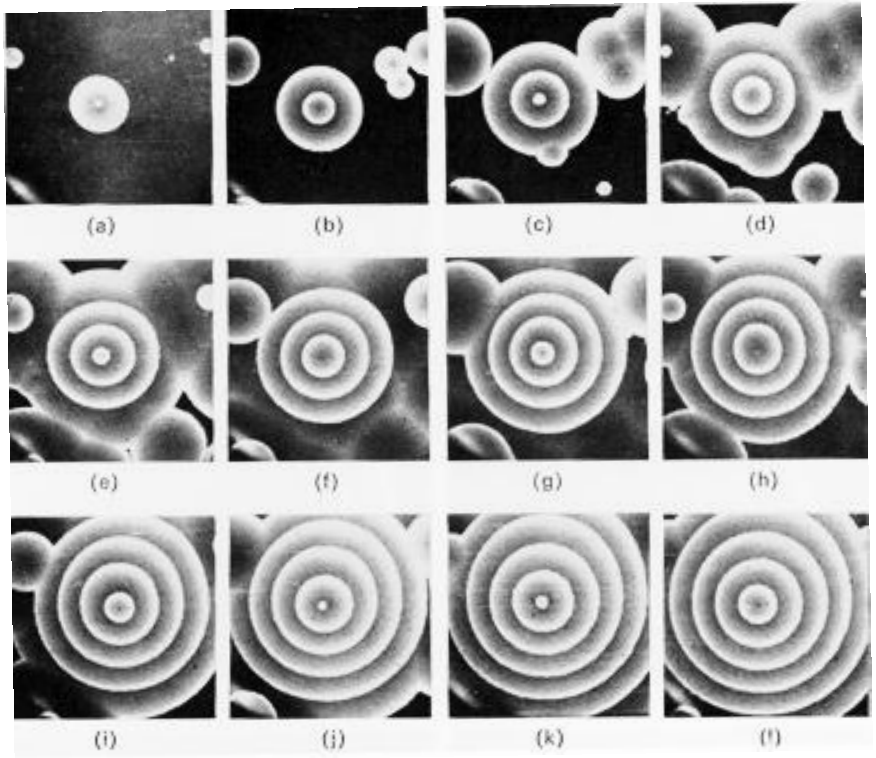
1970

Concentration Wave Propagation in Two-dimensional Liquid-phase Self-oscillating System

A. N. ZAIKIN
A. M. ZHABOTINSKY

Anatol Zhabotinsky
(1938-2008)

Original picture

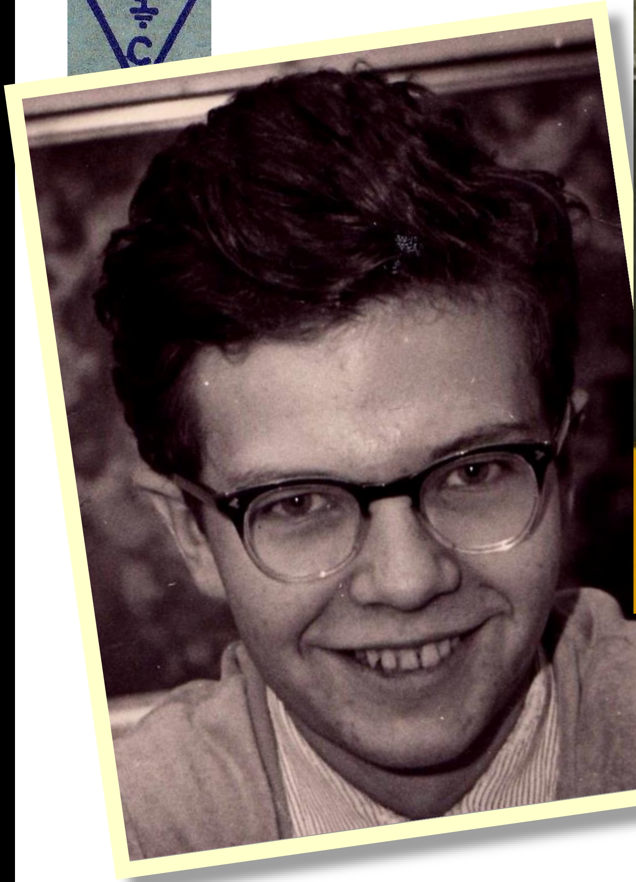


A more recent picture

Et un jour, Otto Rössler
entra dans la dance…

Né le 20 Mai 1940

Deutscher Amateur Radio Club



Franzis

radio PRAKTIKER bücherei

H. SUTANER

Die Wobbelsender

Aufgaben und Schaltungstechnik

GERMANY 1957

DL9KF

To Radio: ....

DOK: P 12

conf. QSO on 3,5/7/14/21/28/144 Mc

Rig here: Tx: .... Wtts, Rx: ....

Pse/tnx QSL om, via Bureau or direct

tnx QSO, Date: ....

time .... MEZ/GMT, cw - fone

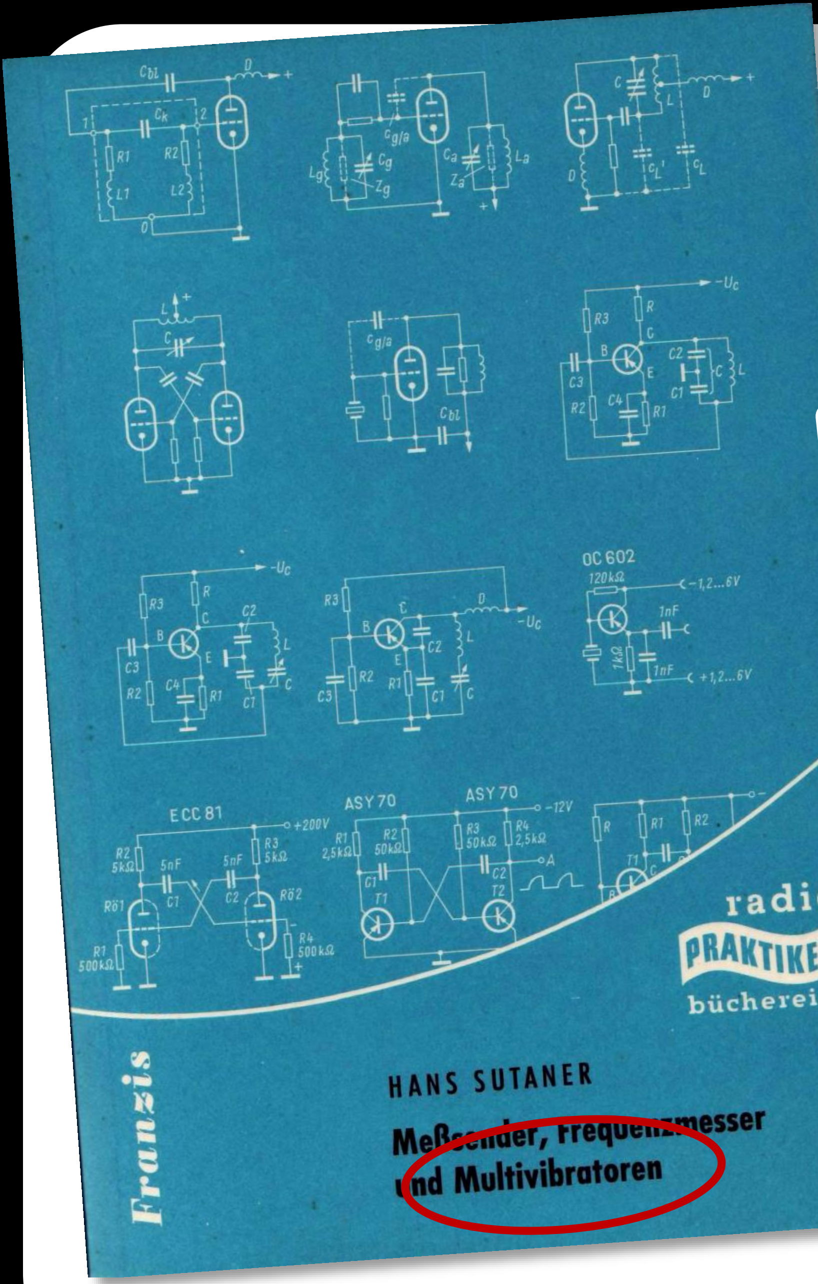
Report cw rst: .... fone r ..... s .....

tnx QSO om! 73! OP: ....

OP: Otto Rössler · QTH: Tübingen/N., Schaffhausenstraße 23

DL1CU print.

DL9: **licence individuelle avec privilèges complets**

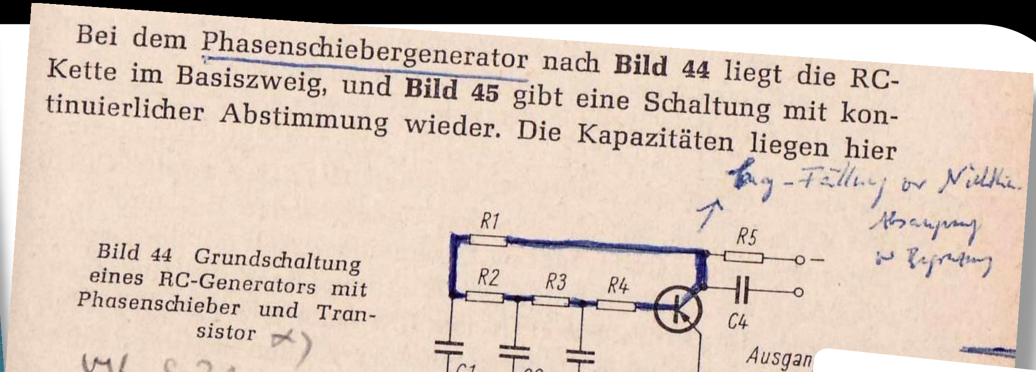


radio
PRAKTIKER
bücherei

Franzis

HANS SUTANER

Meßsender, Frequenzmesser
und Multivibratoren

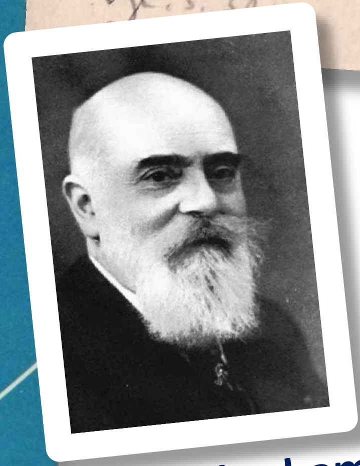


Bei dem Phasenschiebergenerator nach **Bild 44** liegt die RC-Kette im Basiszweig, und **Bild 45** gibt eine Schaltung mit kontinuierlicher Abstimmung wieder. Die Kapazitäten liegen hier

Lag-Fällung or Nillke.
Absaugung
or Begrenzung

Bild 44 Grundschaltung eines RC-Generators mit Phasenschieber und Transistor x)

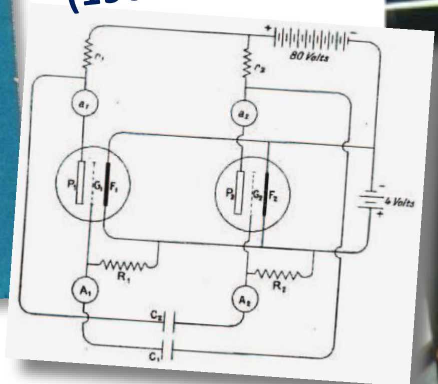
vgl. S. 3



Henri Abraham
(1900-1984)

multi-vibrateur d'Abraham & Bloch

Eugène Bloch
(1878-1944)



MESURE EN VALEUR ABSOLUE DES PÉRIODES DES OSCILLATIONS ÉLECTRIQUES DE HAUTE FRÉQUENCE ;

Par MM. Henri ABRAHAM et Eugène BLOCH.

1917

multivibrateur d'Abraham & Bloch

PHIE MILITAIRE

Secret défense

charges

natifs extrêmement riches e

dire que l'appareil émet er

d'onde. De là, son nom de

© PA

1951-1959: Lycée, Tübingen

1959-1965: Médecine,
Université de Tübingen

1966 : Docteur en médecine
de l'Université de Tübingen

Topic: ***Long-term Immunization of Albino Mice with Bovine Gamma Glob***

1966-1967: Récipiendaire d'une bourse d'étude
à l'Institut Max-Planck Institut pour la Physiologie des comportements, SeeWiesen

1967-1969: Assistant médical Université de Marburg

Konrad Lorenz (1903-1989) **Eric von Holst** (1908-1962)

1969-1970: ***Center for Theoretical Biology***
State University de New York à Buffalo

Robert Rosen
(1934-1998)

1970: Assistant (sous la direction de Friedrich Seelig)
Department for Theoretical Biology, University of Tübingen

1972

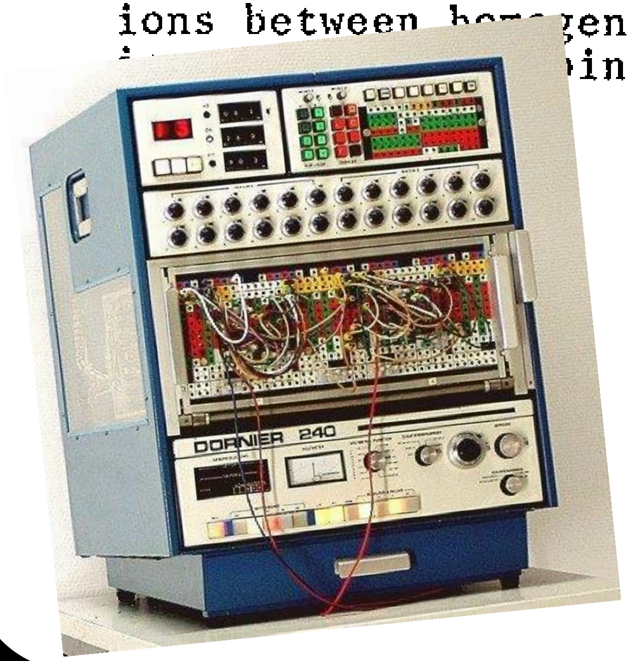
Dietrich Hoffmann

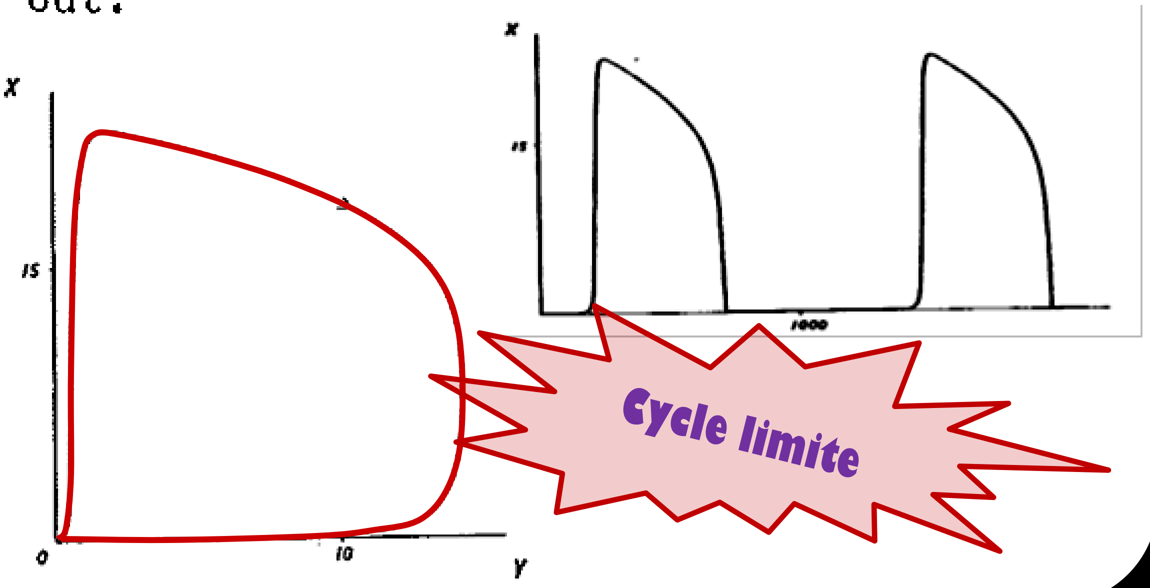
REPETITIVE HARD BIFURCATION IN A HOMOGENEOUS REACTION SYSTEM

O. E. RÖSSLER and D. HOFFMANN
Division of Theoretical Chemistry, University of Tübingen,
West Germany

Analysis and Simulation of Biochemical Systems, 91-102, 1972

This paper consists of three parts. First, theoretical evidence that the Belousov-Zhabotinsky reaction (BZR) is a Bonhoeffer oscillator, i.e. a special type of chemical hysteresis oscillator, is presented. Second, a brief account of the qualitative theory of chemical relaxation oscillators is given, centering around the notion of hard bifurcation. Finally, some connections between homogeneous and nonhomogeneous chemical bifurcati inted out.





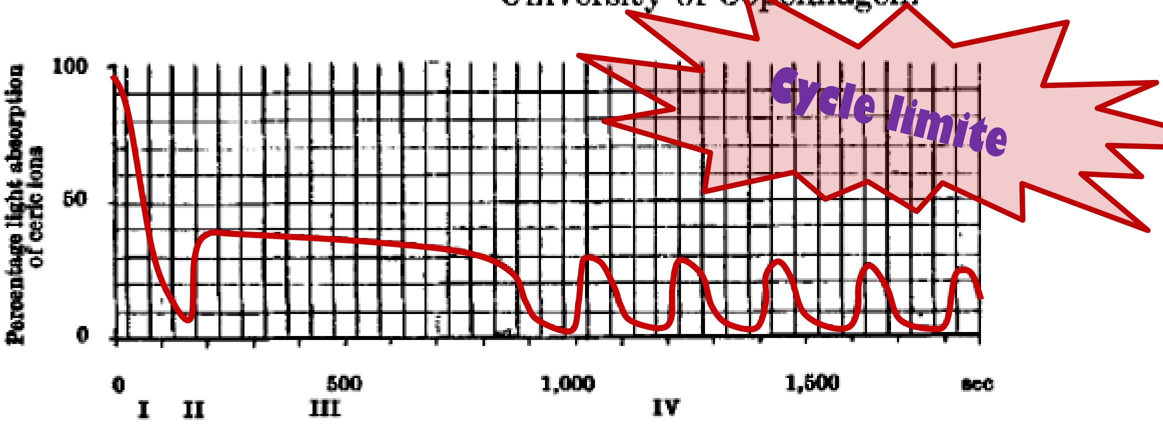
Cycle limite

CHEMISTRY

NATURE, FEBRUARY 11, 1967

Effect of Bromine Derivatives of Malonic Acid on the Oscillating Reaction of Malonic Acid, Cerium Ions and Bromate

Chemistry Laboratory III,
H. C. Ørsted Institute,
University of Copenhagen.



