EADS Innovation Works

Domain decomposition for convective acoustics in 3D

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Outline

Outline

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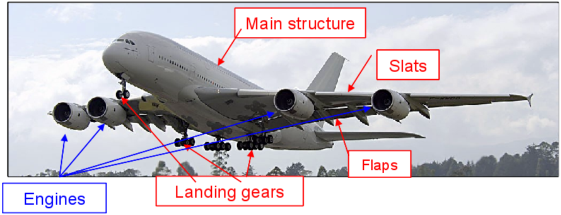
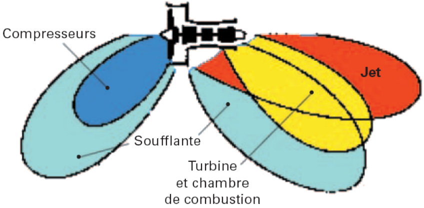
Noise generation on aircrafts

Main noise sources (engine, aircraft manufacturers)





How to propagate the generated noise ?

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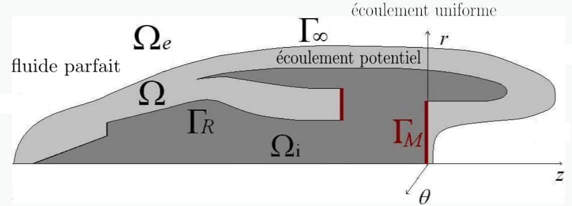
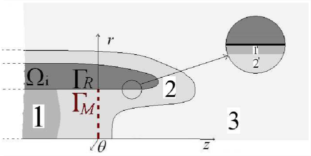
The fluid flow around the jet engine

Test case model



Three levels of modelisation



- Zone 1: visquous complex flow
  → not simulated
- Zone 2: potential flow
- Zone 3: uniform flow

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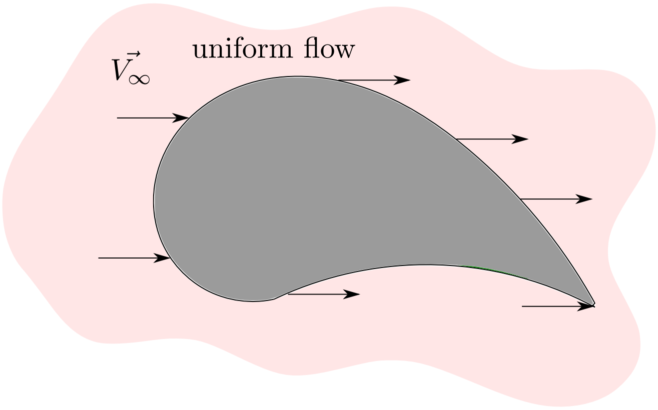
Boundary Finite Element

Boundary Finite Element used to solve acoustic propagation in a fluid

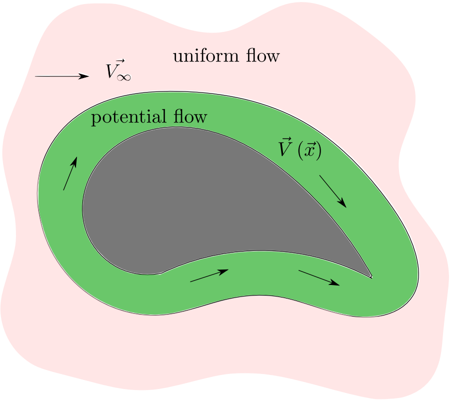
- without flow
- with a uniform flow



$\rightarrow$ problem near the objects

Coupling Boundary and Volumic Finite Elements

Method used to take into account a complex flow near the objects



Compute the acoustic propagation coupling

- 3D-FE in the potential flow
- BEM in the uniform flow

Background and context

Background

- ACTIPOLE software for propagation in the uniform flow
- Bibliography:
  - 1991: V. Levillain, formulation S for coupling FE and IE methods
  - 2005: S. Duprey, PhD on 2D axi-symetric aeroacoustic
  - 2009: E. Peynaud, training student to rediscover Duprey's work
  - 2010: F. Casenave, training student to extend to generic 3D

Context

- AEROSON project, financed by the ANR (French National Research Agency)
- Partners: CERFACS, POEMS (INRIA) and LAUM laboratories

Outline

Hypothesis and comportment equations

Euler equation for non viscous fluid

$$\frac{d}{dt}\vec{v}(\vec{x},t)+\frac{\vec{\vec{\nabla}}p(\vec{x},t)}{\rho(\vec{x},t)}=0$$

