Algorithm Design in Evolutionary Computation: Parameter Identification and Control

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General thoughts on parameters

- Any algorithm has an arbitrary number of parameters

for example: think of all the hidden “1”s
parameters can be used to toggle any algorithms

- First task: agree on the relevant parameters

this is a non-trivial task
on-the-fly definition for *relevant*: a non-trivial tuning improves the performance (sometimes) by more than a factor of two

Classification of parameters

- Off-line parameter identification during the algorithm design

  e.g. based on benchmarks or the designers guts

- ~~Trial-and-error parameter setting by the user~~

  ~~not well-defined (this is not an algorithm)~~

- Setting prior to application, based on known or measured problem properties (features)

  e.g. using the problem dimension

- On-line adaptation
  - Restarts with different parameters (can replace a users trial-and-error procedure)
  - Self-adaptation, CMA, reinforcement, ...

The latter two are an integral part of the algorithm itself

Tuning methods depend essentially on the “class”

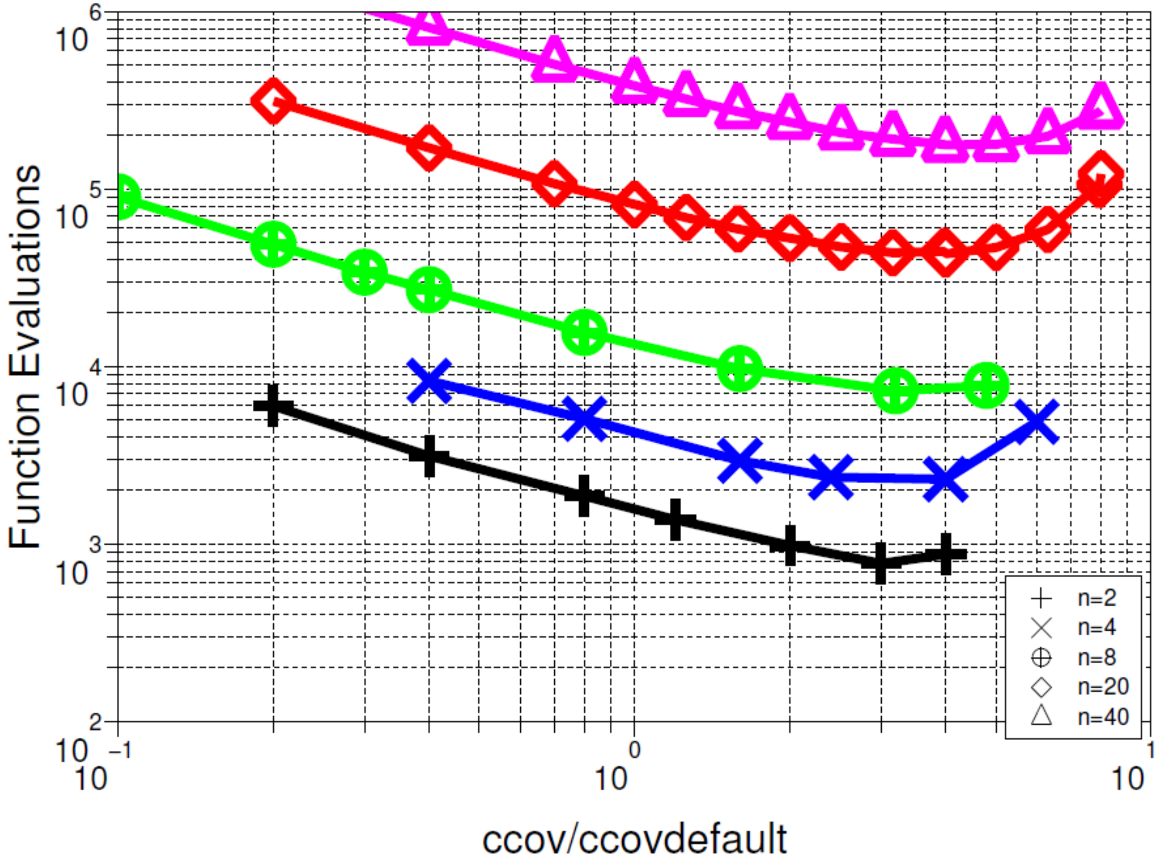
Example for off-line identification

- Learning rate ccov for covariance matrix adaptation (CMA)
  - Using the most simple function that seems to make sense: sphere function $\sum_i x_i^2$ with initially anisotropic covariance matrix
  - Invariance properties and further empirical evidence (in the sense of hypothesis testing) suggest generality of the results

