Mini-Projet : Rayleigh-Bénard convection

Students :
Adviser : Marc-Arthur N'Guessan

1 Introduction

In this mini-projet, we will focus on a very classical problem of instability in fluid mechanics : the Rayleigh-Bénard convection [2]. The objective of the mini-project is threefold. First, relying on a theoretical development proposed in [2] and in many other monographs, the idea is to present a model of such a natural convection instability (Oberbeck-Boussinesq system of equation) and to explain, for a special set of boundary conditions, how we can derive analytical linear stability boundary in terms of Rayleigh number, as well as the most amplified wavenumber such as in the Turing instability we have studied. After these two items, we intend to propose some numerical simulations illustrating the theoretical developments using an in-house code developed by the adviser.

2 Modeling the onset of convection in a fluid heated from below

In this first part, the purpose is to describe what model we should use in order to describe such a phenomenon and why we choose to consider an incompressible limit of the Navier-Stokes equations [1] as in the thermal explosion problem we have studied during the course, and what is the bifurcation parameter we are going to investigate.

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3 Linear stability analysis and Critical Rayleigh number

The purpose of this part is to rely on [2] to propose a normal mode decomposition and to evaluate the critical Rayleigh number for the onset of natural convection, that is when the fluid is sufficiently heated from below in order for the static solution with zero velocity to become unstable. What kind of bifurcation are we then going through?

4 Numerical simulation of the onset of convection

The purpose here is to explain what will be the discretization of the Navier-Stokes equations as well as the numerical schemes (briefly, this is not the essential part of the project) used in order to reproduce numerically the onset of convection at the critical Rayleigh number. Special attention will be devoted to the size of the domain in order to let the most amplified mode develop.

References

[1] T. Dumont, S. Génieys, M. Massot, and V. A. Volpert. Interaction of thermal explosion and natural convection: critical conditions and new oscillating regimes. *SIAM J. Appl. Math.*, 63(1):351–372, 2002.

[2] A. V. Getling. *Rayleigh-Bénard convection*, volume 11 of *Advanced Series in Nonlinear Dynamics*. World Scientific Publishing Co., Inc., River Edge, NJ, 1998. Structures and dynamics.