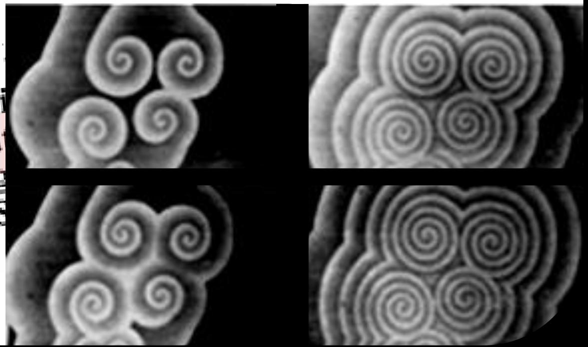
Hans Degn

Spiral Waves of Chemical Activity

Abstract. *The Zhabotinsky-Zaikin reagent propagates waves of chemical activity. Reaction kinetics remain to be fully resolved, but certain features of wave behavior are determined by purely geometrical considerations. If a wave is broken, then spiral waves, resembling involutes of the circle, appear, persist, and eventually exclude all concentric ring waves.*

ARTHUR T. WINFREE
Department of Theoretical Biolog
University of Chicago,
Chicago, Illinois 60637

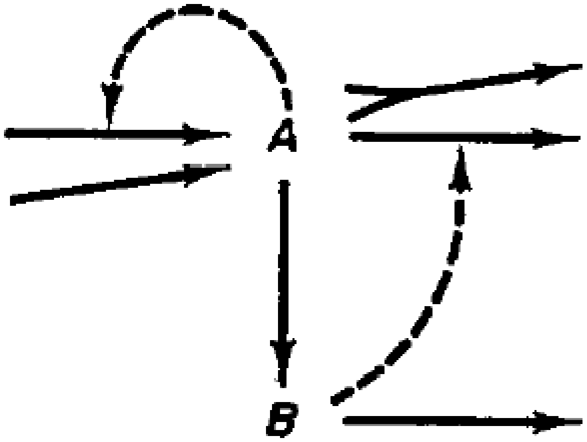
A. Zhabotinsky mentions spiral waves in
oscillating reagent on page 29 of "Investi
tion of homogeneous chemical auto-oscilla
systems" (in Russian) (Institute of Biolog
Physics of the Academy of Sciences, U.S.S
Puschino, 1970).



J. theor. Biol. (1972) 36, 413–417

1972

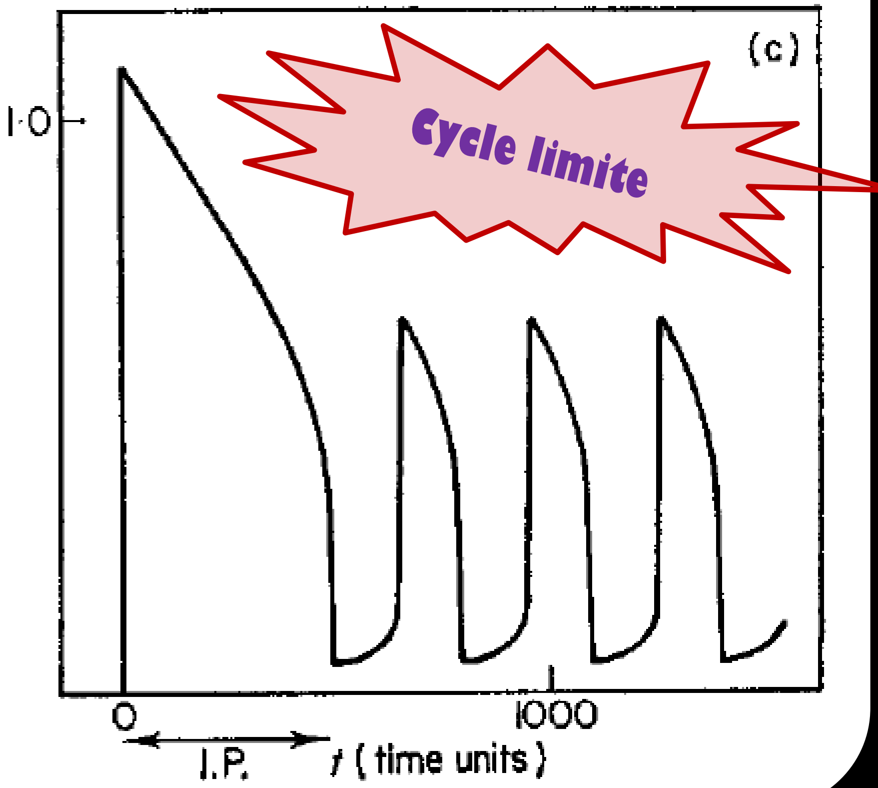
A Principle for Chemical Multivibration



Prototype d'un schéma de réaction

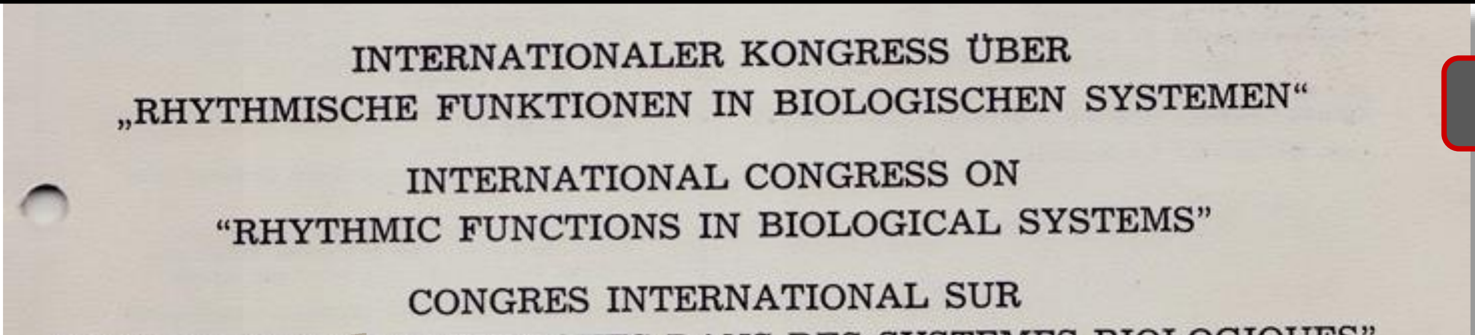
$$\dot{a} = k_1 a - k_2 b \frac{a}{a+K} - k_3 a^2 + k_4$$

$$\dot{b} = k_5 a - k_6 b,$$



Cycle limite

1975

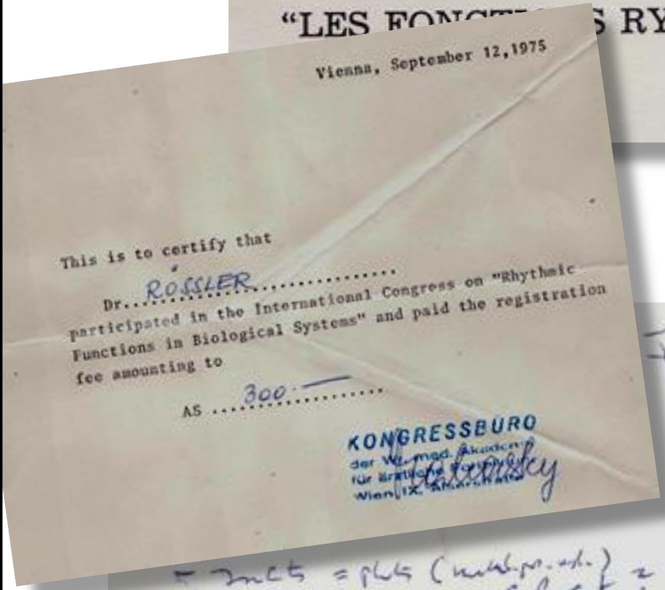


INTERNATIONALER KONGRESS ÜBER
„RHYTHMISCHE FUNKTIONEN IN BIOLOGISCHEN SYSTEMEN“

INTERNATIONAL CONGRESS ON
“RHYTHMIC FUNCTIONS IN BIOLOGICAL SYSTEMS”

CONGRES INTERNATIONAL SUR
“LES FONCT S RYTHMIQUES DANS DES SYSTEMES BIOLOGIQUES”

Wien/Vienna/Vienne, 8.—12. 9. 1975



Vienna, September 12,1975

This is to certify that

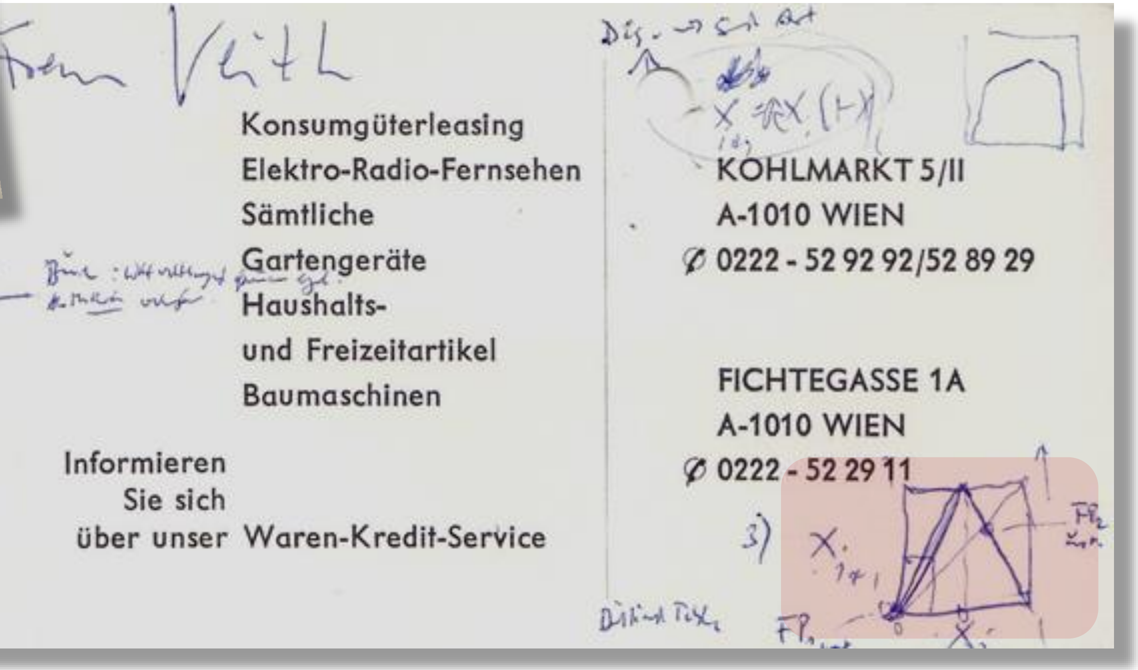
Dr. ROSSLER

participated in the International Congress on "Rhythmic Functions in Biological Systems" and paid the registration fee amounting to

AS 300.—

KONGRESSBURO
der W. med. Akademie
für ärztliche Fortbildung
Wien IX.

Vienne, 8-12 Septembre 1975



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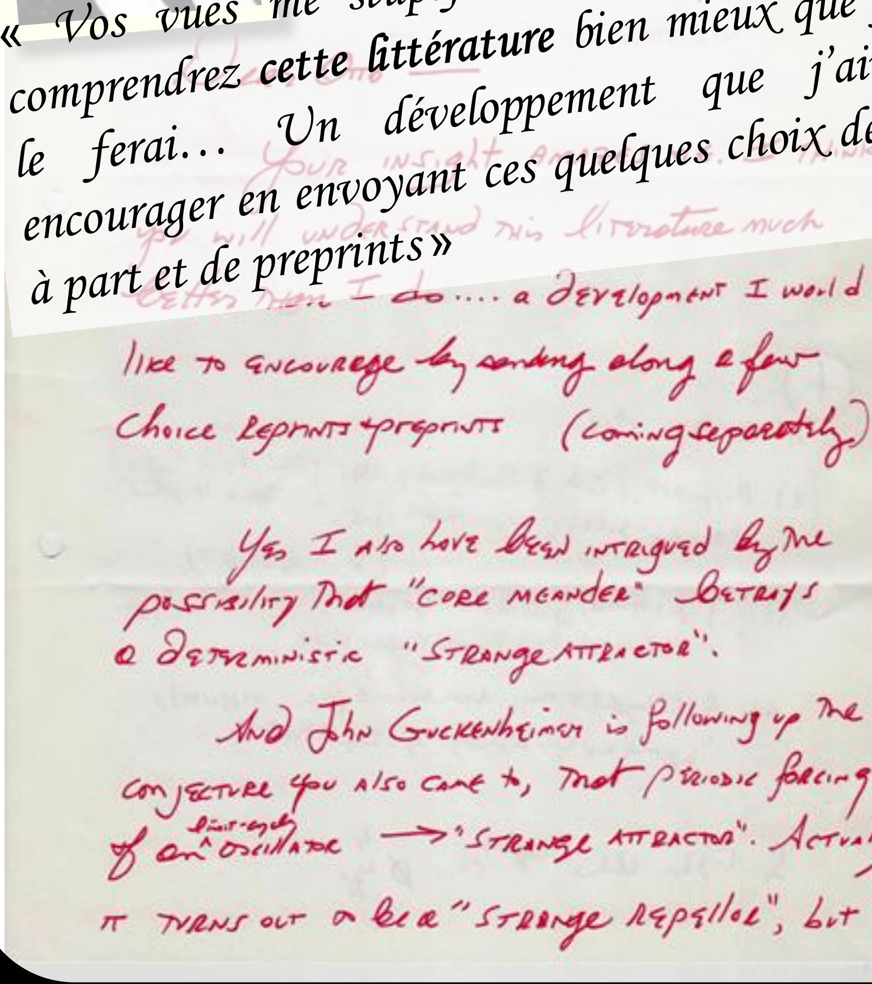
4)

October 7, 1975



PURDUE UNIVERSITY
DEPARTMENT OF BIOLOGICAL SCIENCES
WEST LAFAYETTE, INDIANA 47907

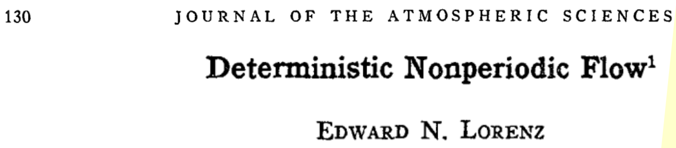
7 October

« *Vos vues me stupéfient. Je pense que vous comprendrez* ***cette littérature*** *bien mieux que je ne le ferai… Un développement que j'aimerai encourager en envoyant ces quelques choix de tirés à part et de preprints* »

you will understand this literature much better than I do.... a development I would like to encourage by sending along a few choice reprints + preprints (coming separately)

Yes I also have been intrigued by the possibility that "core meander" betrays a deterministic "strange attractor".

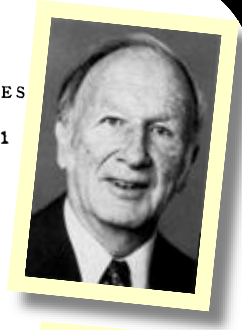
And John Guckenheimer is following up the conjecture you also came to, that periodic forcing of an ^limit-cycle oscillator → "strange attractor". Actually it turns out a bit a "strange repellor", but

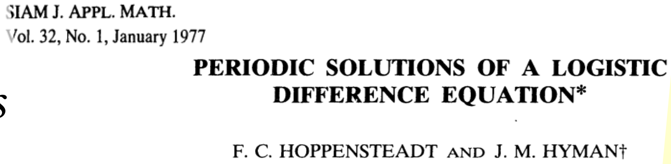


130 JOURNAL OF THE ATMOSPHERIC SCIENCES

Deterministic Nonperiodic Flow[1]

EDWARD N. LORENZ

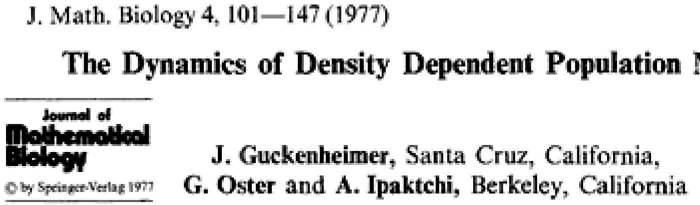




SIAM J. APPL. MATH.
Vol. 32, No. 1, January 1977

PERIODIC SOLUTIONS OF A LOGISTIC DIFFERENCE EQUATION*

F. C. HOPPENSTEADT AND J. M. HYMAN†

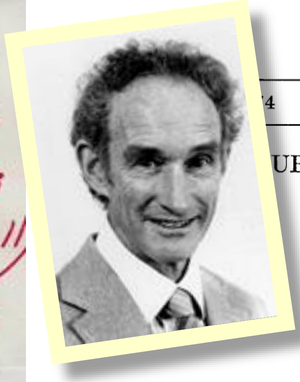


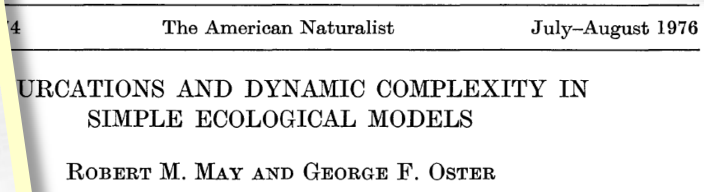
J. Math. Biology 4, 101—147 (1977)

