Convergent finite element approximations for Landau-Lifschitz-Gilbert equation

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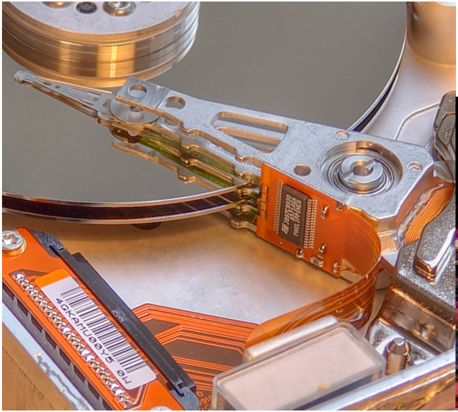
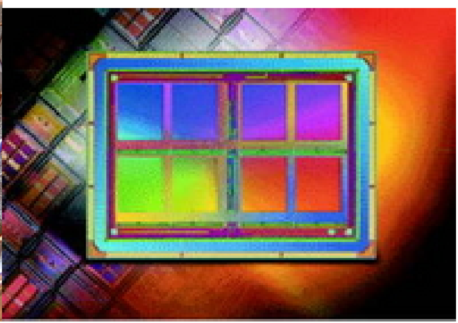
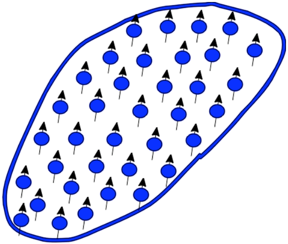
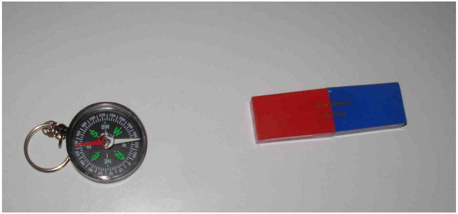
Magnetic storage





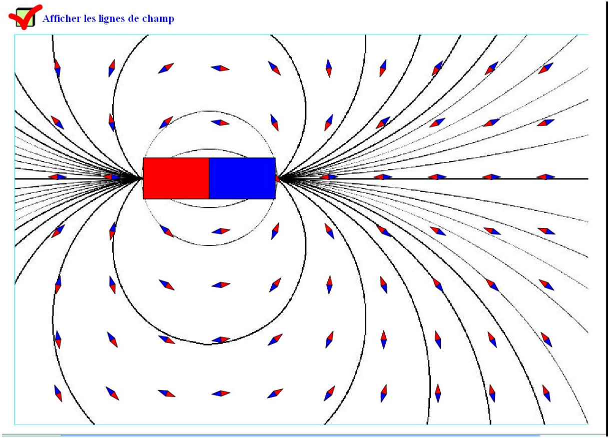
Micromagnetism





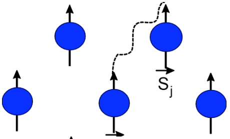
- Continuous medium $\Omega \subset \mathbb{R}^3$
- Magnetization $m : \Omega \to \mathbb{S}^2$

Micromagnetism

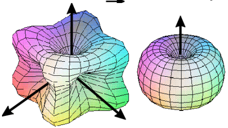


Locally the magnetization is aligned with the applied field.

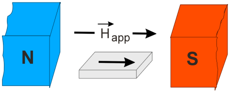
Micromagnetism



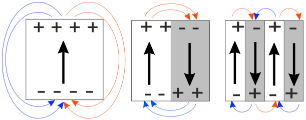
$$\mathcal{E}(m) = A\int_\Omega |\nabla m|^2$$



$$+K\int_\Omega (1-(m\cdot u)^2)$$



$$-\mu_0 M_s\int_\Omega H_{ext}\cdot m$$



$$-\frac{\mu_0 M_s}{2}\int_{\mathbb{R}^3} H_d(m)\cdot m$$

Stray-field

Magnetic field induced by the magnetization distribution

$$H_d(m) = -\nabla\phi(m)$$

where

$$\left[\begin{array}{l} \Delta\phi(m) = M_s \mathrm{div}(m) \text{ in } \Omega \\ \Delta\phi(m) = 0 \text{ outside } \Omega \\ [\phi(m)] = 0 \text{ across } \partial\Omega \\ \left[\frac{\partial\phi(m)}{\partial n}\right] = -m\cdot n \text{ across } \partial\Omega \end{array}\right.$$

$$H_d(m) = -M_s \nabla\Delta^{-1}\mathrm{div}(m) \text{ in } \mathbb{R}^3$$

$H_d(m)$ is the L^2-orthogonal projection of $-M_s m$ on gradient fields

Micromagnetism

Brown's free energy

$$A\int_\Omega |\nabla m|^2 + K\int_\Omega (1-(m\cdot u)^2) - \frac{\mu_0 M_s}{2}\int_{\mathbb{R}^3} H_d(m)\cdot m - \mu_0 M_s \int_\Omega H_{ext}\cdot m$$