$$C \leftarrow (1 - \mathrm{ccov})\, C + \mathrm{ccov} \sum_i^{\mu} z_i z_i^T$$

$$= C + \mathrm{ccov} \times \left(\sum_i^{\mu} z_i z_i^T - C \right)$$

Performance on the sphere



- Trade off: robustness (small learning rate ccov to the left) versus speed (large ccov). Lines (would) smoothly continue to the left.
- Remark: x-axis presentation

Choosing the parameter

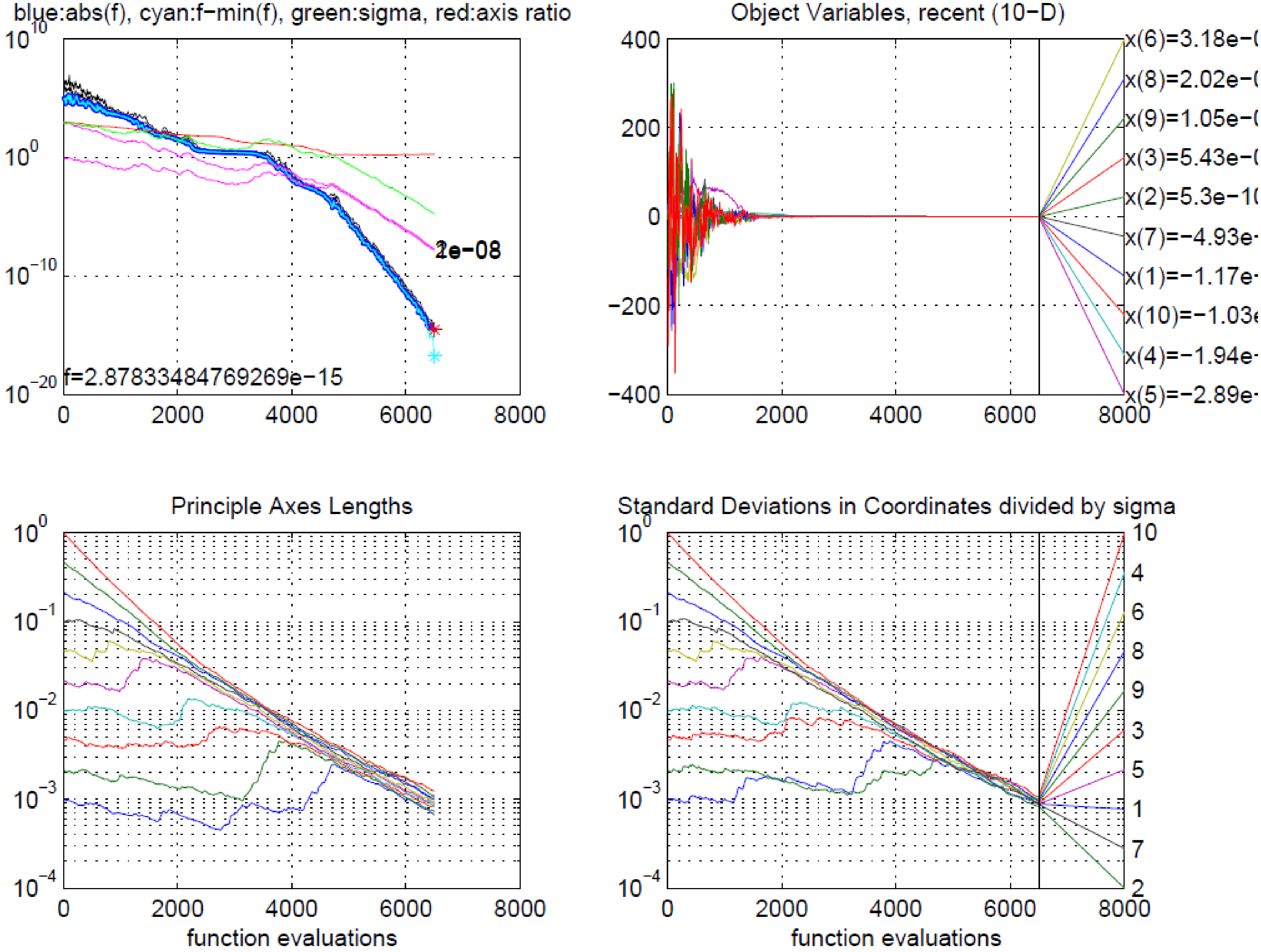
- Robustness (required):
  - increasing ccov three times (faster learning) never leads to a failure
  - increasing ccov two times is better than decreasing ccov two times
- Performance (desirable): the performance loss is less than a factor of three