The Dynamics of Density Dependent Population M

Journal of Mathematical Biology
© by Springer-Verlag 1977

J. Guckenheimer, Santa Cruz, California,
G. Oster and A. Ipaktchi, Berkeley, California



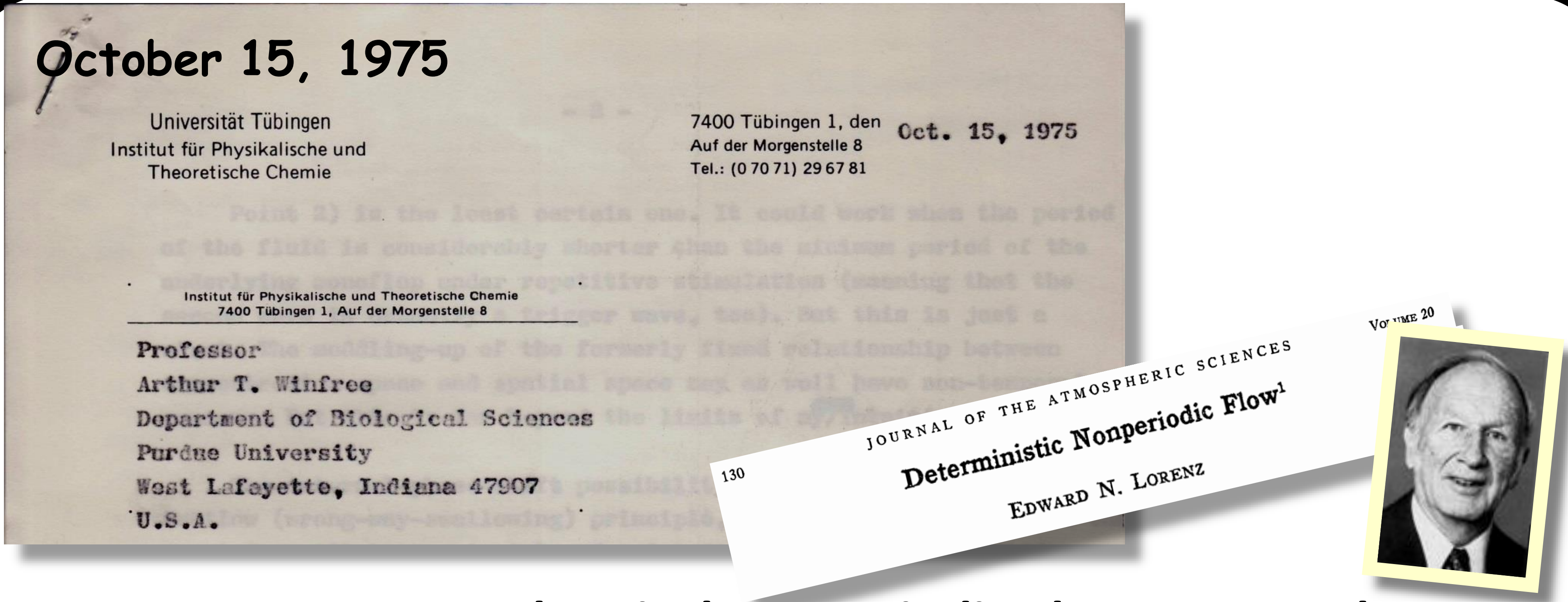


4 The American Naturalist July–August 1976

URCATIONS AND DYNAMIC COMPLEXITY IN SIMPLE ECOLOGICAL MODELS

ROBERT M. MAY AND GEORGE F. OSTER

October 15, 1975



Universität Tübingen
Institut für Physikalische und
Theoretische Chemie

7400 Tübingen 1, den Oct. 15, 1975
Auf der Morgenstelle 8
Tel.: (0 70 71) 29 67 81

Institut für Physikalische und Theoretische Chemie
7400 Tübingen 1, Auf der Morgenstelle 8

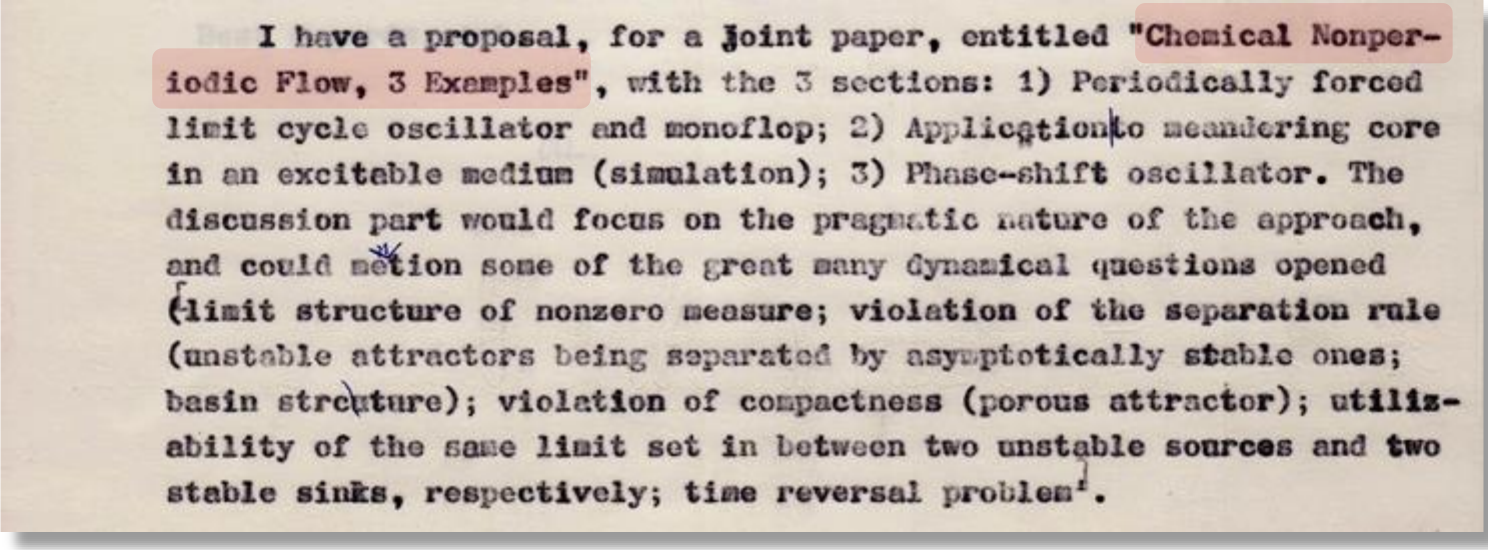
Professor
Arthur T. Winfree
Department of Biological Sciences
Purdue University
West Lafayette, Indiana 47907
U.S.A.

130 JOURNAL OF THE ATMOSPHERIC SCIENCES VOLUME 20

Deterministic Nonperiodic Flow[1]

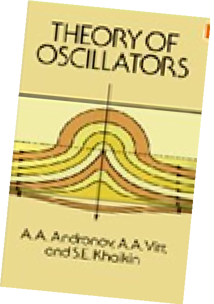
EDWARD N. LORENZ

« Chemical Nonperiodic Flow, 3 examples »



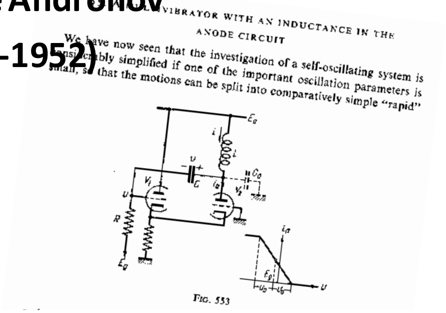
I have a proposal, for a joint paper, entitled "Chemical Nonperiodic Flow, 3 Examples", with the 3 sections: 1) Periodically forced limit cycle oscillator and monoflop; 2) Applicqtion to meandering core in an excitable medium (simulation); 3) Phase-shift oscillator. The discussion part would focus on the pragmatic nature of the approach, and could metion some of the great many dynamical questions opened (limit structure of nonzero measure; violation of the separation rule (unstable attractors being separated by asymptotically stable ones; basin strcuture); violation of compactness (porous attractor); utilizability of the same limit set in between two unstable sources and two stable sinks, respectively; time reversal problem'.

Le circuit universel



Alexandre Andronov

(1901-1952)

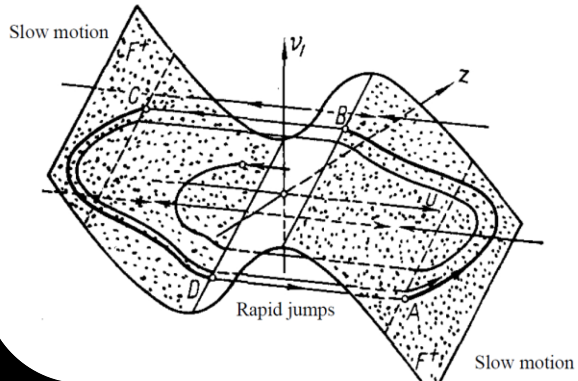


VIBRATOR WITH AN INDUCTANCE IN THE ANODE CIRCUIT

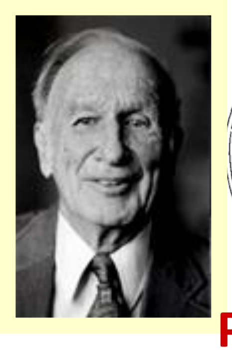
We have now seen that the investigation of a self-oscillating system is considerably simplified if one of the important oscillation parameters is small, so that the motions can be split into comparatively simple "rapid"

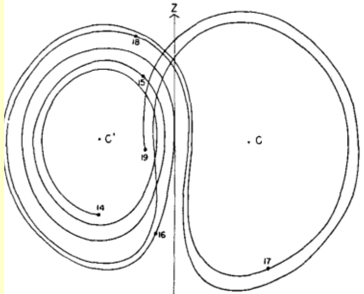
Fig. 553

$$\left\{\begin{aligned} \mu\dot u &= E_a - Ri_a(u) - \left(1+\frac{R}{\beta r}\right)u + (1-\beta)\frac{R}{\beta r}z - v_1 \\ \dot v_1 &= z \\ \dot z &= \frac{C_1}{\beta(1-\beta)C_2}n - \left(1+\frac{C_1}{\beta C_2}\right)\frac{z}{1-\beta} \end{aligned}\right.$$

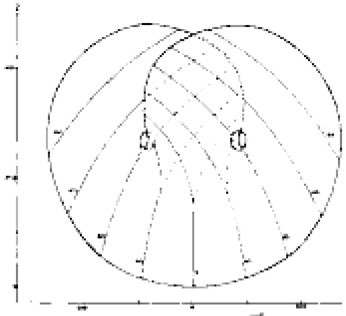


Lorenz 1963

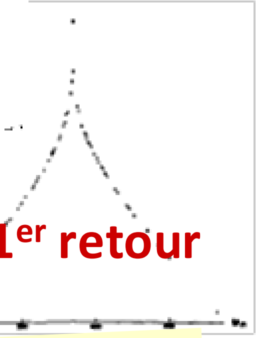




Portrait des états



Topologie



Application de 1^{er} retour

Royal McBee LGP 300

PERIOD THREE IMPLIES CHAOS

TIEN-YIEN LI AND JAMES A. YORKE

Theorem 1. *Let J be an interval and let $F: J \to J$ be continuous.* *which the points $b = F(a)$, $c = F^2(a)$ and $d = F^3(a)$, satisfy*

$$d \leqq a < b < c \text{ (or } d \geqq a > b > c).$$

Then

T1: *for every $k = 1, 2, \cdots$ there is a periodic point in J havin*

Furthermore,

T2: *there is an uncountable set $S \subset J$ (containing no periodic* *the following conditions:*

(A) *For every $p, q \in S$ with $p \neq q$,*

$$\limsup_{n\to\infty} |F^n(p) - F^n(q)| > 0 \tag{2.1}$$

and

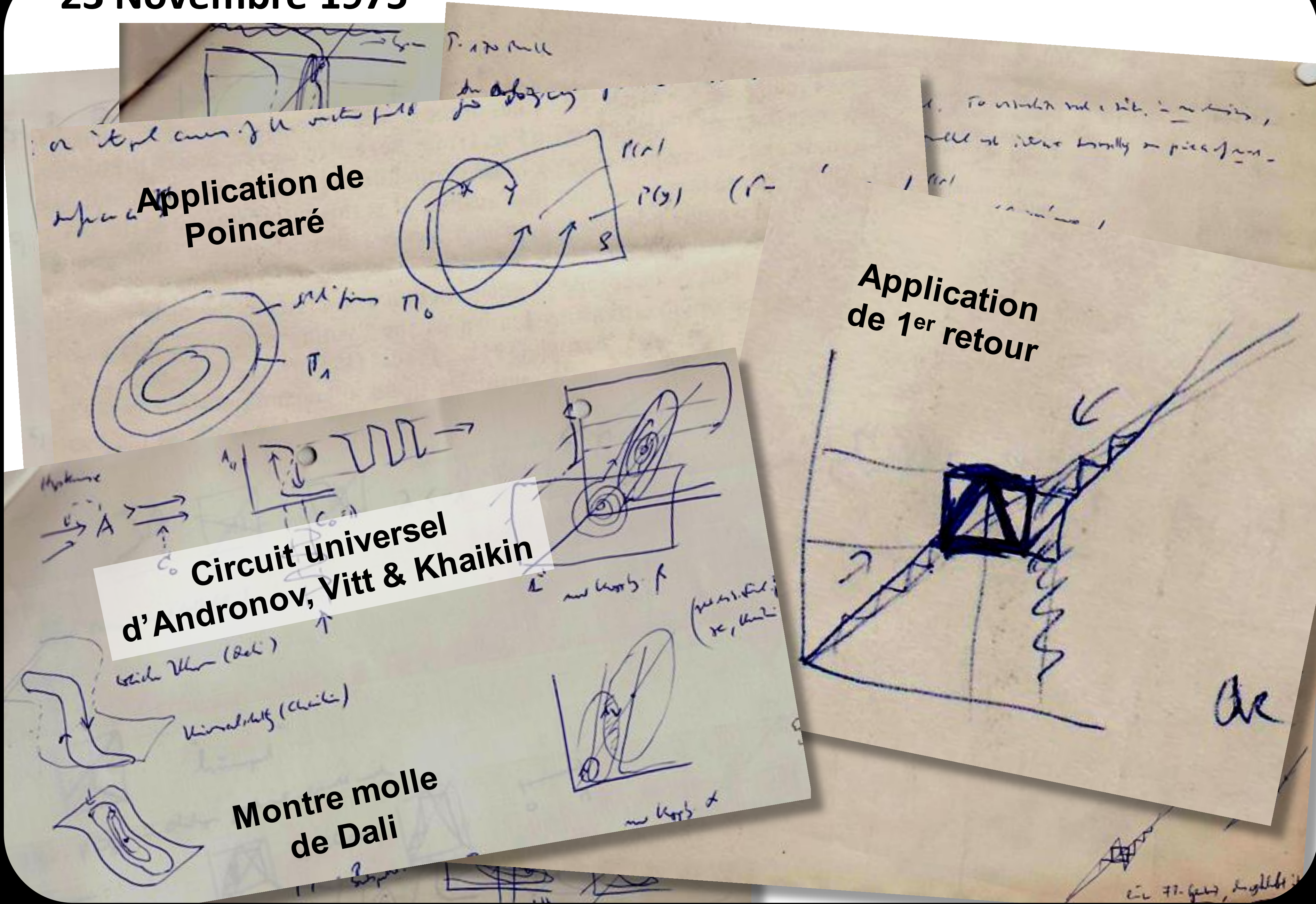
$$\liminf_{n\to\infty} |F^n(p) - F^n(q)| = 0. \tag{2.2}$$

(B) *For every $p \in S$ and periodic point $q \in J$,*

$$\limsup_{n\to\infty} |F^n(p) - F^n(q)| > 0.$$

Remarks. Notice that if there is a periodic point with period 3, th will be satisfied.

23 Novembre 1975

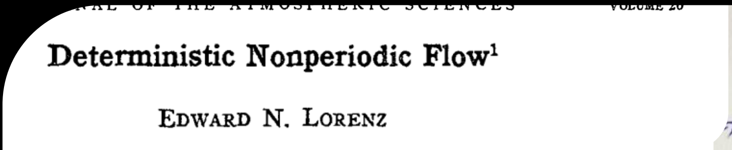


Application de Poincaré

Application de 1er retour

Circuit universel d'Andronov, Vitt & Khaikin

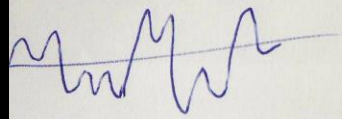
Montre molle de Dali

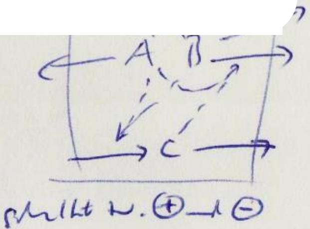


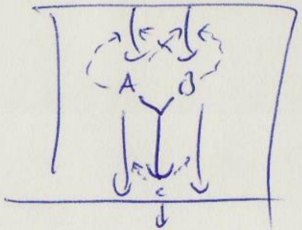
NAL OF THE ATMOSPHERIC SCIENCES VOLUME 20

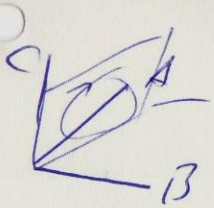
Deterministic Nonperiodic Flow[1]

EDWARD N. LORENZ



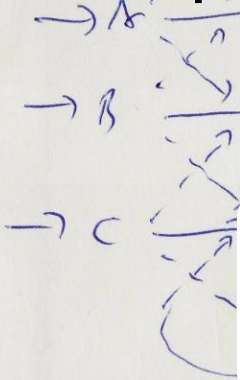


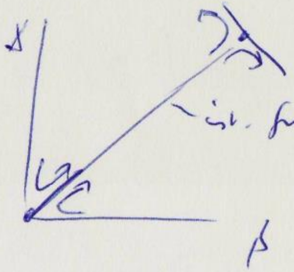


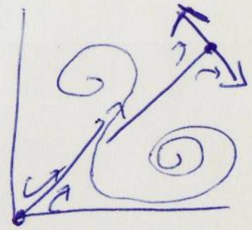


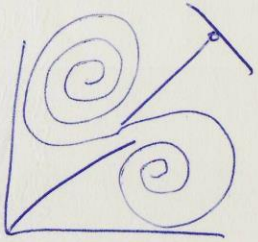
Essais de construction d'une réaction chimique « à la Lorenz »

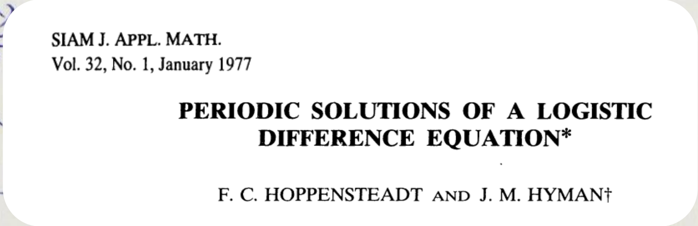
Suche:











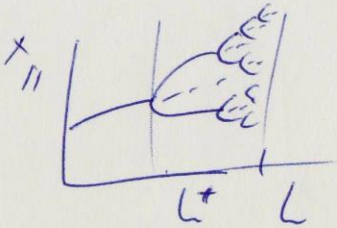
SIAM J. APPL. MATH.
Vol. 32, No. 1, January 1977