Euler-Lagrange equations (remember $|m| = 1$)

$$H_{eff} = \frac{2A}{\mu_0 M_s}\Delta m + \frac{2K}{\mu_0 M_s}(m\cdot u)u + H_d(m) + H_{ext} = \lambda m,$$

where $\lambda = \lambda(x)$ is a Lagrange multiplier

$$H_{eff} = -\frac{1}{\mu_0 M_s}\frac{\partial \mathcal{E}(m)}{\partial m} = \text{ Effective field}$$

Micromagnetism

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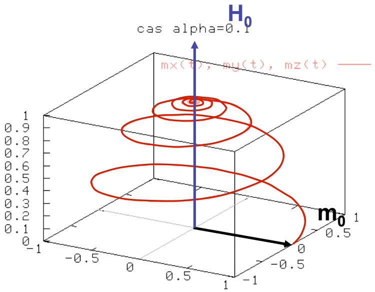
Landau-Lifschitz equation

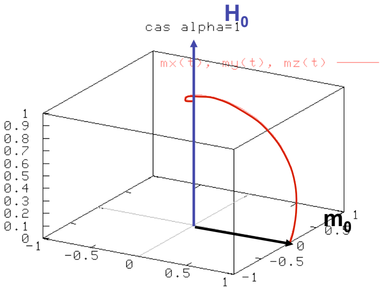
- describes the evolution of the magnetization inside a ferromagnetic material

$$\frac{\partial m}{\partial t} = -\gamma\mu_0 m \times H_{eff} + \alpha m \times \frac{\partial m}{\partial t} \text{ in } \Omega$$

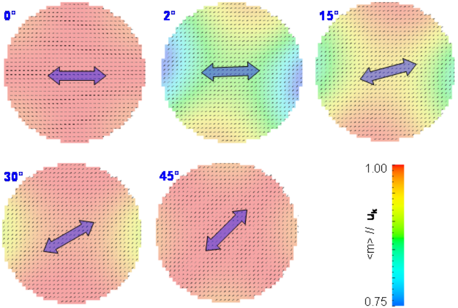
$$\frac{\partial m}{\partial n} = 0 \text{ on } \partial\Omega$$

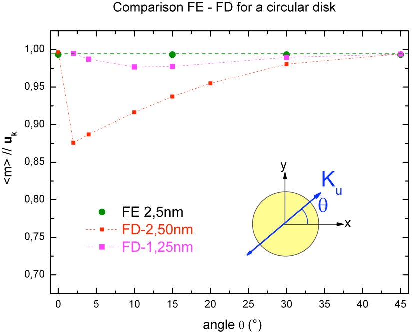
- H_{eff} is the effective field, $\alpha > 0$ damping parameter, γ gyromagnetic constant





Need for finite element formulations





NiFe nanodot : 100 nm thick and 10 nm height

Properties

- $\frac{\partial m}{\partial t} = -\gamma\mu_0 m \times H_{eff} + \alpha m \times \frac{\partial m}{\partial t}$
- $H_{eff} = \frac{2A}{\mu_0 M_s}\Delta m + \frac{2K}{\mu_0 M_s}(m \cdot u)u + H_d(m) + H_{ext}$
- $|m(x,t)| = 1$ is preserved

Non linear PDE, with non local terms and a non convex constraint...

What does LLG equation look like ?

Forget constants $H_{eff} = \Delta m + l.o.t.$...

$$\frac{\partial m}{\partial t} = -m \times \Delta m + \alpha m \times \frac{\partial m}{\partial t} \text{ dans } \Omega$$

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What does LLG equation look like ?

Several equivalent forms (formal)

Gilbert form

$$m_t - \alpha m \times m_t = -m \times \Delta m$$

$$m \times m_t = -m \times (m \times \Delta m) + \alpha m \times (m \times m_t)$$

Unused form

$$\alpha m_t + m \times m_t = (\Delta m - (\Delta m \cdot m)m)$$

Multiplying by m_t and integrating, we arrive at

$$\alpha \int |m_t|^2 = -\frac{1}{2}\frac{d}{dt}\int |\nabla m|^2$$

Landau-Lifshitz form

$$(1 + \alpha^2)m_t = -m \times \Delta m + \alpha(\Delta m - (\Delta m \cdot m)m)$$

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Known mathematical results

- Local existence of strong solutions [Carbou-Fabrie]
- Global existence of strong solutions for small energy initial data (2D) [Carbou-Fabrie]
- Global existence of strong solutions for small energy initial data (3D only on ellipsoids) [Beauchard-A.]
- Global existence of weak solutions [Visintin, Soyeur-A.]
- Nonuniqueness of weak solutions (only exchange) [Soyeur-A.]