for ccov=0 the loss factor is roughly 1000

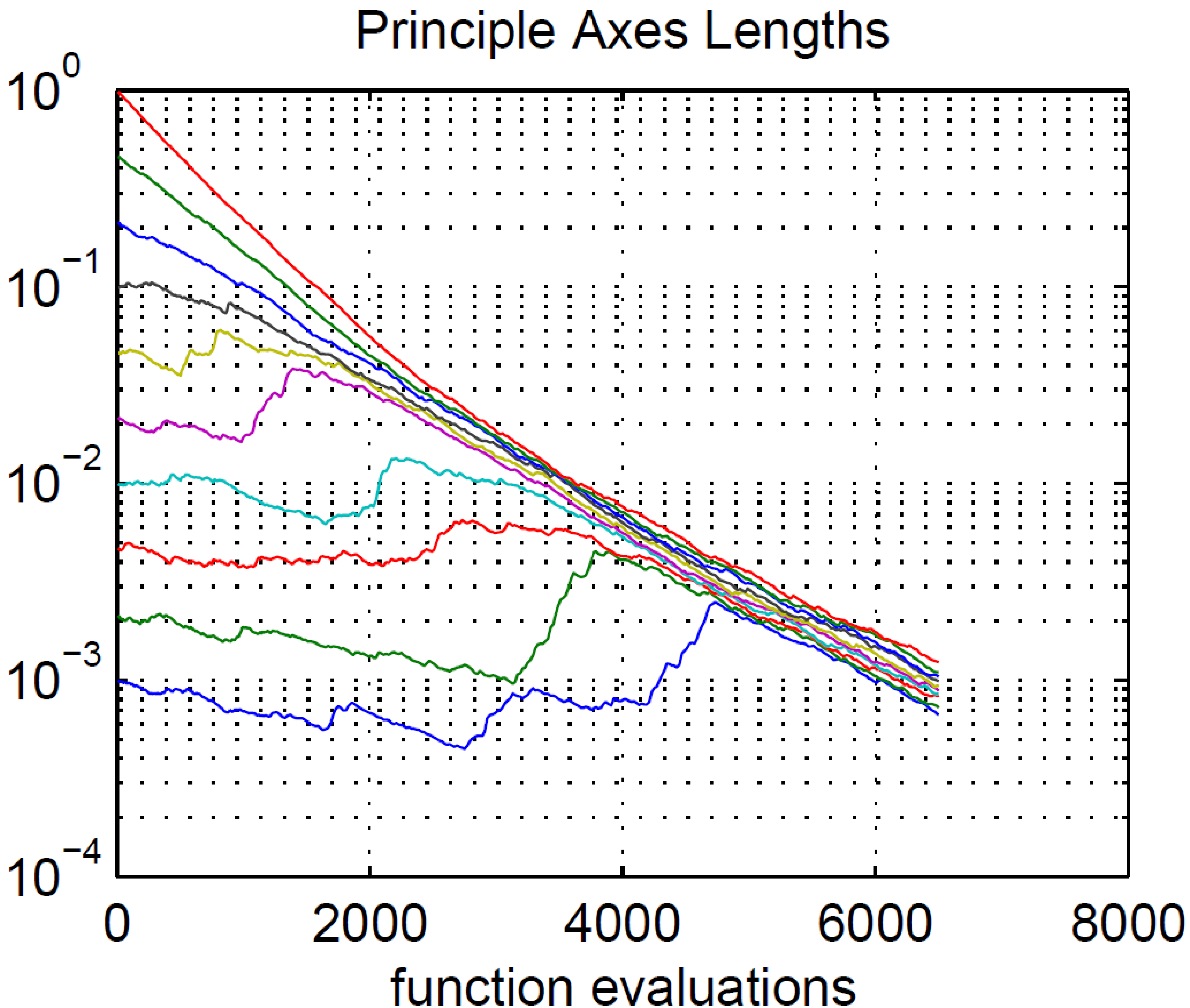
- Adaptation quality (learning accuracy, related to robustness): final condition number of the covariance matrix is smaller than ten.