Mass conservation

$$\frac{\partial\rho}{\partial t}(\vec{x},t)+div\left(\rho(\vec{x},t)\vec{v}(\vec{x},t)\right)=0$$

Perfect gas isentropy

$$\frac{p(\vec{x},t)}{\rho(\vec{x},t)^{\gamma}}=K$$

Fluid irrotationality

$$\vec{v}(\vec{x},t)=\vec{\vec{\nabla}}\psi(\vec{x},t)$$

Some hypothesis and notations

Fluid and acoustic dissociation

- $$\underbrace{\psi(\vec{x},t)}_{\text{total flow}} = \underbrace{\psi_0(\vec{x},t)}_{\text{carrier flow}} + \underbrace{\psi_a(\vec{x},t)}_{\text{acoustic contribution}}$$
- acoustic is an order 1 perturbation of the fluid:

$$\psi_a \ll \psi_0$$

Some hypothesis

- harmonic acoustic perturbation

$$\psi_a(\vec{x},t) = \psi_a(\vec{x})e^{-i\omega t}$$

- stationary carrier flow

$$\psi_0(\vec{x},t) = \psi_0(\vec{x})$$

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Equations verified by the acoustic potential

Euler equation linearisation

Zero flow (Helmholtz equation)

$$\Delta\psi_a + k_\infty^2\psi_a = 0$$

Uniform flow

$$\Delta\psi_a + k_\infty^2\psi_a + 2ik_\infty\overrightarrow{M}_\infty\cdot\overrightarrow{\nabla}\psi_a - \overrightarrow{M}_\infty\cdot\overrightarrow{\nabla}\left(\overrightarrow{M}_\infty\cdot\overrightarrow{\nabla}\psi_a\right) = 0$$

Potential flow

$$\rho_0\left(k_0^2\psi_a + ik_0\vec{M}_0\cdot\overrightarrow{\nabla}\psi_a\right) + div\left[\rho_0\left(\overrightarrow{\nabla}\psi_a - \left(\vec{M}_0\cdot\overrightarrow{\nabla}\psi_a\right)\vec{M}_0 + ik_0\psi_a\vec{M}_0\right)\right] = 0$$

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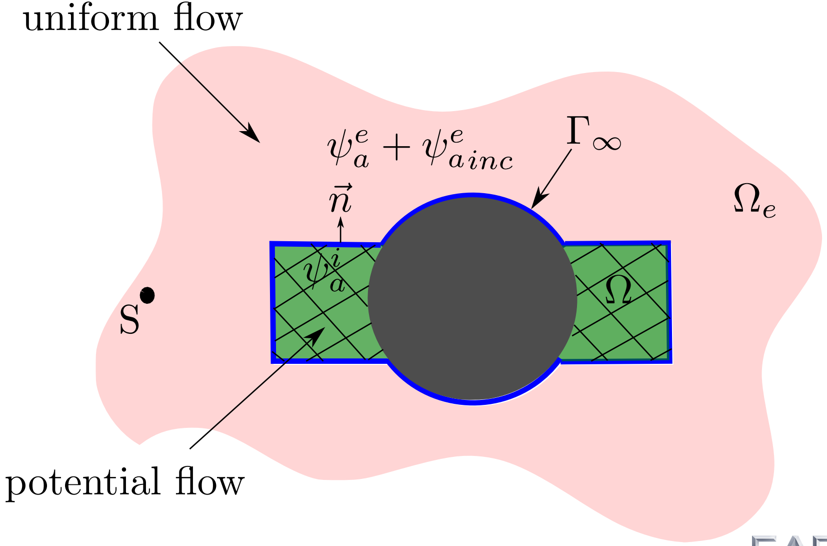
Description and notations



Equations

- Interior problem

$$\left\{\begin{array}{l} \rho_0 \left(k_0^2 \psi_a + ik_0 \vec{M_0} \cdot \vec{\nabla} \psi_a\right) \\ \quad + div \left[\rho_0 \left(\vec{\nabla} \psi_a - \left(\vec{M_0} \cdot \vec{\nabla} \psi_a\right) \vec{M_0} + ik_0 \psi_a \vec{M_0}\right)\right] = 0 \qquad \Omega \\ CL \text{ on } \Gamma_\infty \end{array}\right.$$