Strong=twice differentiable, Weak = only once differentiable

$m \in H^1(\Omega \times [0,T], S^2)$ is a weak solution of (LLG) if

- $\forall \phi \in H^1(\Omega \times [0,T])$

$$\int m_t \cdot \phi - \alpha \int m \times m_t \cdot \phi = \int \sum_i m \times \frac{\partial m}{\partial x_i} \cdot \frac{\partial \phi}{\partial x_i}$$

  This is due to the fact that
  $-\sum_i \frac{\partial}{\partial x_i}\left(m \times \frac{\partial m}{\partial x_i}\right) = -\sum_i m \times \frac{\partial^2 m}{\partial x_i^2} = -m \times \Delta m\,.$

- $\frac{1}{2}\int |\nabla m(T)|^2 + \alpha \int_0^T \int \left|\frac{\partial m}{\partial t}\right|^2 \leq \frac{1}{2}\int |\nabla m(0)|^2$

What about the discretization

- A lot of existing things (Finite differences, finite volumes, finite elements, etc.). How to deal with the constraint $|m| = 1$?
- How to have a weak formulation ? (FE)
- Convergence towards a solution of LLG as $\delta t, \delta x \to 0$?
- Stability, consistency of the scheme ? (Explicit vs implicit)
- Implementation (robustness, speed, efficiency, etc.)
- Algorithmic issues (FFT or FMM for stray field, linear vs non-linear systems)
- Scientific computing (accuracy, e.g. NIST benchs), dissipation but not overdissipation (α small)...
- ...

A first explicit scheme

Idea 1 : Test with a function which is orthogonal to m at every point (tangent plane formulation)

$$\alpha m_t + m \times m_t = (\Delta m - (\Delta m \cdot m) m)$$

$m^n \sim m(n\delta t)$, $m^n = \sum_i m_i^n \phi_i$ with $\forall i$, $|m_i^n| = 1$,

$K_n = \{w = \sum_i w_i \phi_i,\ \ w_i \cdot m_i^n = 0\}$.

- For all $n \geq 0$, Find $v^n \in K_n$ such that $\forall w \in K_n$

$$(*) \quad \alpha \int v^n \cdot w + \int m^n \times v^n \cdot w = - \int \nabla m^n \cdot \nabla w$$

- Set $m^{n+1} = \sum_i m_i^{n+1} \phi_i$, with $m_i^{n+1} = \dfrac{m_i^n + \delta t v_i^n}{|m_i^n + \delta t v_i^n|}$

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Properties

- The problem (*) is linear. It possesses always a unique solution v^n
- After time and space linear interpolation, the solution converges weakly to a weak solution of (LL) when $\delta t \to 0$, $\delta x \to 0$, and $\frac{\delta t}{\delta x^2} \to 0$
- Like an explicit scheme for the heat equation. Difficult to use in practice (δt very small...)

$\rightarrow$ Implicit schemes

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$\longrightarrow$ Implicit schemes

Implicit schemes

- Some have been proposed by [Bartels-Prohl]
- Non linear iteration
- Although unconditionally stable, the convergence of the Newton method is guaranteed only if $\frac{\delta t}{\delta x^2}$ is sufficiently small

$\longrightarrow$ Need for a implicit, unconditionally stable scheme with a linear iteration

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A new implicit scheme

- Idea 2 : $\forall n \geq 0$, Find $v^n \in K_n$ such that $\forall w \in K_n$

$$\alpha \int v^n \cdot w + \int m^{n+1} \times v^n \cdot w = - \int \nabla m^{n+1} \cdot \nabla w$$

  Too difficult... (non linear)

- but $m^{n+1} = \dfrac{m^n + \delta t v^n}{|m^n + \delta t v^n|} \sim m^n + \delta t v^n + O(\delta t^2)$

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The $\theta-$scheme

Take $\theta \in [0, 1]$

- $\forall n \geq 0$, Find $v^n \in K_n$ such that $\forall w \in K_n$

$$\int \alpha v^n \cdot w + m^n \times v^n \cdot w + \theta\delta t \int \nabla v^n \cdot \nabla w = - \int \nabla m^n \cdot \nabla w$$

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Energy decay and renormalization

- For $w \in H^1$ being such that $|w| \geq 1$ a.e. one has

$$\int \left|\nabla \frac{w}{|w|}\right|^2 \leq \int |\nabla w|^2$$

- Q : Is it still true after discretization ? For $w = \sum_i w_i \phi_i$ with $|w_i| \geq 1$ do we have

$$\int \left|\nabla \sum_i \frac{w_i}{|w_i|} \phi_i\right|^2 \leq \int |\nabla w|^2 ?$$

- Answer [Bartels] : Yes, for P^1, if the mesh is Delaunay (2D) or has diedral angles less than $\frac{\pi}{2}$ (3D) (**)

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Convergence result

- The $\theta-$scheme is well defined. It needs only to solve linear problems and converges (weakly) after interpolation (and subsequence extraction) to a weak solution of (LL) when $\delta t \to 0$ and $\delta x \to 0$ provided $\theta > \frac{1}{2}$ and the meshes satisfy (**)
- When $\theta = \frac{1}{2}$ same result if moreover $\delta t/\delta x \to 0$.
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2nd order in time...