This (a similar) list works in many cases cf. [Brockhoff et al 2010. Mirrored sampling and sequential selection for evolution strategies, PPSN XI]

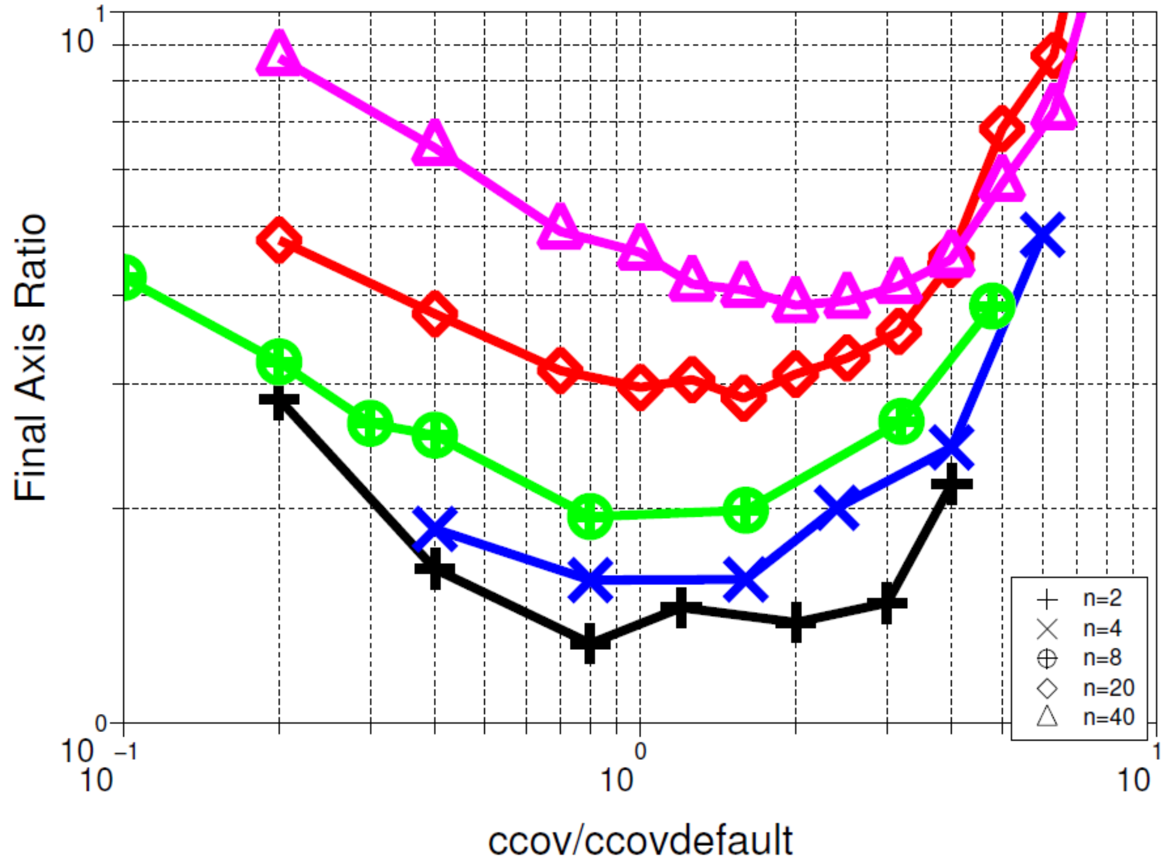
CMA-ES on the sphere function



CMA-ES on the sphere function



Adaptation quality



Axis ratio of mutation ellipsoid = sqrt(condition number of covariance matrix)