- Exterior problem

$$\left\{\begin{array}{l} \Delta \psi_a^e + k_\infty^2 \psi_a^e + 2ik_\infty \vec{M}_\infty \cdot \vec{\nabla} \psi_a^e - \vec{M}_\infty \cdot \vec{\nabla} \left(\vec{M}_\infty \cdot \vec{\nabla} \psi_a^e\right) = 0 \qquad \Omega_e \\ \lim_{r \to +\infty} r \left(\frac{\partial \psi_a^e}{\partial r} - ik_\infty \psi_a^e\right) = 0 \\ CL \text{ on } \Gamma_\infty \end{array}\right.$$

with

$$CL = \left\{\begin{array}{ll} \psi_a|_\Omega(x) = \psi_a|_{\Omega_e}(x) & \forall x \in \Gamma_\infty \\ \frac{\partial \psi_a}{\partial n}|_\Omega(x) = \frac{\partial \psi_a}{\partial n}|_{\Omega_e}(x) & \forall x \in \Gamma_\infty \end{array}\right.$$

Need to transform the equations

Problem

No framework to use the integral representation theorem

Lorentz transformation

- Helmholtz equation in the exterior domain
- transformation of the boundary integral to be compatible with the coupling
- but complicate the equation in the interior domain

Lorentz Transformation

Coordinate change

$$\vec{x'} = L(\vec{x}) = \frac{\left(\vec{x}\cdot\vec{M}_\infty\right)\vec{M}_\infty}{M_\infty^2\sqrt{1-M_\infty^2}} + \left[\vec{x} - \frac{\left(\vec{x}\cdot\vec{M}_\infty\right)\vec{M}_\infty}{M_\infty^2}\right]$$

Phase difference : change of unknown function

$$\psi_a(\vec{x}) = f\left(L(\vec{x})\right) e^{-\frac{ik_\infty \vec{M}_\infty\cdot L(\vec{x})}{\sqrt{1-M_\infty^2}}} = f\left(\vec{x'}\right) e^{-\frac{ik_\infty \vec{M}_\infty\cdot\vec{x'}}{\sqrt{1-M_\infty^2}}}$$

→ new unknown f and new coordinate system x'

Exterior domain

Lorentz transformation on the potential equation → Helmholtz equation

$$k'^2_\infty f + \Delta' f = 0 \quad \text{with} \quad k'_\infty = \frac{k_\infty}{\sqrt{1 - M_\infty^2}}$$

Framework for integral representation theorem and Calderón formulas

Find $(f, \frac{\partial f}{\partial n}) \in H^1(\Omega') \times H^{-\frac{1}{2}}(\Gamma'_\infty)$ s.t. $\forall (q^t, \lambda^t) \in H^1(\Omega') \times H^{-\frac{1}{2}}(\Gamma'_\infty)$

$$\begin{cases} \int_{\Gamma'_\infty} S'\left(\frac{\partial f}{\partial n}\right)\lambda^t - \int_{\Gamma'_\infty} D'(f)\,\lambda^t + \frac{1}{2}\int_{\Gamma'_\infty} f\,\lambda^t = \int_{\Gamma'_\infty} f^e_{inc}\lambda^t \\ \int_{\Gamma'_\infty} D'^*\left(\frac{\partial f}{\partial n}\right) q^t - \int_{\Gamma'_\infty} N'(f)\,q^t + \frac{1}{2}\int_{\Gamma'_\infty} \frac{\partial f}{\partial n} q^t = \int_{\Gamma'_\infty} \frac{\partial f^e_{inc}}{\partial n} q^t \end{cases}$$

where S, D, D^* et N are integral operators defined with the Green fonction of the problem

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Interior domain

Variational formulation before Lorentz transformation

Find $\psi_a \in H^1(\Omega)$ such that $\forall q^t \in H^1(\Omega)$

$$\int_\Omega \rho_0 \left[\vec{\nabla}\psi_a \cdot \vec{\nabla} q^t - k_0^2 \psi_a q^t - ik_0 \left(\left(\vec{M}_0 \cdot \vec{\nabla}\psi_a\right) q^t - \left(\vec{M}_0 \cdot \vec{\nabla} q^t\right)\psi_a\right)\right]$$
$$- \int_\Omega \rho_0 \left(\vec{M}_0 \cdot \vec{\nabla}\psi_a\right)\left(\vec{M}_0 \cdot \vec{\nabla} q^t\right)$$
$$- \int_{\Gamma_\infty} \rho_0 \left(\vec{\nabla}\psi_a \cdot \vec{n} - \left(-ik_0\psi_a + \vec{M}_0 \cdot \vec{\nabla}\psi_a\right)\vec{M}_0 \cdot \vec{n}\right) q^t = 0$$