PERIODIC SOLUTIONS OF A LOGISTIC DIFFERENCE EQUATION*

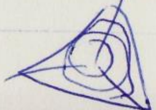
F. C. HOPPENSTEADT AND J. M. HYMAN†

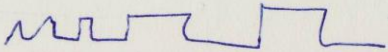
$$\dot{x}_i = x_i f(x_1, \ldots),$$

$$\frac{\partial f}{\partial x_j} < 0$$

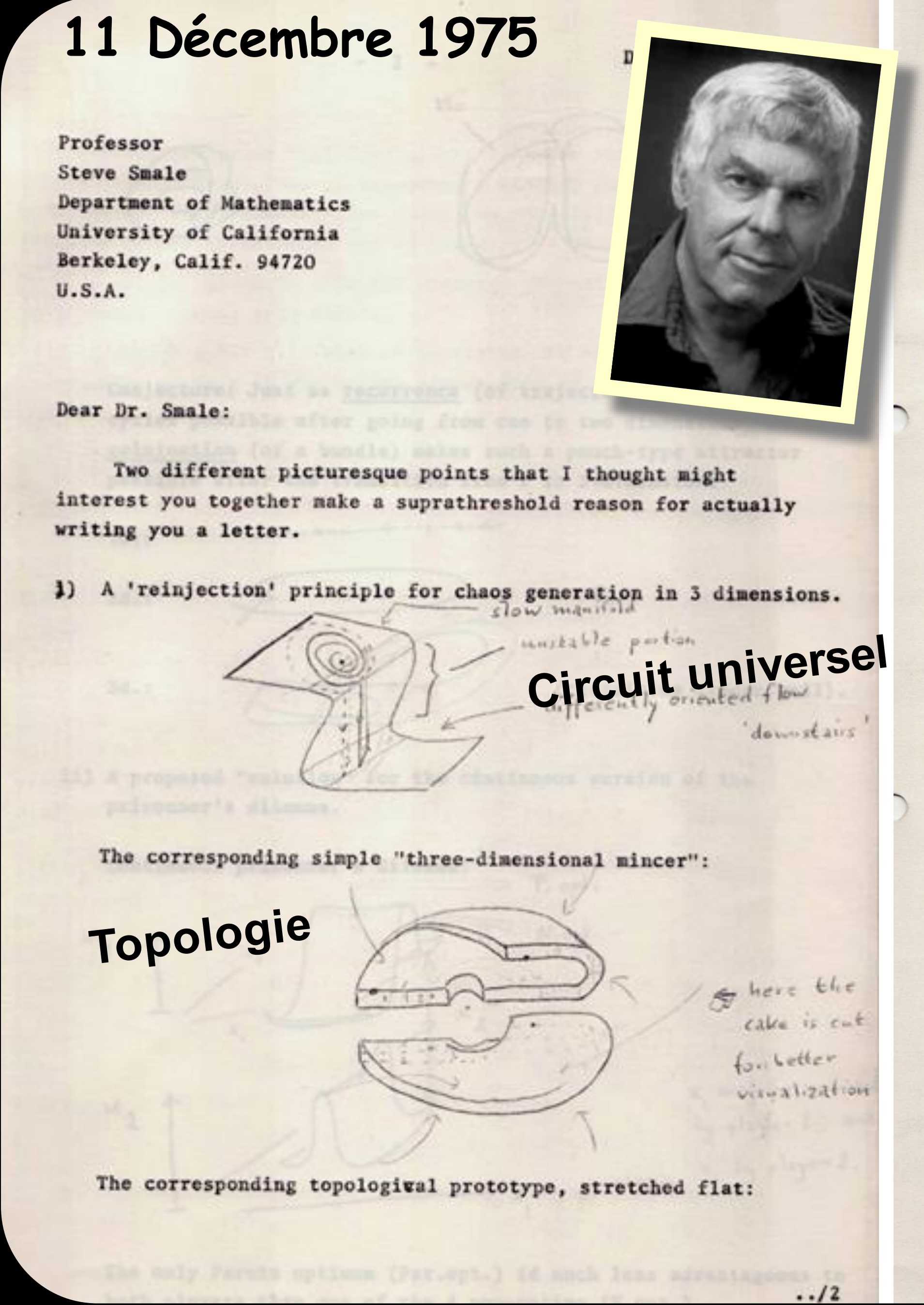


$$N_i = N_i\left(1 - \sum_j \alpha_{ij} N_j\right)$$





11 Décembre 1975



D

Professor
Steve Smale
Department of Mathematics
University of California
Berkeley, Calif. 94720
U.S.A.

Dear Dr. Smale:

Two different picturesque points that I thought might interest you together make a suprathreshold reason for actually writing you a letter.

i) A 'reinjection' principle for chaos generation in 3 dimensions.

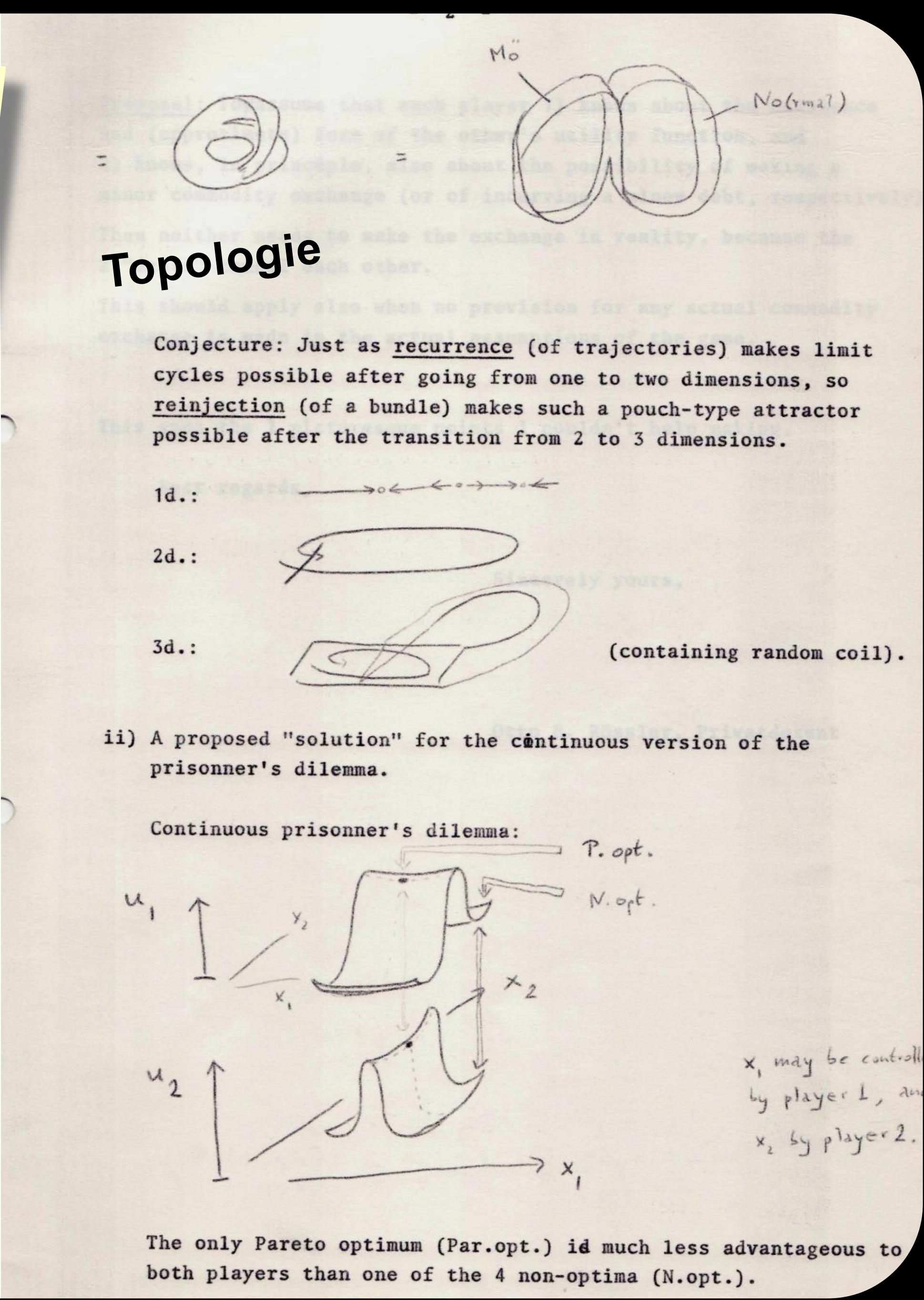
Circuit universel

The corresponding simple "three-dimensional mincer":

Topologie

The corresponding topological prototype, stretched flat:

../2



Topologie

Conjecture: Just as recurrence (of trajectories) makes limit cycles possible after going from one to two dimensions, so reinjection (of a bundle) makes such a pouch-type attractor possible after the transition from 2 to 3 dimensions.

1d.:

2d.:

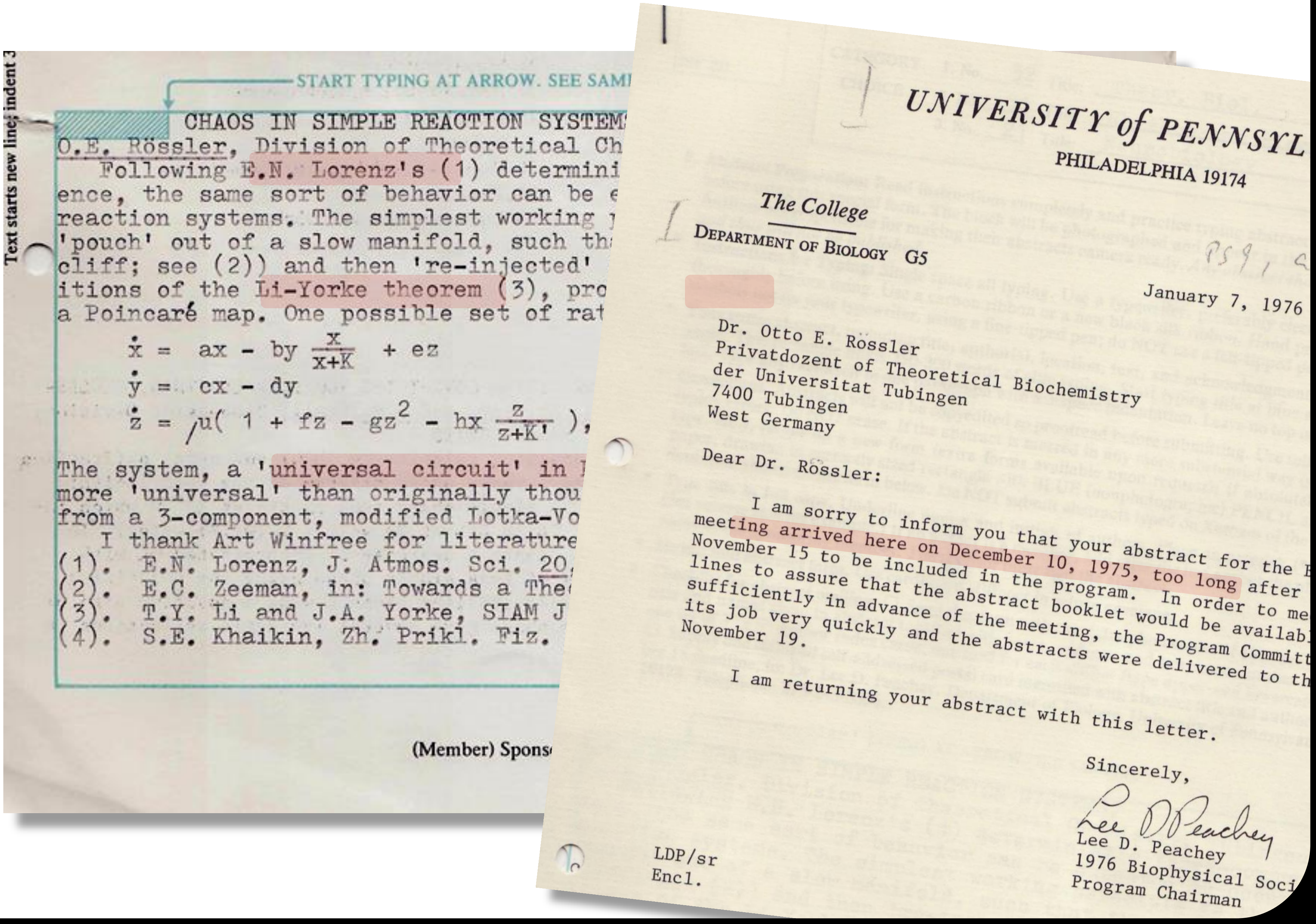
3d.: (containing random coil).

ii) A proposed "solution" for the continuous version of the prisonner's dilemma.

Continuous prisonner's dilemma:

The only Pareto optimum (Par.opt.) id much less advantageous to both players than one of the 4 non-optima (N.opt.).

1er Décembre 1975



Text starts new line; indent 3

START TYPING AT ARROW. SEE SAM

CHAOS IN SIMPLE REACTION SYSTEM

O.E. Rössler, Division of Theoretical Ch

Following E.N. Lorenz's (1) determini
ence, the same sort of behavior can be e
reaction systems. The simplest working
'pouch' out of a slow manifold, such th
cliff; see (2)) and then 're-injected'
itions of the Li-Yorke theorem (3), pro
a Poincaré map. One possible set of rat

$$\dot{x} = ax - by\frac{x}{x+K} + ez$$

$$\dot{y} = cx - dy$$

$$\dot{z} = \mu(1 + fz - gz^2 - hx\frac{z}{z+K'}),$$

The system, a 'universal circuit' in
more 'universal' than originally thou
from a 3-component, modified Lotka-Vo
I thank Art Winfree for literature

(1). E.N. Lorenz, J. Atmos. Sci. 20,
(2). E.C. Zeeman, in: Towards a The
(3). T.Y. Li and J.A. Yorke, SIAM J
(4). S.E. Khaikin, Zh. Prikl. Fiz.

(Member) Spons

I

UNIVERSITY of PENNSYL
PHILADELPHIA 19174

The College
DEPARTMENT OF BIOLOGY G5

I

PS 9, 1

January 7, 1976

Dr. Otto E. Rössler
Privatdozent of Theoretical Biochemistry
der Universitat Tubingen
7400 Tubingen
West Germany

Dear Dr. Rössler:

I am sorry to inform you that your abstract for the B
meeting arrived here on December 10, 1975, too long after
November 15 to be included in the program. In order to me
lines to assure that the abstract booklet would be availab
sufficiently in advance of the meeting, the Program Committ
its job very quickly and the abstracts were delivered to th
November 19.

I am returning your abstract with this letter.

Sincerely,

Lee D Peachey

Lee D. Peachey
1976 Biophysical Soci
Program Chairman

LDP/sr
Encl.

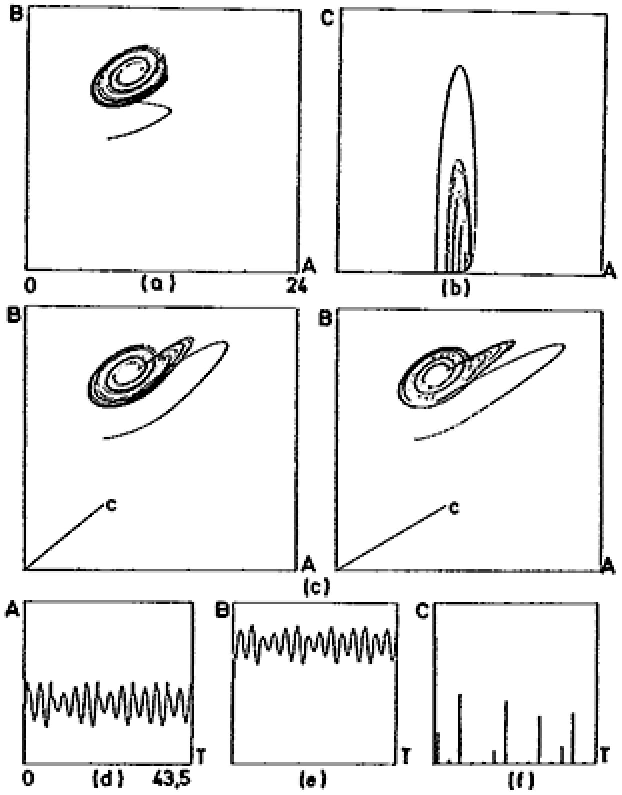
Chaotic Behavior in Simple Reaction Systems

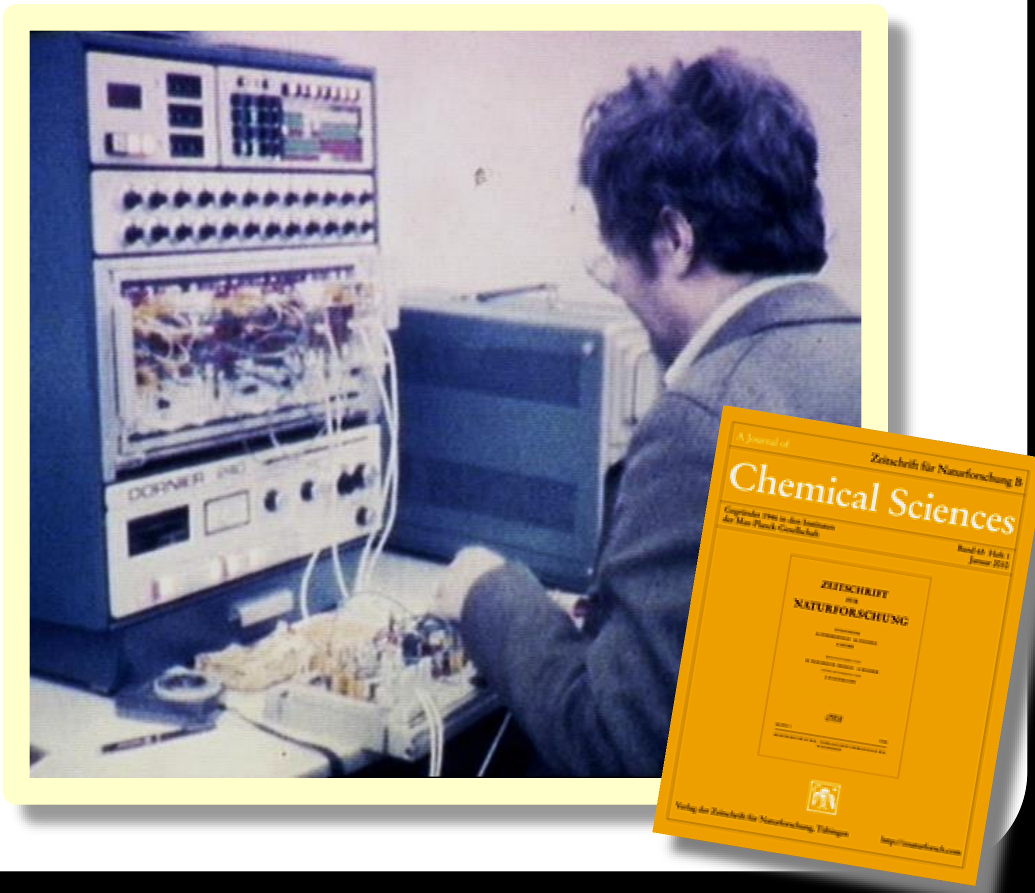
Otto E. Rössler

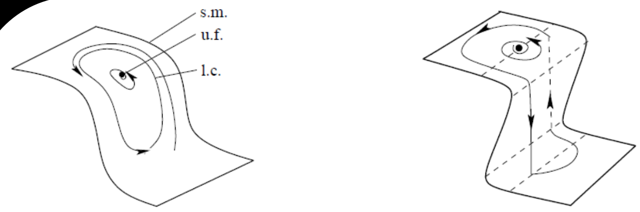
Institut für Physikalische und Theoretische Chemie der Universität Tübingen