A priori the renormalization stage forbids an order 2 formulation $(|m + \delta t\, v| = 1 + \frac{\delta t^2}{2}|v|^2 + O(\delta t^4))$

Idea 3 : Look for $v \perp m$ such that

$$\frac{m + \delta t\, v}{|m + \delta t\, v|} = m(\delta t) + O(\delta t^3)$$

be 2nd order precise.

One finds $v = m_t + \frac{\delta t}{2}\,\Pi_{m^\perp} m_{tt}$

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- For $s = 0$ the scheme still needs a linear problem to be solved but is not robust. We can not prove existence (and uniqueness of a solution)

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$$-\frac{\delta t}{2} \int |\nabla m^{n+s}|^2 v^n \cdot w = -\int \nabla m^n \cdot \nabla w$$

- Set $m^{n+1} = \sum_i m_i^{n+1} \phi_i$, with $m_i^{n+1} = \dfrac{m_i^n + \delta t\ v_i^n}{|m_i^n + \delta t\ v_i^n|}$

- Energy decay control for $s = 1$, to the price of a slight non linearity

- For $s = 0$ the scheme still needs a linear problem to be solved but is not robust. We can not prove existence (and uniqueness of a solution)

Order 2...

- $\forall n \geq 0$, Find $v^n \in K_n$ such that $\forall w \in K_n$

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$$- \frac{\delta t}{2} \int |\nabla m^{n+s}|^2 v^n \cdot w = - \int \nabla m^n \cdot \nabla w$$

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- Energy decay control for $s = 1$, to the price of a slight non linearity
- For $s = 0$ the scheme still needs a linear problem to be solved but is not robust. We can not prove existence (and uniqueness of a solution)

Idea 4

- $\forall n \geq 0$, Find $v^n \in K_n$ such that $\forall w \in K_n$

$$\int \frac{\alpha}{1 + \frac{\delta t}{2\alpha}|\nabla m^n|^2} v^n \cdot w + m^n \times v^n \cdot w + \frac{\delta t}{2} \int \nabla v^n \cdot \nabla w$$
$$= - \int \nabla m^n \cdot \nabla w$$

- Set $m^{n+1} = \sum_i m_i^{n+1} \phi_i$, with $m_i^{n+1} = \dfrac{m_i^n + \delta t \; v_i^n}{|m_i^n + \delta t \; v_i^n|}$
- Energy decays along iterations
- Linear iteration (existence and uniqueness of a solution)
- convergence (with minor modifications)

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- Energy decays along iterations
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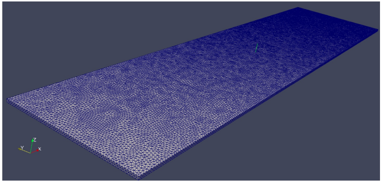
Conclusions

- 1st order in time with cv proof
- 2nd order in time stability (cv ?) proof
- 1st order in space
- FMM or NUFFT for stray field
- Preconditionning of linear systems

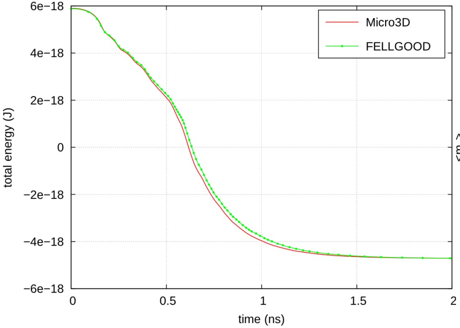
Applications

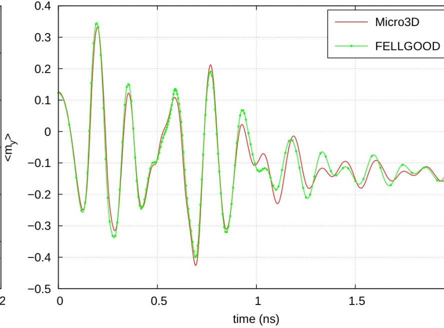
- Comparison with finite difference/finite volumes codes and experiments
- Statics and Dynamics
- NIST benchmark problems
- nanodots
- spin oscillators
- ...

NIST Problem #4



33k Nodes, 180K Elements





Spin oscillators