Lorentz transformation

- Surface integral the coupling term will appear
- Volume integral will be complexified

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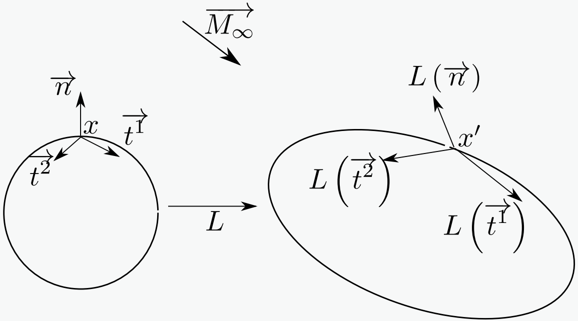
Surface integral transformation

Change of variable : jacobian of the surface-restrained tranformation



$$\left| Jac\left(L|_{\partial\Omega}^{-1}\right)\left(L|_{\partial\Omega}^{-1}(x')\right)\right| = \sqrt{\frac{1-M_\infty^2}{1-\left(\vec{n}\left(L^{-1}(x')\right)\cdot \vec{M_\infty}\right)^2}}$$

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Surface integral transformation

Variational formulation becomes

$$\int_{\Omega} \rho_0 \left[\vec{\nabla}\psi_a \cdot \vec{\nabla} q^t - k_0^2 \psi_a q^t - ik_0 \left(\left(\vec{M}_0 \cdot \vec{\nabla}\psi_a \right) q^t - \left(\vec{M}_0 \cdot \vec{\nabla} q^t \right) \psi_a \right) \right]$$
$$- \int_{\Omega} \rho_0 \left(\vec{M}_0 \cdot \vec{\nabla}\psi_a \right) \left(\vec{M}_0 \cdot \vec{\nabla} q^t \right)$$
$$- \rho_\infty \sqrt{1 - M_\infty^2} \int_{\Gamma'_\infty} \frac{\partial f}{\partial n'} f^t = 0$$

Coupling term to identify in the Calderón system.

Volume term transformation

Gradients tranformation

$$\left(\vec{\mathcal{L}'_{\pm}} f\right) = \vec{\nabla}' f + \left[C \left(\vec{M}_\infty \cdot \vec{\nabla}' f \right) \pm \frac{i k_\infty}{1 - M_\infty^2} f \right] \vec{M}_\infty$$

Transformed interior domain variational equation

Find $f \in H^1(\Omega')$ such that $\forall f^t \in H^1(\Omega')$

$$\int_{\Omega'} \frac{\rho_0}{\rho_\infty} \left[\left(\vec{\mathcal{L}'_{-}} f\right) \cdot \left(\vec{\mathcal{L}'_{+}} f^t\right) - {k_0}^2 f f^t \right]$$
$$- \int_{\Omega'} \frac{\rho_0}{\rho_\infty} i k_0 \left[\left(\vec{M}_0 \cdot \left(\vec{\mathcal{L}'_{-}} f\right)\right) f^t - \left(\vec{M}_0 \cdot \left(\vec{\mathcal{L}'_{+}} f^t\right)\right) f \right]$$
$$- \int_{\Omega'} \frac{\rho_0}{\rho_\infty} \left(\vec{M}_0 \cdot \left(\vec{\mathcal{L}'_{-}} f\right)\right) \left(\vec{M}_0 \cdot \left(\vec{\mathcal{L}'_{+}} f^t\right)\right)$$
$$- \int_{\Gamma'_\infty} \frac{\partial f}{\partial n'} f^t = 0$$

Coupling the interior and the exterior problems

Global variational formulation

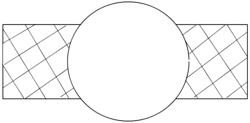
Find $(q,\lambda) \in H^1(\Omega') \times H^{-\frac{1}{2}}(\Gamma'_\infty)$ s.t. $\forall (q^t,\lambda^t) \in H^1(\Omega') \times H^{-\frac{1}{2}}(\Gamma'_\infty)$