(Z. Naturforsch. 31 a, 259—264 [1976]; received January 5, 1976)

Chemical system theory, exotic kinetics, nonperiodic oscillation, 3-variable dynamical systems, strange attractors

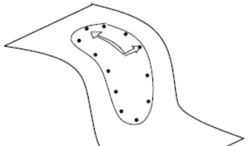


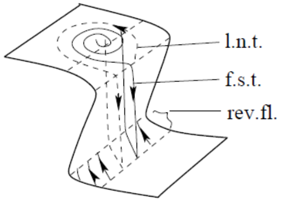




(a) Nearly linear mode.
(= limit cycle)

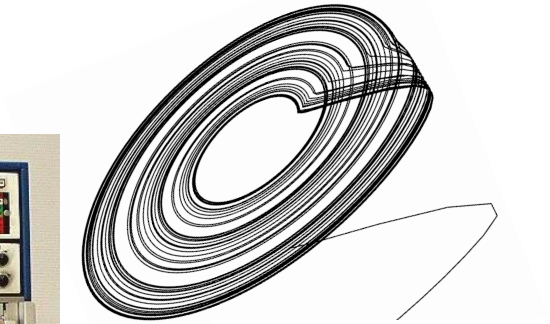
(b) Relaxation mode.
(= limit cycle)





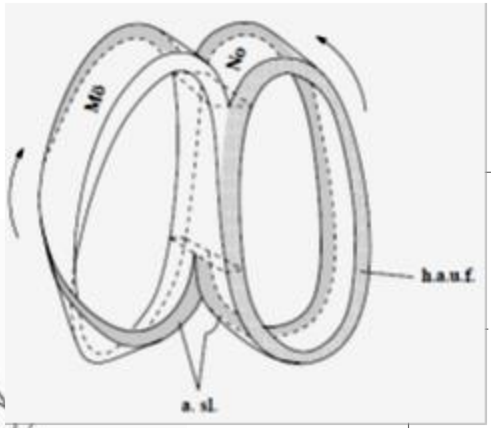
(d) Chaos-producing mode (see text).

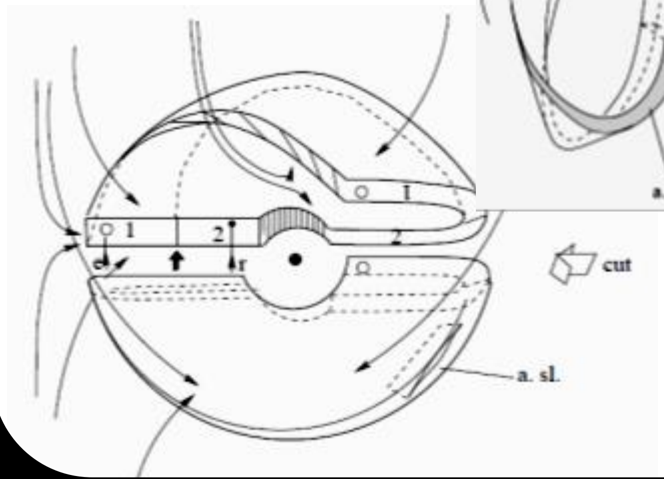
ircuit. s.m.= slow manifold, u.f.= unstable
low manifold in (b) and (d) is unstable, f.s.t.
ched trajectory", rev.fl.= reversed direction

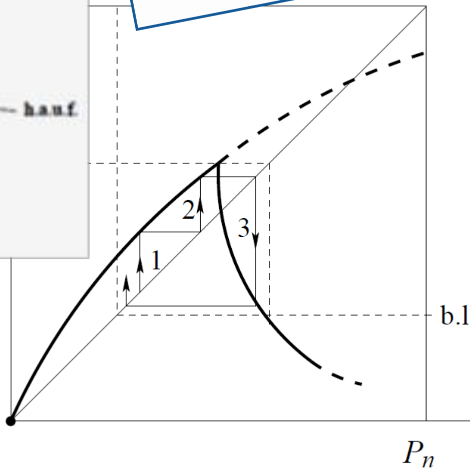


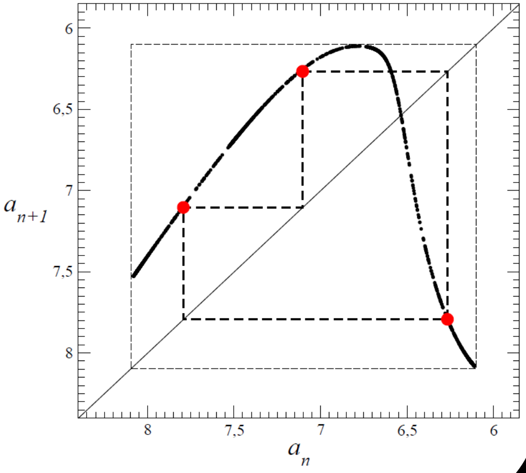
$$\begin{cases} \dot{a} &= k_1 + k_2 a - \dfrac{(k_3 b + k_4 c)a}{a+K} \\ \dot{b} &= k_5 a - k_6 b \\ \mu\dot{c} &= k_7 a + k_8 c - k_9 c^2 - \dfrac{k_{10} c}{c+K'} \end{cases}$$

PERIOD THREE IMPLIES CHAOS

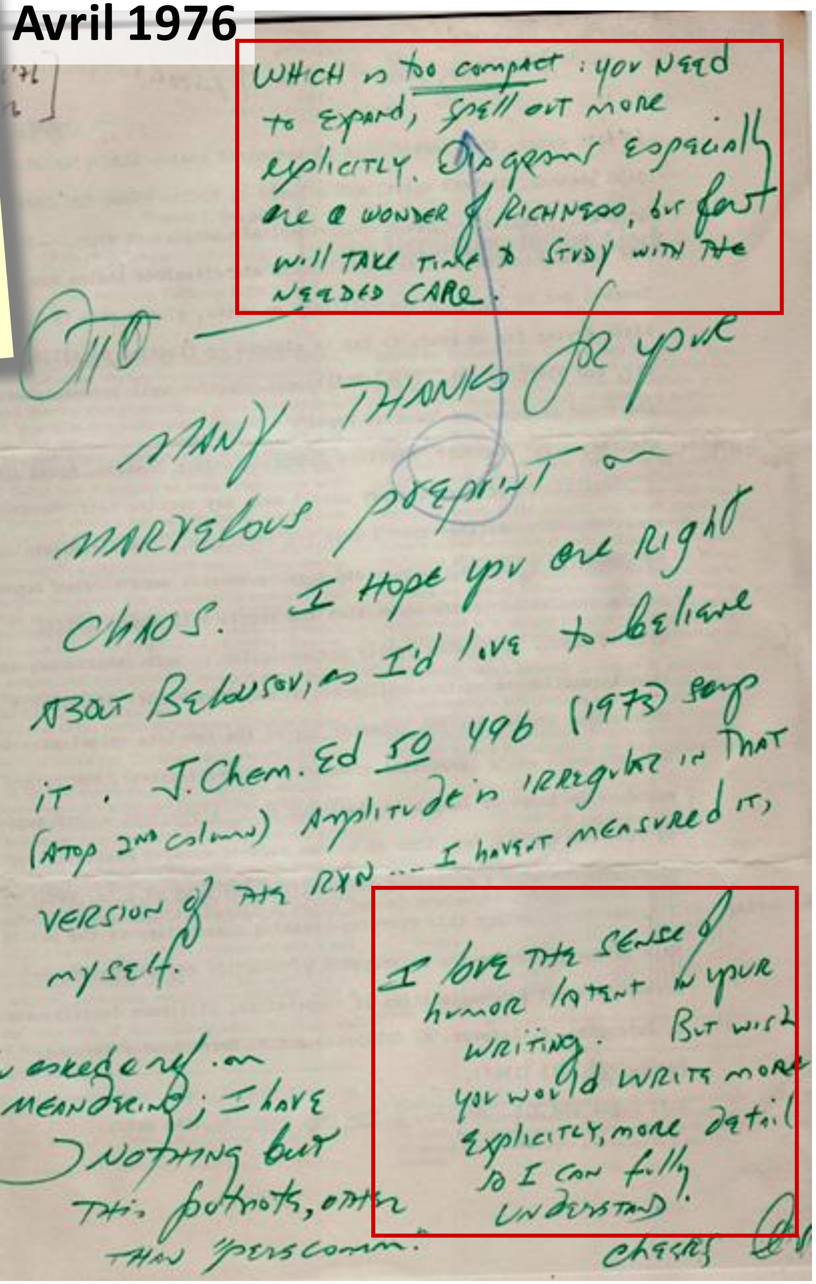








Avril 1976



WHICH is too compact: you need to expand, spell out more explicitly. Diagrams especially are a wonder of richness, but [illegible] will take time to study with the needed care.

Otto — Many thanks for your marvelous preprint on chaos. I hope you are right about Belousov, as I'd love to believe it. J. Chem. Ed 50 496 (1973) says (atop 2nd column) amplitude is irregular in that version of the rxn ... I haven't measured it myself.

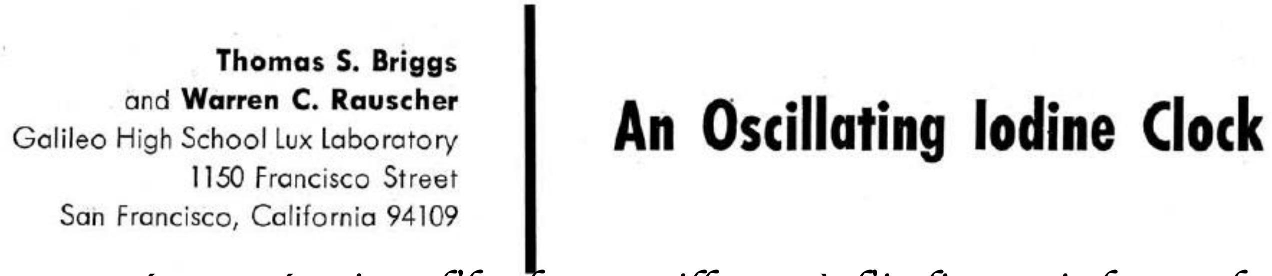
You asked a ref. on meandering; I have nothing but this footnote, other than "pers comm."

I love the sense of humor latent in your writing. But wish you would write more explicitly, more detail so I can fully understand.

Cheers

« qui est trop compact: vous avez besoin de détailler, d'expliquer clairement et plus explicitement. Les diagrammes sont plus spécialement un émerveillement de richesses, mais ils demanderont du temps pour être étudiés avec l'attention requise. »

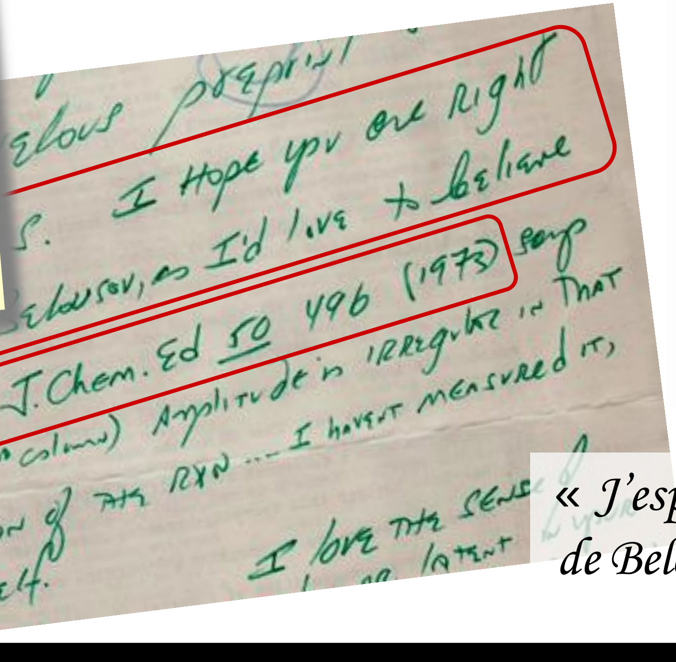
« J'adore le sens d'humour latent dans votre écriture. Mais voudriez-vous écrire plus explicitement, avec plus de détails de manière à ce que je puisse comprendre complètement? »



Thomas S. Briggs
and **Warren C. Rauscher**
Galileo High School Lux Laboratory
1150 Francisco Street
San Francisco, California 94109

An Oscillating Iodine Clock

« Nous avons trouvé une réaction d'horloge oscillante à l'iodine qui donne des variations saisissantes de couleurs, de l'or au bleu, à l'aide de simples réactifs.[…] Plusieurs variations de cette réaction existe. Nous avons observé des oscillations de faible durée de vie en substituant le 2,4-pentanedione pour de l'acide malonique. Le cérium peut être utilisé à la place du manganèse, ce qui donne des oscillations de hautes fréquences et d'amplitudes variable »



elous preprint

S. I Hope you are Right

About Belousov, as I'd love to believe

it. J. Chem. Ed 50 496 (1973) says

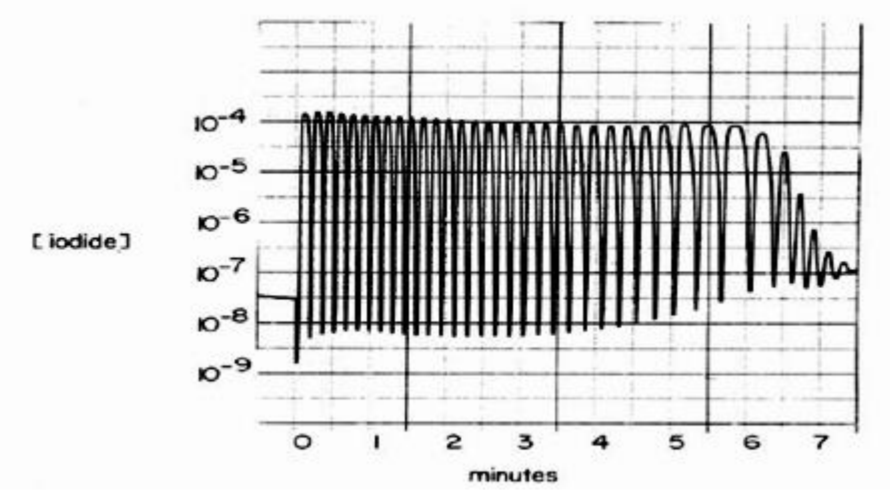
(Atop 2nd column) Amplitude is irregular in that

version of the RXN ... I haven't measured it,

myself.

I love the sense

latent

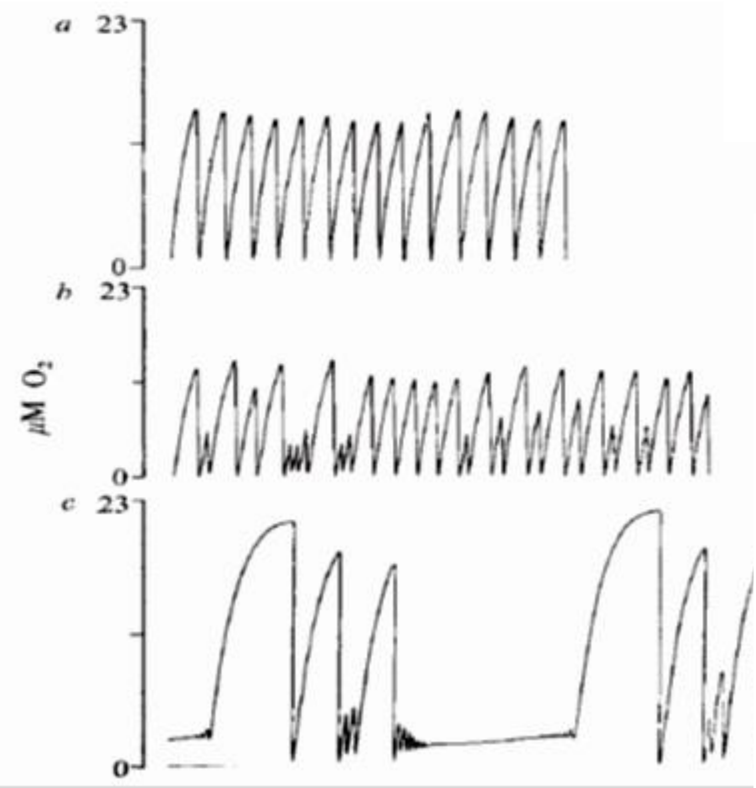


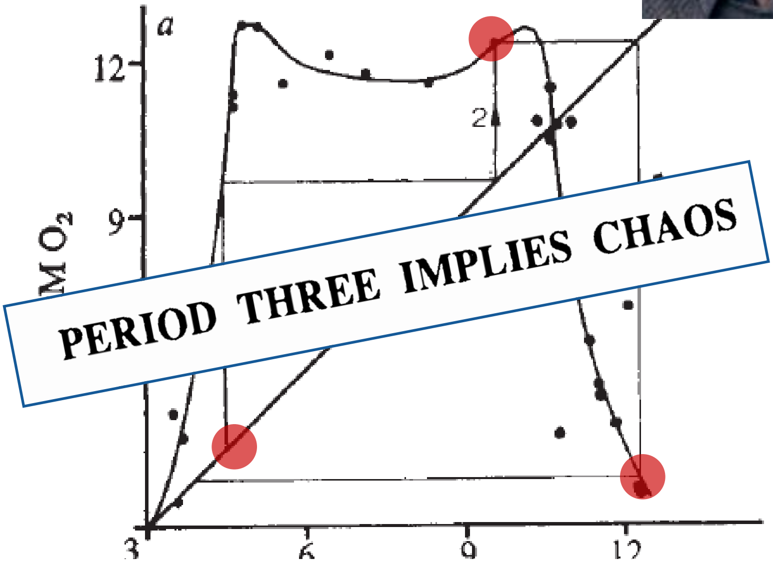
« J'espère que vous avez raison à propos de Belousov, comme j'aimerai le croire. »

