The influence of the wall on the motion of micro-swimmer

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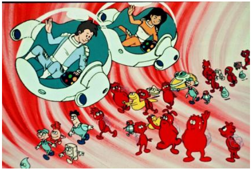
Motivations

- Developing self-propulsion at micro-scales?
- Application in human diagnostics and therapy...



Model swimmer/fluid

The swimmer is described by the vector (ξ, p) such as :

- ξ is a function which defines the shape of the swimmer.
- $p = (c, R) \in \mathbb{R}^3 \times SO(3)$ parametrizes the swimmer's position.

The swimmer changes its shape $\Longrightarrow \xi(t)$ pushes the fluid.
The fluid reacts, under the Stokes Equation

$$\left[\begin{array}{l} -\nu\Delta u + \nabla p = f, \\ \mathrm{div} u = 0. \end{array}\right.$$

Self-propulsion constraints $\Longrightarrow \begin{cases} \sum \text{Forces} & = 0 \\ \text{Torque} & = 0 \end{cases}$

$$\Longleftrightarrow \begin{cases} \int_{\partial\Omega} DN_{p,\xi} \left((\partial_p \Phi)\dot{p} + (\partial_\xi \Phi)\dot{\xi} \right) dx_0 = 0 \\ \int_{\partial\Omega} x_0 \times DN_{p,\xi} \left((\partial_p \Phi)\dot{p} + (\partial_\xi \Phi)\dot{\xi} \right) dx_0 = 0. \end{cases}$$

As a result the swimmer moves, under the ODE

$$\dot{p} = V(p, \xi)\dot{\xi}.$$

[Dal Maso, Desimone, and Marandotti, 2010]

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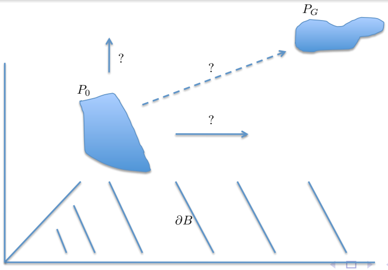
Controllability issues

$$\begin{cases} \dot{p} = V(p,\xi)\dot{\xi} \\ p_0 \end{cases}$$

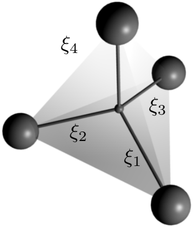
Questions

- Is it possible to control the state of the system (ξ and p) using as controls only the rate of shape changes $\frac{d}{dt}\xi$?
- Does the boundary have an effect on the controllability of the swimmer?

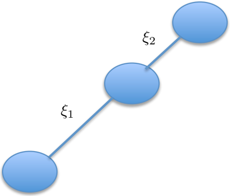


The swimmers

The swimmer that we consider consists of n spheres connected by the swimmer's arm.
The change of the swimmer's shape consists in changing the length of its arms $(\xi_i)_i$.



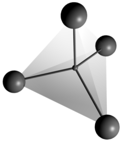
Four sphere swimmer



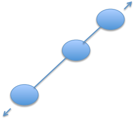
Three sphere swimmer
[Golestanian, Najafi 2004]

Example of stroke

Controllability's result in $\mathbb{R}^3$ [Alouges, DeSimone, Heltai, Lefevbre, Merlet (Preprint)]

- 

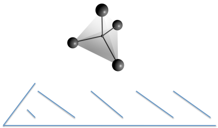
  The 4-sphere swimmer is globally controllable on $\mathbb{R}^3$.

- 

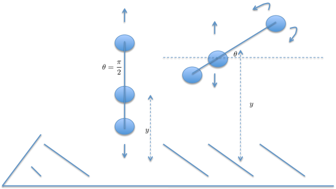
  The 3-sphere swimmer is globally controllable on $\mathbb{R}$.

- Does the presence of a wall modify the swimmer's reachable set?

Influence of the wall - Main results [Alouges, G]



The 4-spheres swimmer is globally controllable on an dense open set.



- For any initial condition (y_0, θ_0) such that $\theta_0 \neq \frac{\pi}{2}$, the swimmer can reach every (y_G, θ_G) given $(\theta_G \neq \frac{\pi}{2})$.
- If $\theta_0 = \frac{\pi}{2}$ then the swimmer cannot change its angle and it moves only on a straight line defined by itself. (i.e., the dimension of $Lie_{(\xi_1, \xi_2, y, \frac{\pi}{2})}(V_1, V_2)$ is equal to 3.

Outline of the proof

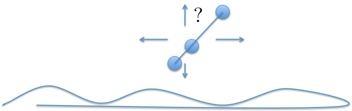
$$\dot{p} = \sum_{i=1}^{M} V_i(p,\xi)\dot{\xi}$$

By studying the dimension of the subspace $Lie_{(p,\xi)}((V_i)_{i=1..M})$ which denotes the set of all tangent vectors $V(p,\xi)$ in $Lie((V_i)_{i=1..M})$.

- By using the limit and the case without wall
- By calculation of Lie Brackets and application of Nagano (1966) Hermann (1963) theorem [Lobry 1970], we show that there are two kinds of orbit :
  - the orbit with a 3 dimensional Lie space (if $\theta_0 = \frac{\pi}{2}$).
  - the others such that the dimension is equal to 4.

Conclusion and outlook

- Influence of the boundary.



- Optimal strokes.