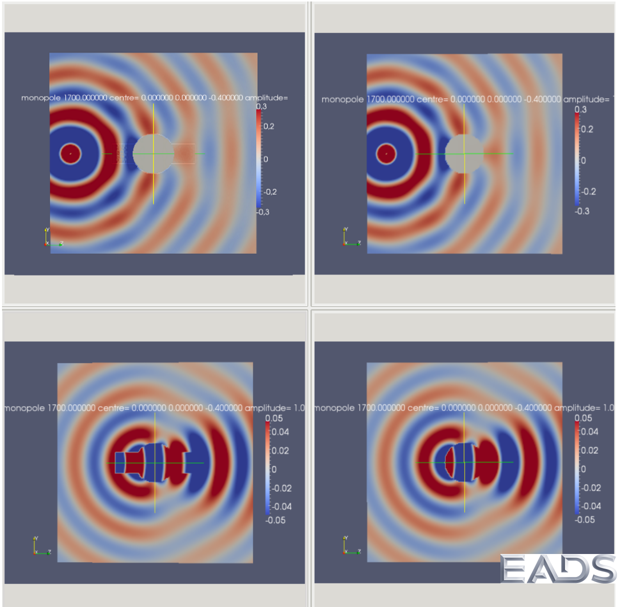
$$\left\{ \begin{array}{l} \frac{1}{ik_\infty \rho_\infty} \int_{\Omega'} \rho_0 \left[\vec{\mathcal{L}'_-} q \cdot \vec{\mathcal{L}'_+} q^t - {k_0}^2 q q^t \right] \\ \quad - \frac{1}{k_\infty \rho_\infty} \int_{\Omega'} \rho_0 k_0 \left[\left(\vec{M_0} \cdot \vec{\mathcal{L}'_-} q \right) q^t - \left(\vec{M_0} \cdot \vec{\mathcal{L}'_+} q^t \right) q \right] \\ \quad - \frac{1}{ik_\infty \rho_\infty} \int_{\Omega'} \rho_0 \left(\vec{M_0} \cdot \vec{\mathcal{L}'_-} q \right) \left(\vec{M_0} \cdot \vec{\mathcal{L}'_+} q^t \right) \\ \quad\quad - \int_{\Gamma'_\infty} D'^* (\lambda)\, q^t - \frac{1}{ik_\infty} \int_{\Gamma'_\infty} N'(q)\, q^t + \frac{1}{2} \int_{\Gamma'_\infty} \lambda q^t \\ \quad\quad = - \frac{1}{ik_\infty} \int_{\Gamma'_\infty} \frac{\partial f^e_{inc}}{\partial n} q^t \\ \\ -ik_\infty \int_{\Gamma'_\infty} S'(\lambda)\, \lambda^t - \int_{\Gamma'_\infty} D'(q)\, \lambda^t + \frac{1}{2} \int_{\Gamma'_\infty} q \lambda^t = - \int_{\Gamma'_\infty} f^e_{inc} \lambda^t \end{array} \right.$$