Chaos in an enzyme reaction

LARS FOLKE OLSEN
HANS DEGN

Institute of Biochemistry,
Odense University,
Odense, Denmark



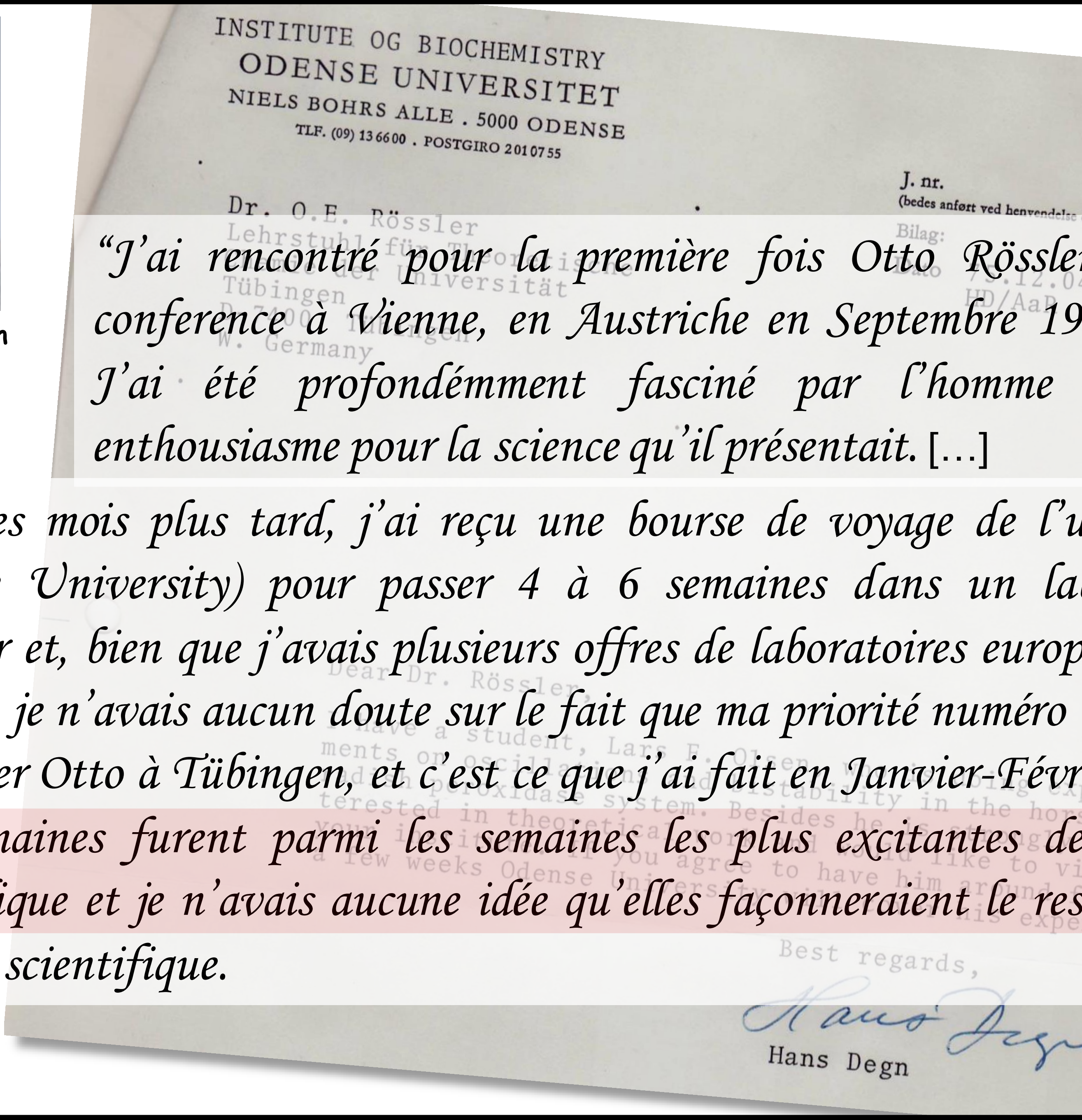


PERIOD THREE IMPLIES CHAOS

solutions. The argument is based on a theorem by Li and Yorke[3]. Here we report the finding of chaotic behaviour as an experimental result in an enzyme system (peroxidase). Like Rössler[2] we base our identification of chaos on the theorem by Li and Yorke[3].

Nature Vol. 267 12 May 1977

Lars Folke Olsen



INSTITUTE OG BIOCHEMISTRY
ODENSE UNIVERSITET
NIELS BOHRS ALLE . 5000 ODENSE
TLF. (09) 13 66 00 . POSTGIRO 2010755

Dr. O.E. Rössler
Lehrstuhl für Theoretische
Chemie der Universität
Tübingen
D-7400 Tübingen
W. Germany

J. nr.
(bedes anført ved henvendelse om denne sag)
Bilag:
Dato 75.12.04
HD/AaP

Dear Dr. Rössler,

Best regards,

Hans Degn

“J’ai rencontré pour la première fois Otto Rössler à une conference à Vienne, en Austriche en Septembre 1975. […] *J’ai été profondémment fasciné par l’homme et son enthousiasme pour la science qu’il présentait.* […]

Quelques mois plus tard, j’ai reçu une bourse de voyage de l’université (Odense University) pour passer 4 à 6 semaines dans un laboratoire étranger et, bien que j’avais plusieurs offres de laboratoires européens très réputés, je n’avais aucun doute sur le fait que ma priorité numéro une était de visiter Otto à Tübingen, et c’est ce que j’ai fait en Janvier-Février 1976. Ces semaines furent parmi les semaines les plus excitantes de ma vie scientifique et je n’avais aucune idée qu’elles façonneraient le reste de ma carrière scientifique.

The Chemical Engineering Journal, 12 (1976) 9–21
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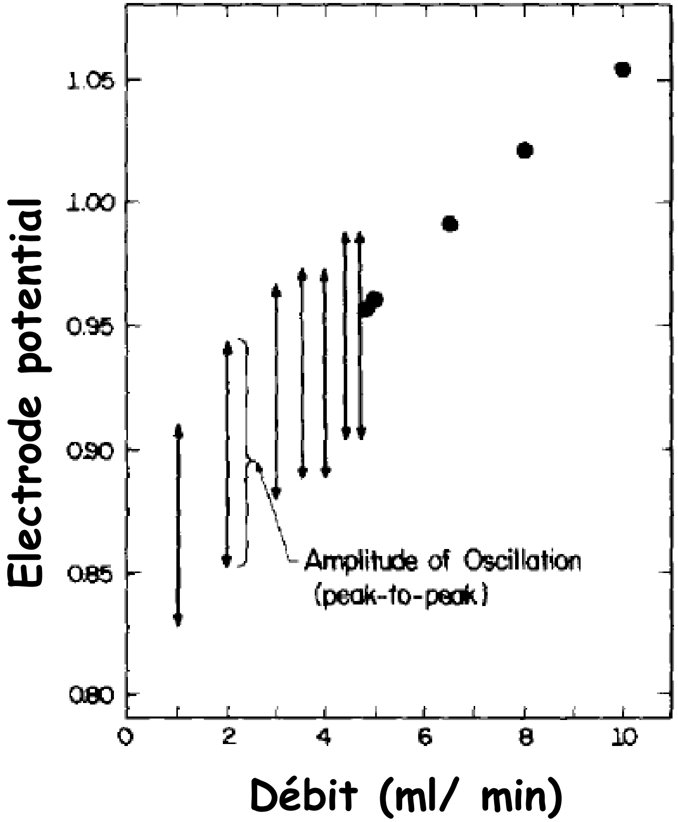
The Belousov–Zhabotinskii Reaction in

K. R. GRAZIANI*, J. L. HUDSON** and R.


1976



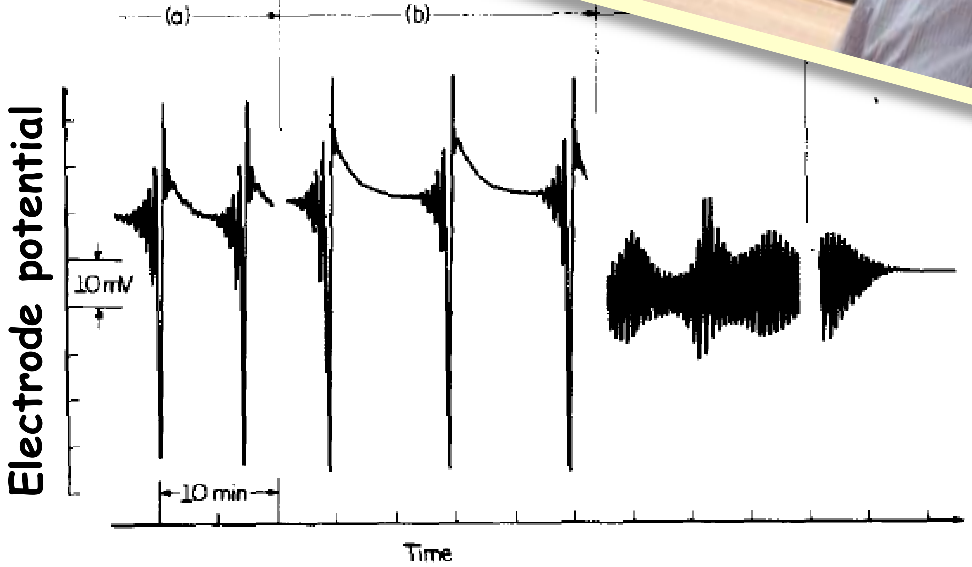


Fig. 3. Effect of sodium bromate feed rate on oscillations: (a) 1.28 ml min^{-1}; (b) 1.32 ml min^{-1}, the normal flow rate of bromate for a total flow of 4.7 ml min^{-1}; (c) 1.33 ml min^{-1}; (d) 1.36 ml min^{-1}.

Fluctuations de pic à pic des oscillations entretenues

1977

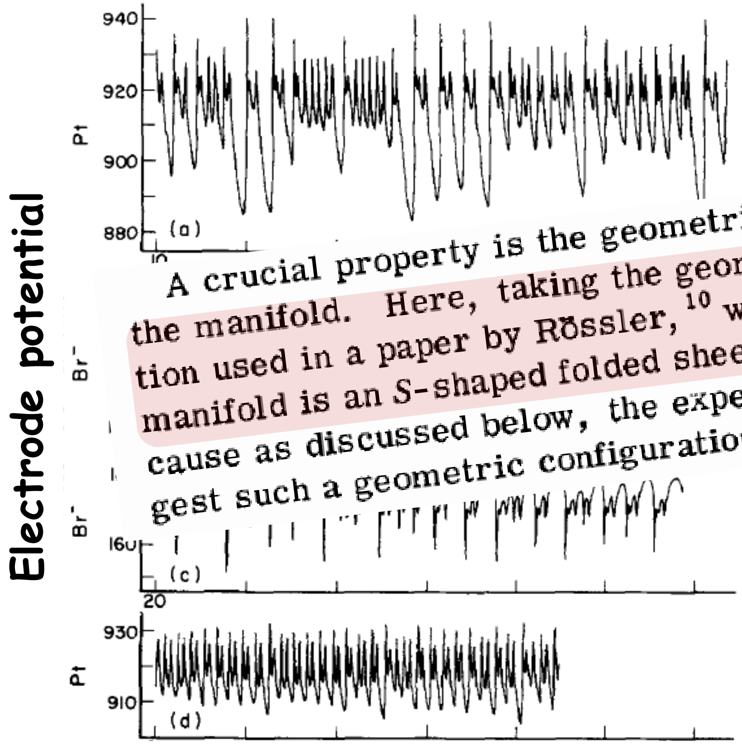
Experimental evidence of chaotic states in the Belousov–Zhabotinskii reaction

R. A. Schmitz, K. R. Graziani,[a] and J. L. Hudson[b]

Department of Chemical Engineering, University of Illinois, Urbana, Illinois 61801

(Received 3 May 1977)

Experimental results are reported which show strong evidence that the Belousov–Zhabotinskii reaction proceeds in an intrinsic chaotic (sustained time-dependent, nonperiodic) manner over a range of residence



A crucial property is the geometric configuration of the manifold. Here, taking the geometric representation used in a paper by Rössler,[10] we suppose that the manifold is an S-shaped folded sheet (see Fig. 4), because as discussed below, the experimental results suggest such a geometric configuration.

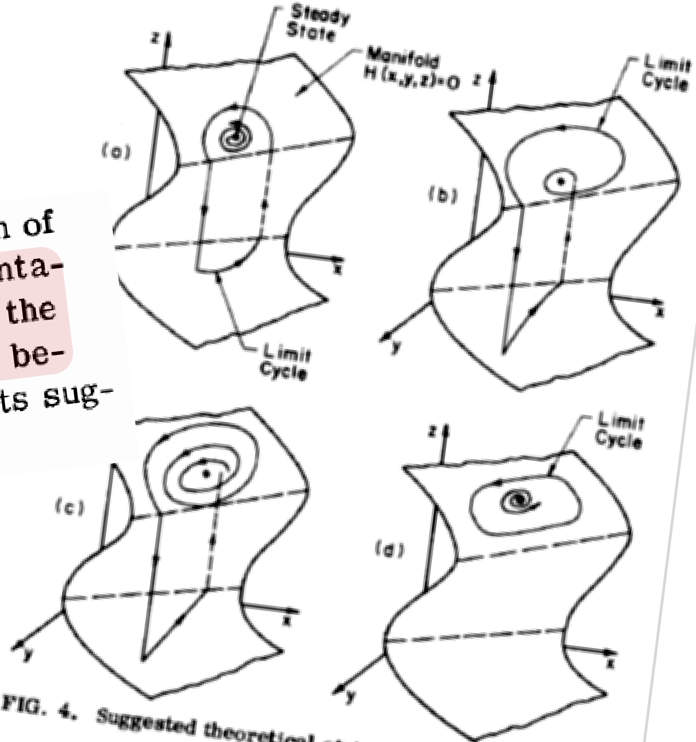


FIG. 4. Suggested theoretical state space trajectories.

3040 The Journal of Chemical Physics, Vol. 67, No. 7, 1 October 1977

1978

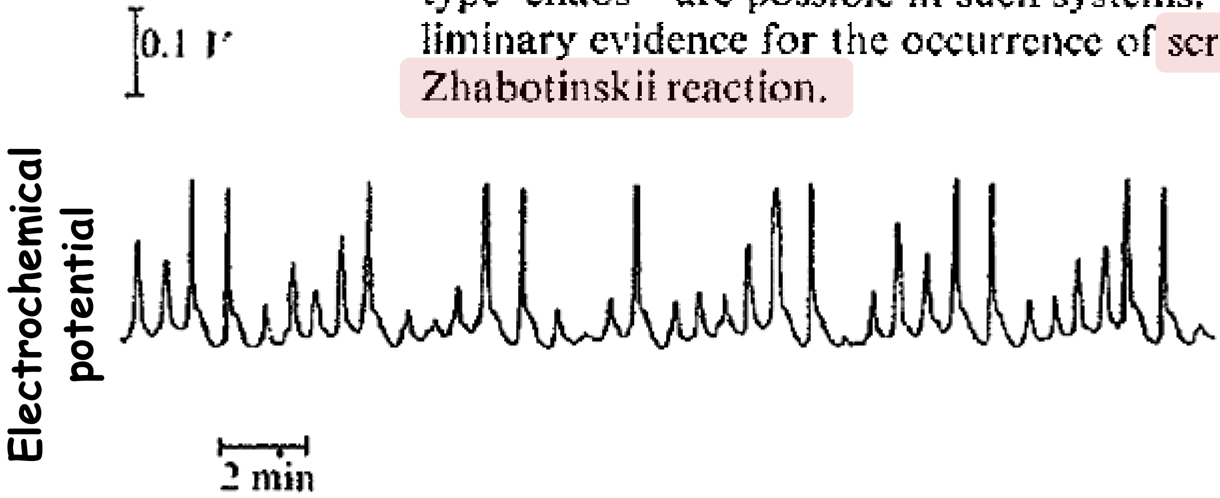
Chaos in the Zhabotinskii reaction

OTTO E. ROSSLER
sical and Theoretical Chemistry,
bingen,
Theoretical Physics,
ttgart

KLAUS WEGMANN
mical Plant Physiology,
bingen, 7400 Tübingen, FRG

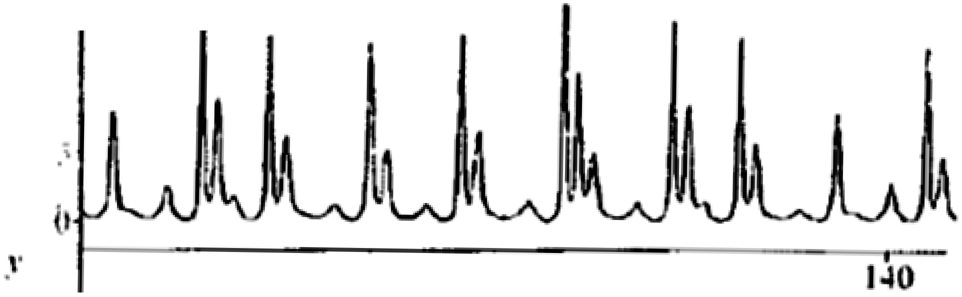
THE Belousov–Zhabotinskii reaction is a chemical Bonhoeffer–van der Pol circuit, that is, a relaxation oscillator that can be run as both an astable and a monostable ‘flip-flop’[1–3]. Apparently

type’ chaos[20] are possible in such systems. We present here preliminary evidence for the occurrence of screw-type chaos in the Zhabotinskii reaction.



compared to

$$
\begin{cases}
\dot{x} = -y - z \\
\dot{y} = x + 0.55\, y \\
\dot{z} = 2 - 4z + xz
\end{cases}
$$