Final result for ccov

- Identification needs to be done depending on the population size

more precisely: the amount of input information

- A trial-and-error process leads to

$$\mathrm{ccov} = 2\,\frac{\mu_{\mathrm{eff}} - 1 + 1/\mu_{\mathrm{eff}}}{(n+2)^2 + \mu_{\mathrm{eff}}}$$

  - Inverse proportional to the degrees of freedom
  - Proportional to the amount of input information
  - Correction for small values

Off-line identification = algorithm design

- Major pitfall: we must seek for good generalization performance, i.e. prevent overtuning to test problems (= design for a too narrow problem class)

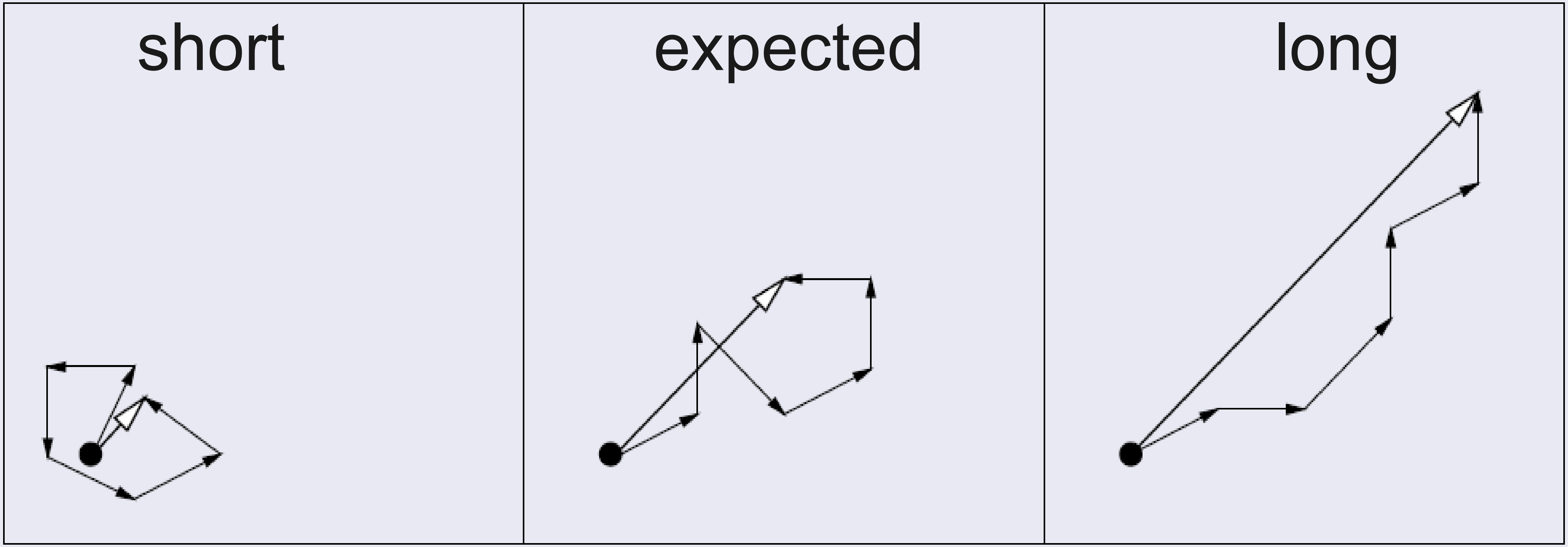
an optimal parameter setting on a single problem is rarely of practical interest

- Using a single function works(!) and leaves many functions for hypothesis testing

Example: Evolution Path

- An evolution path (search path) is extensively used for on-line parameter adaptation in CMA-ES

conceptually, the path is the difference vector of the evolving population mean m



evolution (search) paths with cumulation of six time steps in 2-D

Example: Evolution Path

- An evolution path (search path) is extensively used for on-line parameter adaptation in CMA-ES

conceptually, the path is the difference vector of the evolving population mean m