Outline

Test case : sphere_in_pave

Uniform case without flow





$$M_0 = M_\infty = 0$$

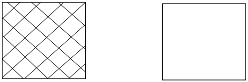
$$c_0 = c_\infty = 340 m.s^{-1}$$

$$\rho_0 = \rho_\infty = 1.2 kg.m^{-3}$$

Difference (norm 2): 1.1%

Test case : two_cubes

Uniform flow

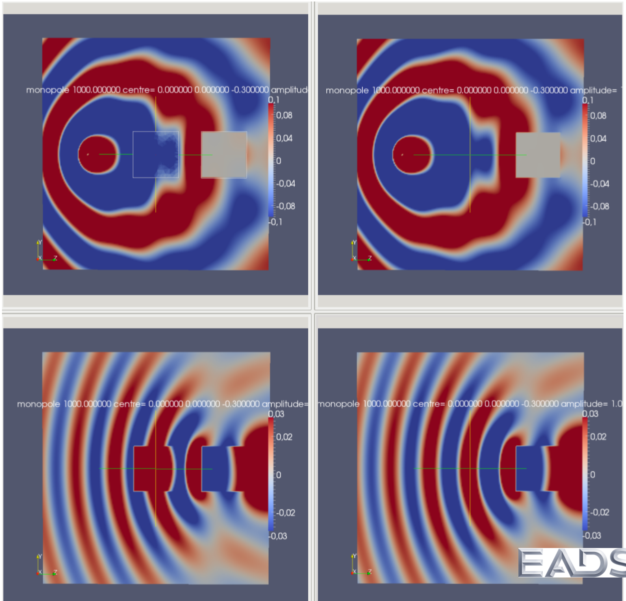


$$M_0 = M_\infty = 0.5$$

$$c_0 = c_\infty = 340 m.s^{-1}$$

$$\rho_0 = \rho_\infty = 1.2 kg.m^{-3}$$

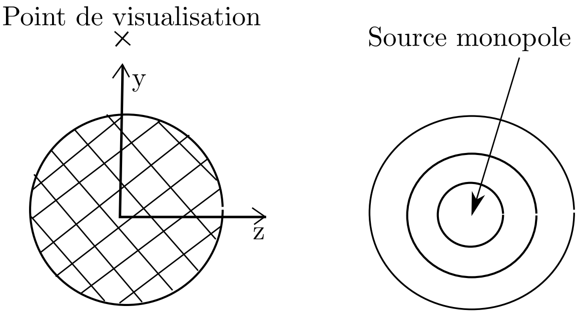
Difference (norm 2): 3.5%



Validation non-uniform test case

Formulation for a potential flow

Analytic solution by Mie series



$M_0 = M_\infty = 0,\ c_0 = 2c_\infty,\ \rho_0 = \rho_\infty$

Validation non-uniform test case

Formulation for a potential flow

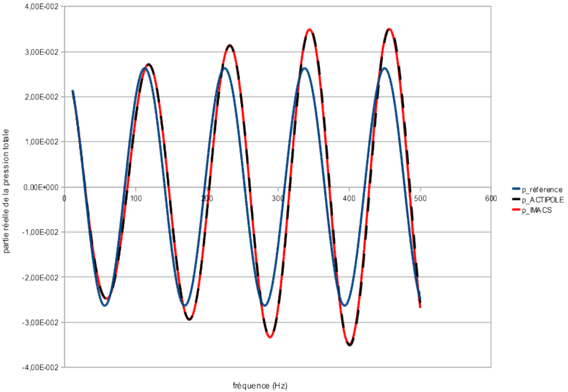
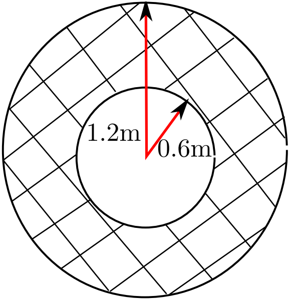


Figure: Real part of the pressure at a point in function of the frequency

error: $< 0.25\%$ for $f < 85Hz$, and $< 5\%$ up to 500Hz (infinite norm, mesh done for 85Hz)

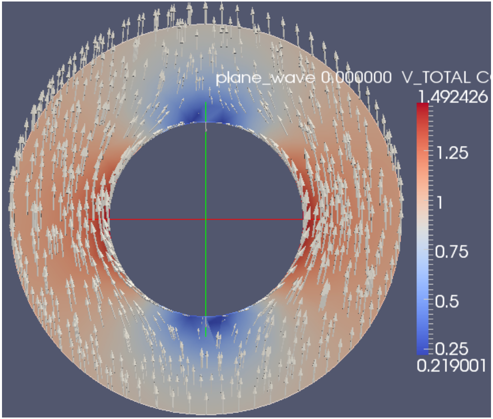
Test case : sphere_in_ball

A ‘more‘ realistic potential flow



$M_\infty = 0.4$
incompressible potential flow

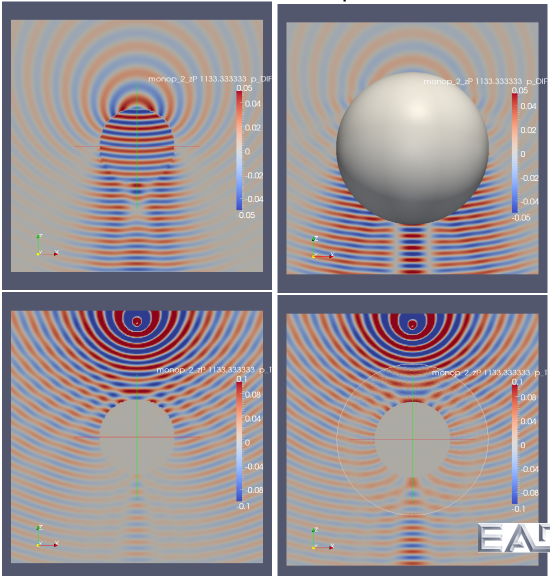
f=1133.333Hz
1.5M tetraedres
250k dl vol, 100k dl surf
residual $< 10^{-3}$