Nature Vol. 271 5 January 1978

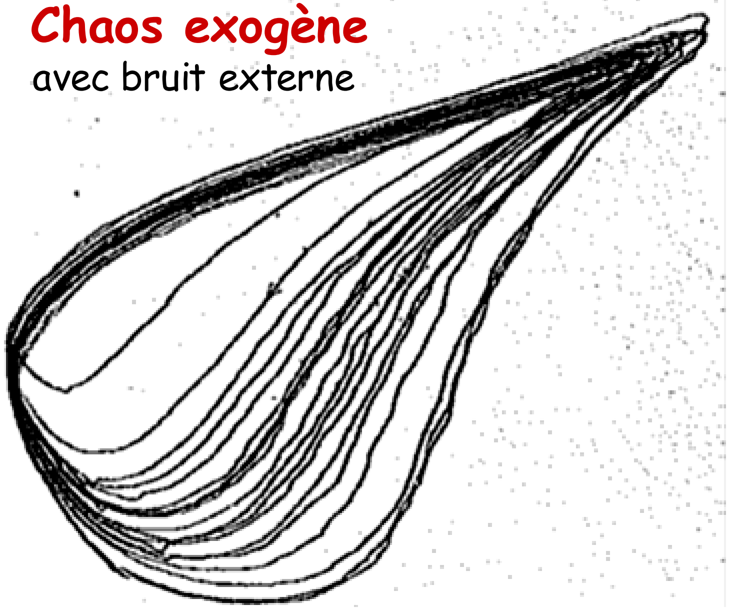
Different Kinds of Chaotic Oscillations in the Belousov-Zhabotinskii Reaction

Klaus Wegmann
Institut für Chemische Pflanzenphysiologie der Universität

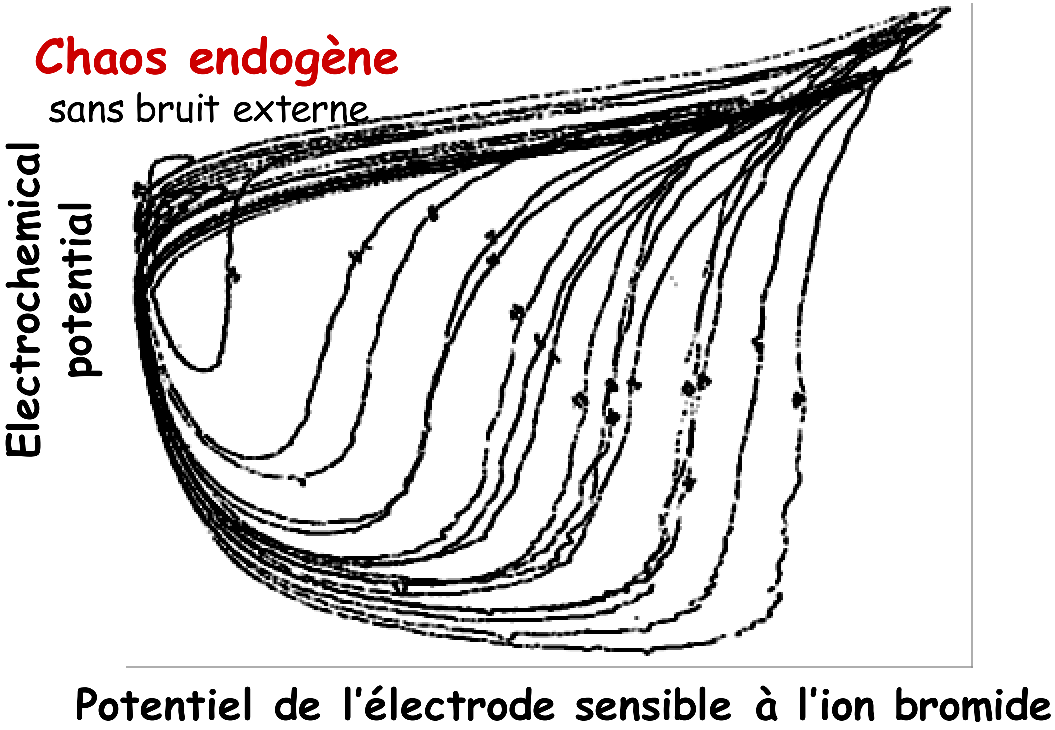
and

Otto E. Rössler
Institut für Physikalische und Theoretische Chemie der Ur
Institut für Theoretische Physik der Universität Stuttgart

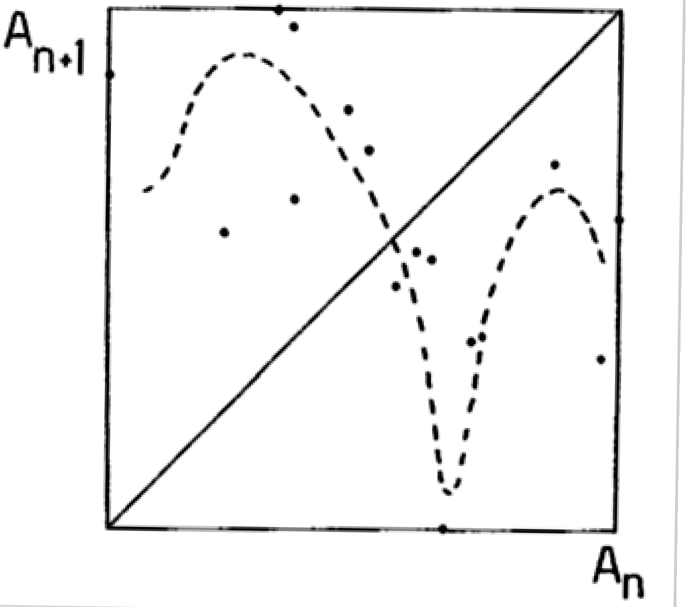
Chaos exogène
avec bruit externe



Chaos endogène
sans bruit externe



Application de 1er retour



Z. Naturforsch. 33 a, 1179—1183 (1978); received July 19, 1978

Different Types of Chaos in Two Simple Differential Equations*

Otto E. Rössler

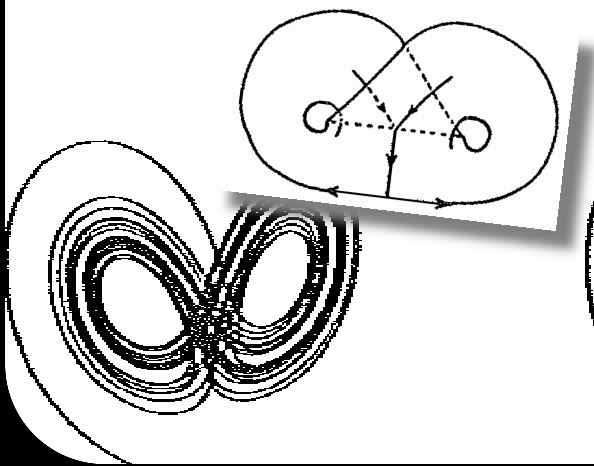
Institute for Physical and Theoretical Chemistry, Division of Theoretical Chemistry,
University of Tübingen

(Z. Naturforsch. 31 a, 1664—1670 [1976]; received November 10, 1976)

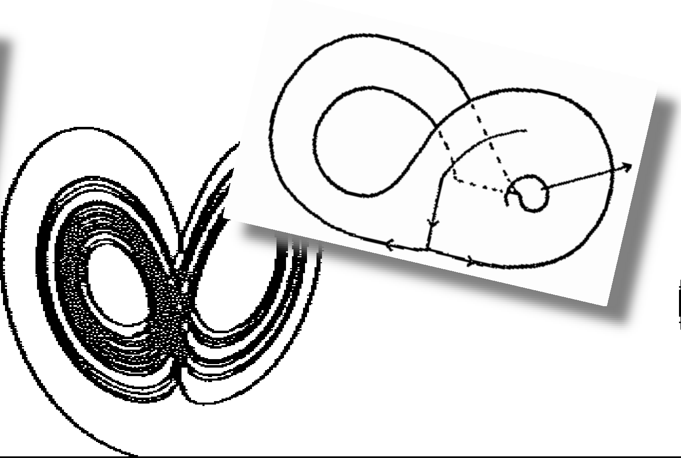
1976

Different types of chaotic flow are possible in the 3-dimensional state spaces of two simple non-linear differential equations. The first equation consists of a 2-variable, double-focus subsystem complemented by a linearly coupled third variable. It produces at least three types of chaos: Lorenzian chaos, "sandwich" chaos, and "horseshoe" chaos. Two figure 8-shaped chaotic regimes of the latter type are possible simultaneously, running through each other like 2 links of a chain. In the second equation, a transition between two different types of horseshoe chaos (spiral chaos and screw chaos) is possible. While sandwich chaos allows for a genuine strange attractor, the same has not yet been demonstrated for horseshoe chaos. Unlike the situation in the analogous 1-dimensional case, an emergent period-3 solution is not necessarily stable in the horseshoe. Since chaos is a "super-oscillation" (emergent with the third dimension), the existence of "super-chaos" is postulated for the nect level.

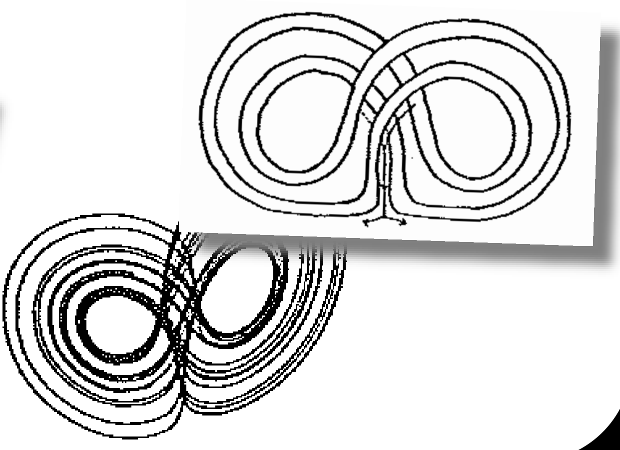
Chaos lorenzien



Chaos sandwich

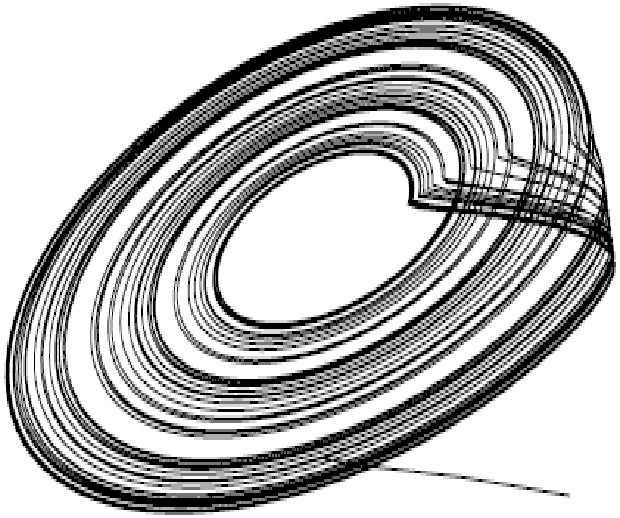


Chaos fer-à-cheval

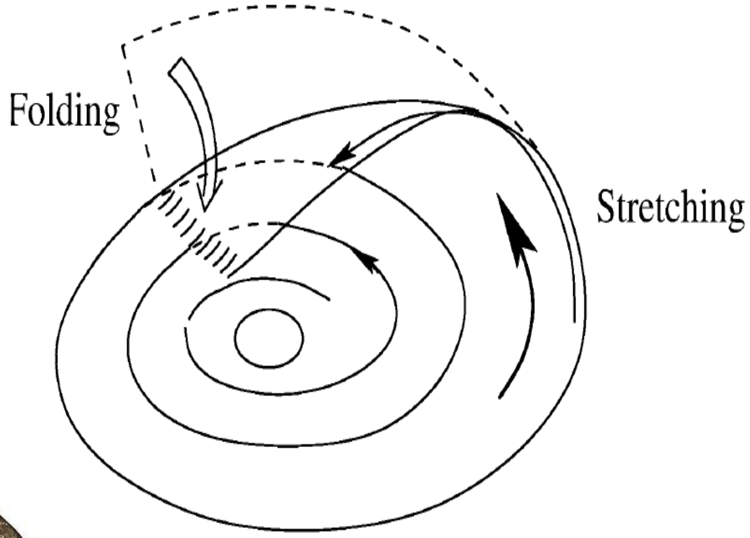


Chaos spiral (fer-à-cheval)

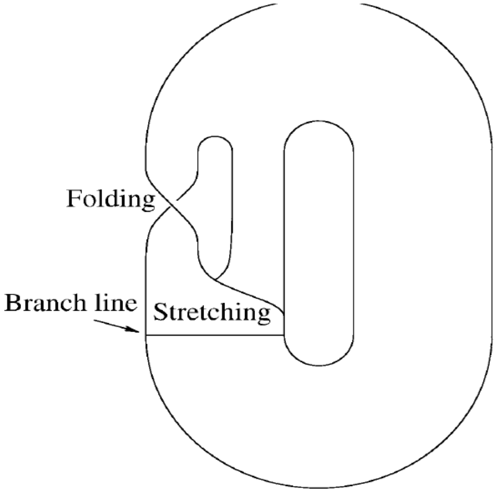
1. Espace des états



2. « modèle de papier »

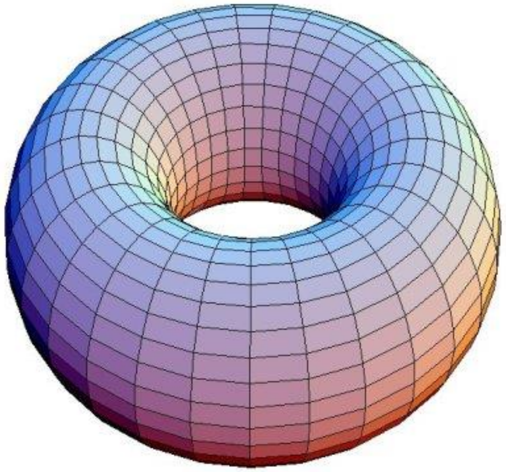


3. Gabarit



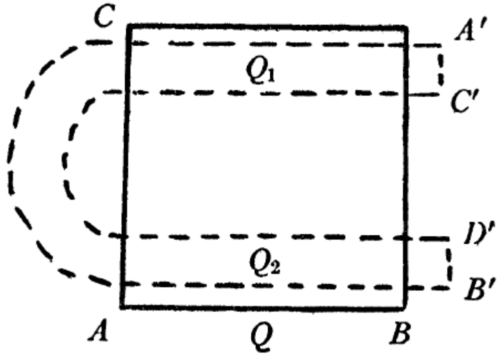
Où
est le
fer-à-cheval ?

4. Frontière toroïdale



DIFFERENTIABLE DYNAMICAL SYSTEMS[1]

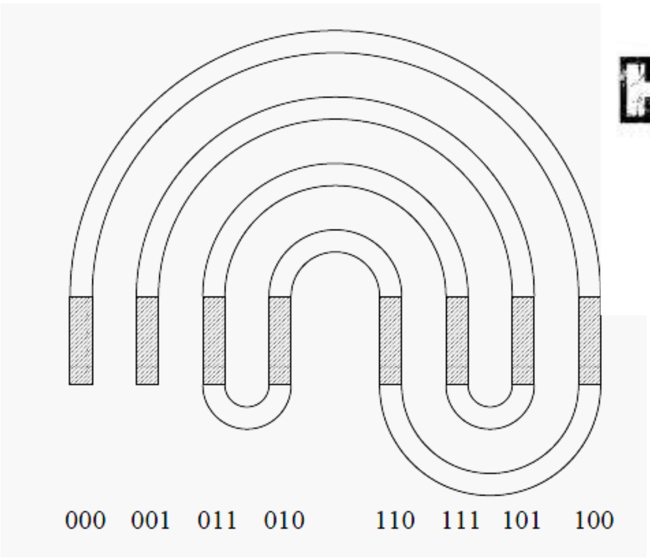
BY S. SMALE



1967

MULTIPLE-VALUED STATIONARY STATE AND ITS INSTABILITY OF THE TRANSMITTED LIGHT BY A RING CAVITY SYSTEM

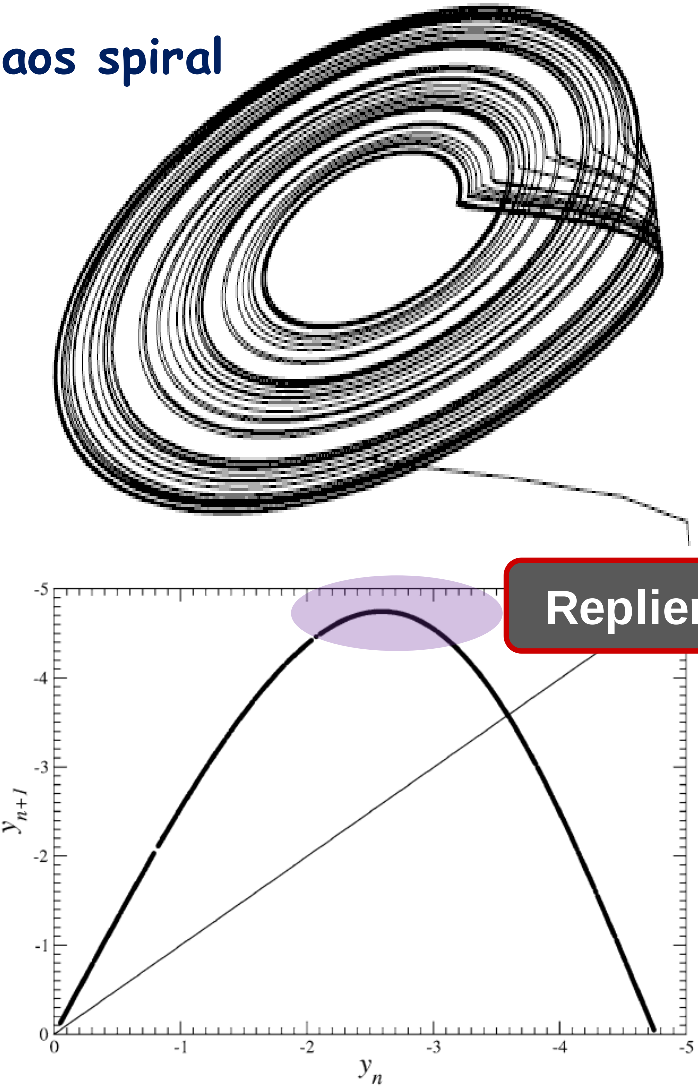
Kensuke IKEDA



HORSE SHOE GANG

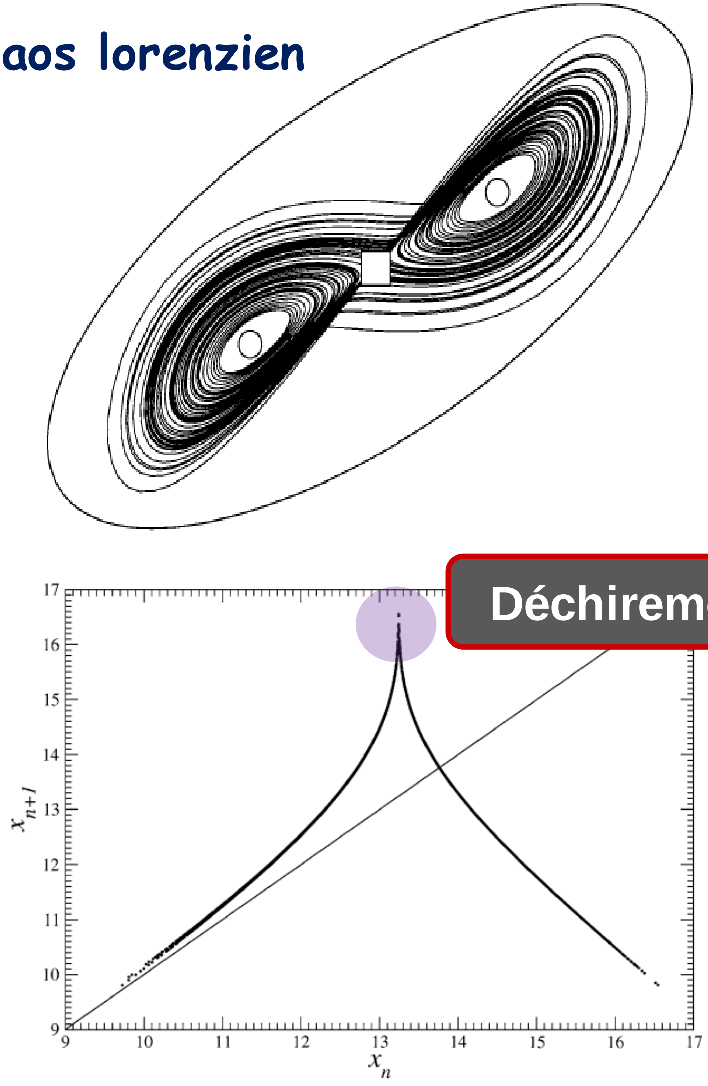
Chaos unimodal

Chaos spiral



Repliement

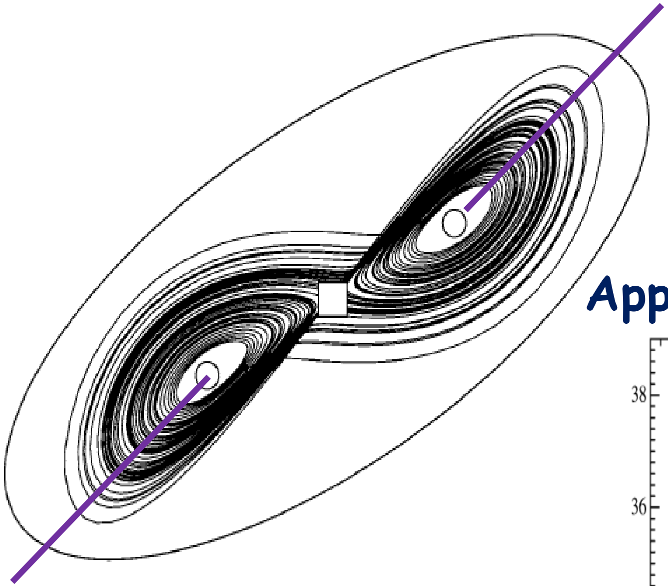
Chaos lorenzien



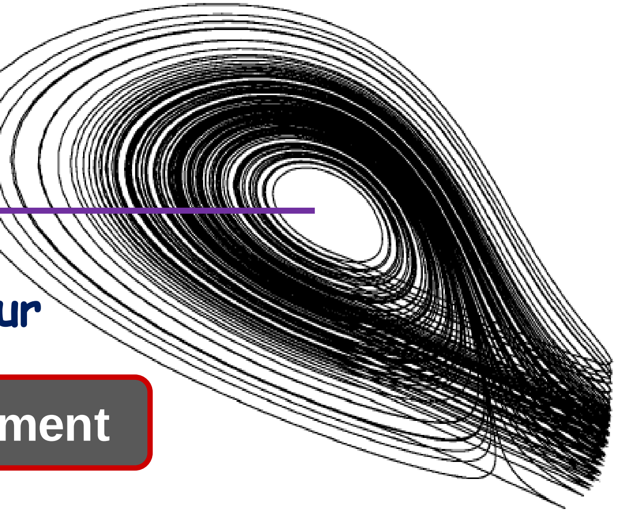
Déchirement

Chaos lorenzien

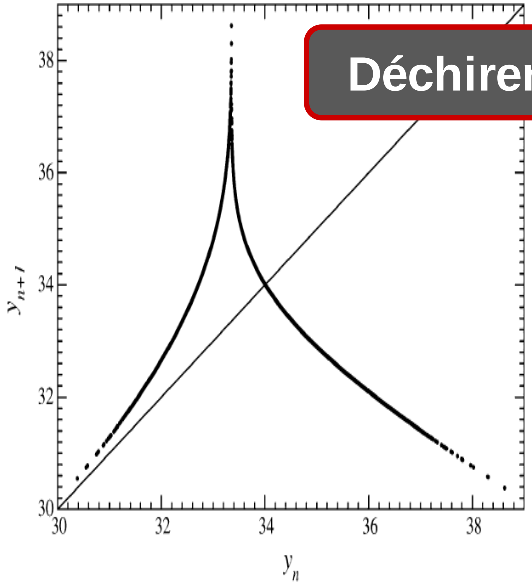
Avec une symétrie de rotation



Sans aucune symétrie



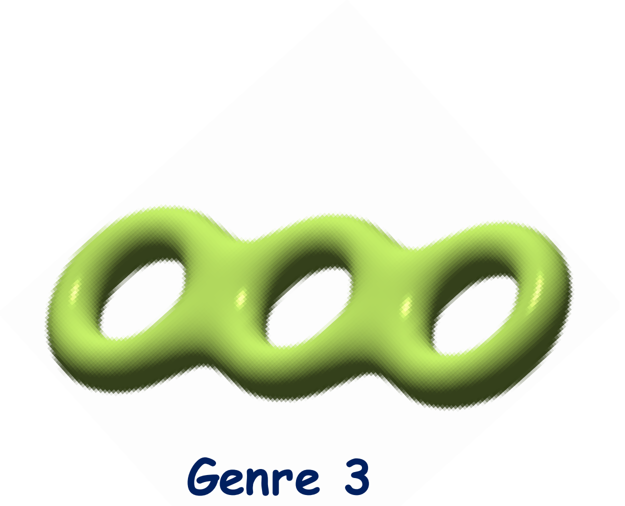
Application de 1er retour



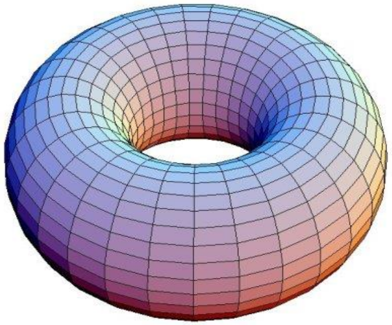
Déchirement

$$\begin{cases} \dot{x} = ax + by - cxy - \dfrac{(dz+e)x}{x+K_1} \\ \dot{y} = f + gz - hy - \dfrac{jxy}{y+K_2} \\ \dot{z} = k + lxz - mz \end{cases}$$

Rössler & Ortoleva (1978)

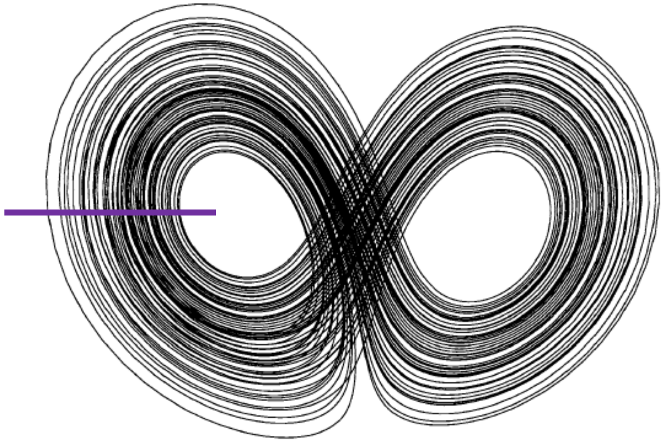


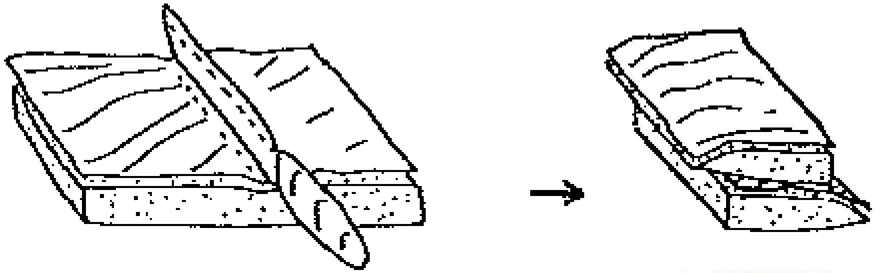
Genre 3



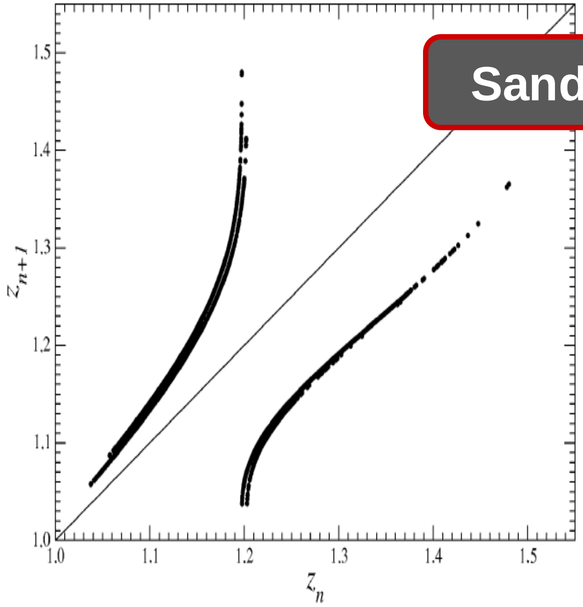
Genre 1

Chaos sandwich





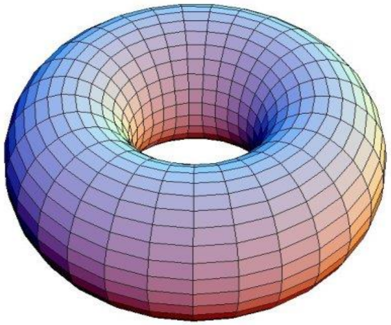
Application de 1^{er} retour



Sandwich

$$\begin{cases}\dot{x} = x - xy - z\\ \dot{y} = x^2 - ay\\ \dot{z} = bx - cz + d\end{cases}$$

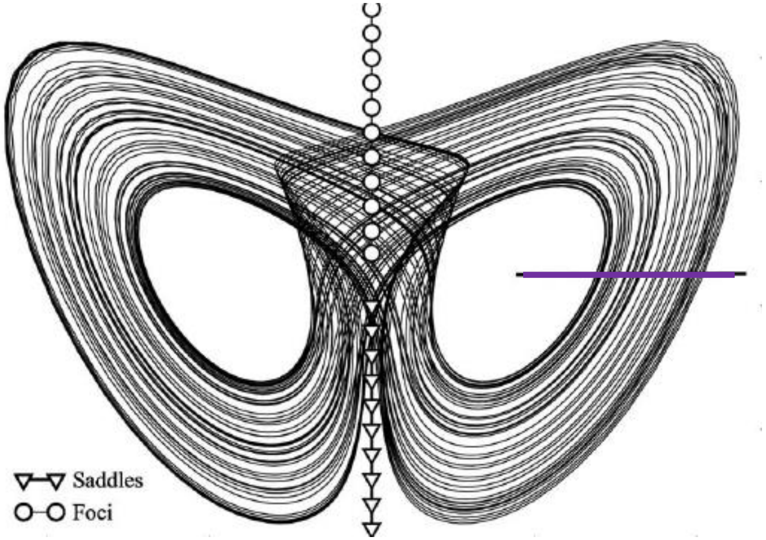
Rössler (1976)



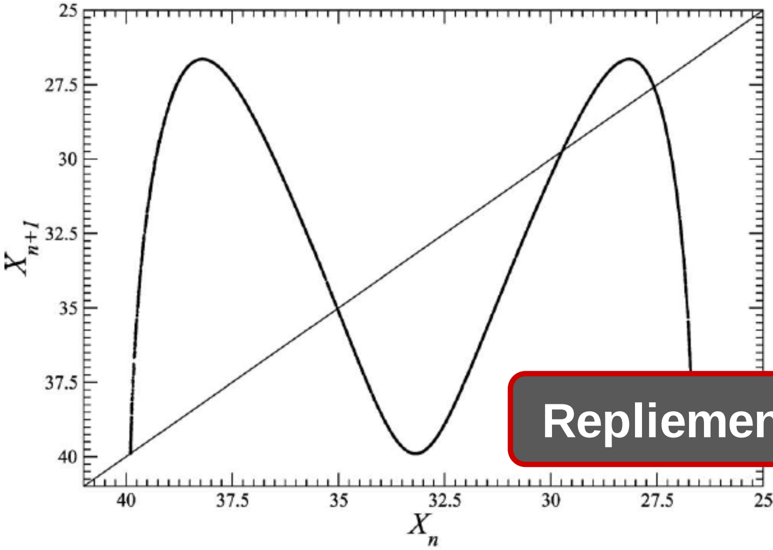
Genre 1

Chaos double fer-à-cheval

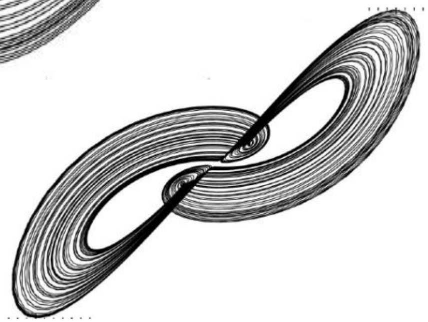
In the Lorenz system

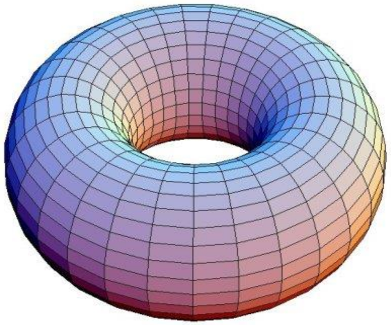


Application de 1^{er} retour



Repliements





Genre 1

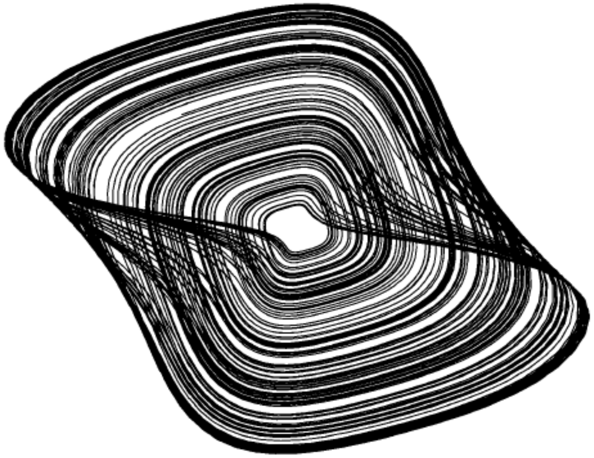
Another example

PHYSICAL REVIEW E, VOLUME 63, 016206

Covering dynamical systems: Twofold covers

Christophe Letellier
CORIA UMR 6614—Université de Rouen, Place Emile Blondel, F-76821 Mont Saint-Aignan Cedex, France

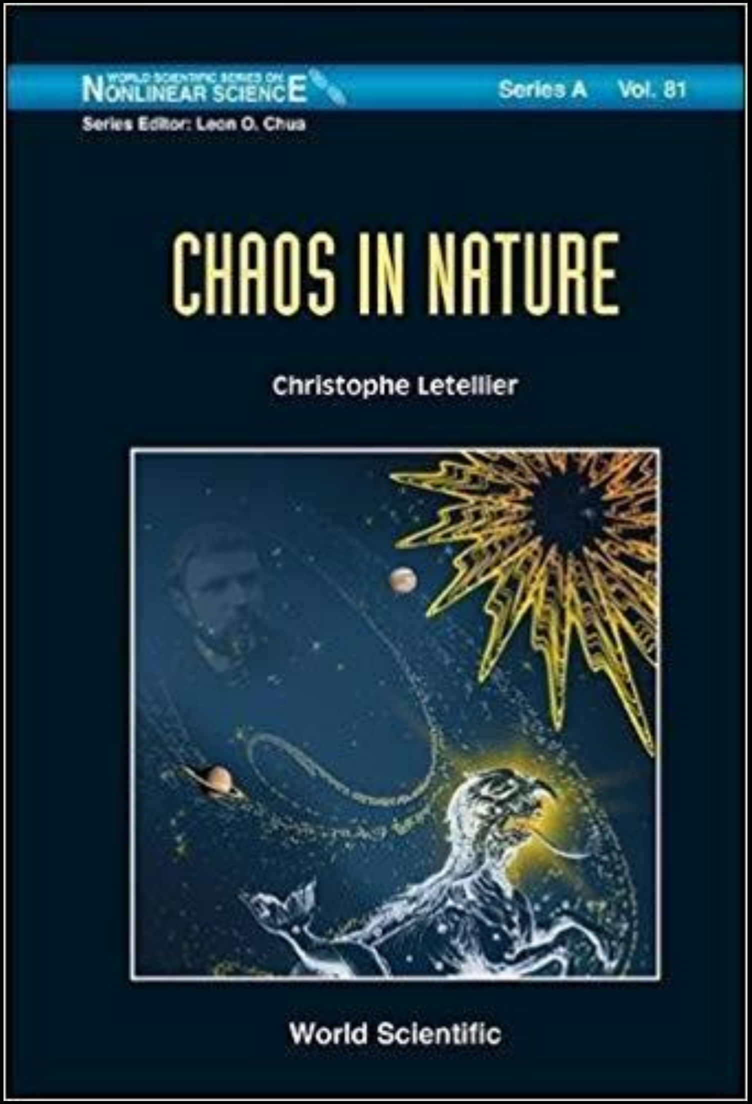
Robert Gilmore[†]
Laboratoire de Physique des Lasers Atomes et Molécules UMR CNRS 8523, Centre d'Études et de Recherches Lasers et Applications, Université des Sciences et Technologies de Lille, F-59655 Villeneuve d'Ascq Cedex, France



The End

Réalisé par
Christophe Letellier

Normandie Université
2011



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Series A Vol. 81

Series Editor: Leon O. Chua

CHAOS IN NATURE

Christophe Letellier

World Scientific

2nd Edition

À paraître en 2019