Test case : sphere_in_ball

Upper source : comparison with a uniform flow

With uniform flow — With potential flow



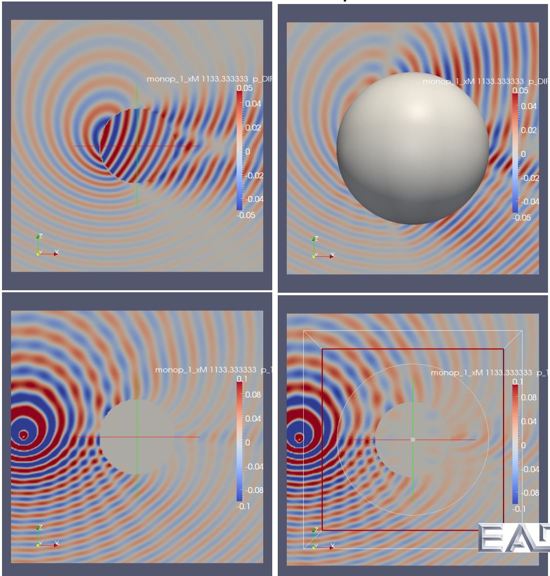
top : diffracted potential
bottom : total potential

Under-estimated diffraction phenomena with uniform flow

Test case : sphere_in_ball

Side source : comparison with a uniform flow

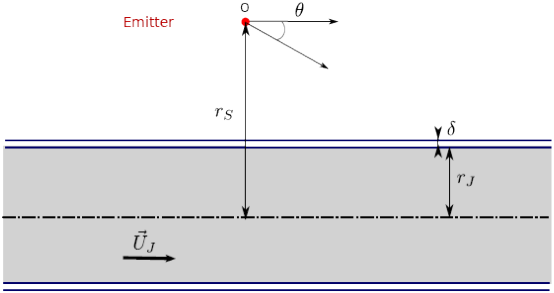
With uniform flow With potential flow

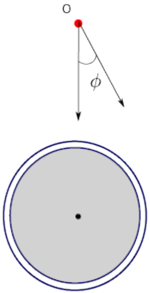


top : diffracted potential
bottom : total potential

Under-estimated diffraction and distortion with uniform flow

Tube shaped jet flow test case





	Exterior domain	Jet
celerity	340.2 m/s	545.4 m/s
rho	1.23 kg/m^3	0.48 kg/m^3
Mach	0	0.69

and linear interpolation in the thickness δ. Measures at 10m for $0 < \theta < 120$ and $0 < \phi < 360$.

Tube shaped jet flow test case

Results

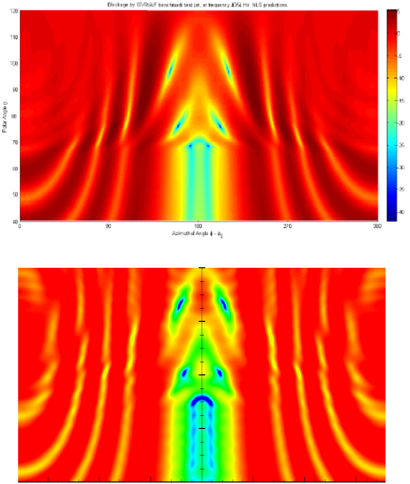


Figure: Comparison of pressure intensity w.r.t. ψ and θ

top : ISVR, bottom : ACTIPOLE volumic

Information on the computations

Some computations done

- Volumic part: 1 677 767 elements, 258 486 unknowns
- Surfacic part: 343 609 elements, 154 895 unknowns
- ≈3 hours on 64 processors

Solution method

- Schur complement on the volumic part (MUMPS)
- Direct or FMM solver, parallelized with MPI
- remark: same solver used for EM applications with several millions of volumic unknowns
  - $(nb\_unknowns/tetra)_{EM} \approx 10(nb\_unknowns/tetra)_{AC}$

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Outline

Conclusion

What has been done

- first 3D code coupling LEE with BEM to our knowledge (integrated in ACTIPOLE software)
- validations have been conducted (with autotests data)

What we still have to do

- more realistic validation test cases (object, flow)
- implement direct coupling between modal sources and FE domain