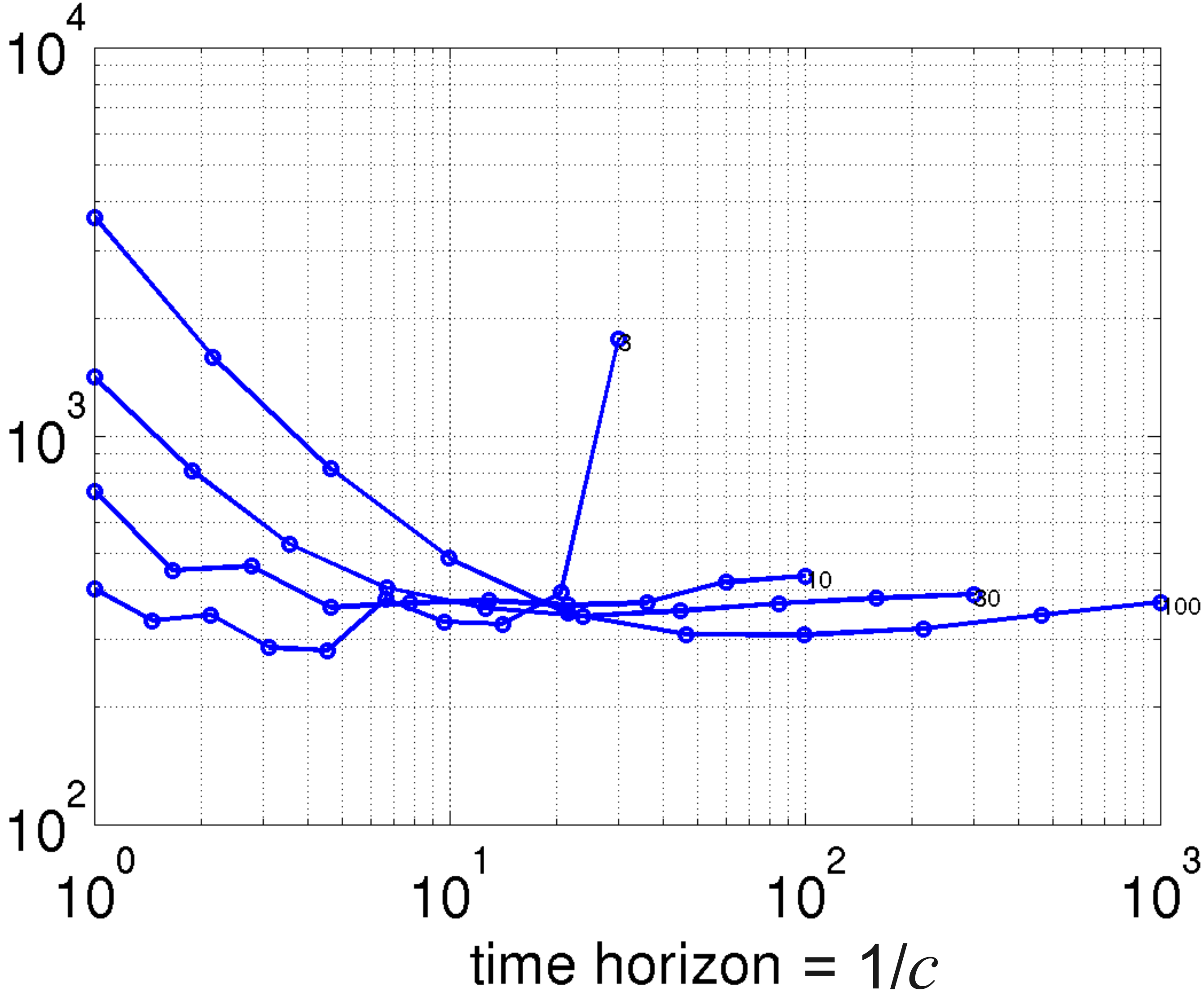
- Question: How many time steps should an evolution path comprise?

$$p \leftarrow (1-c)\,p + \sqrt{c(2-c)}\,\Delta m^t$$

$$\approx \sqrt{c}\,(m^t - m^{t-\lfloor 1/c \rfloor}) =: \frac{1}{\sqrt{t_c}}(m^t - m^{t-t_c})$$

$1/c$ is roughly the effective number of time steps

Cumulation time horizon

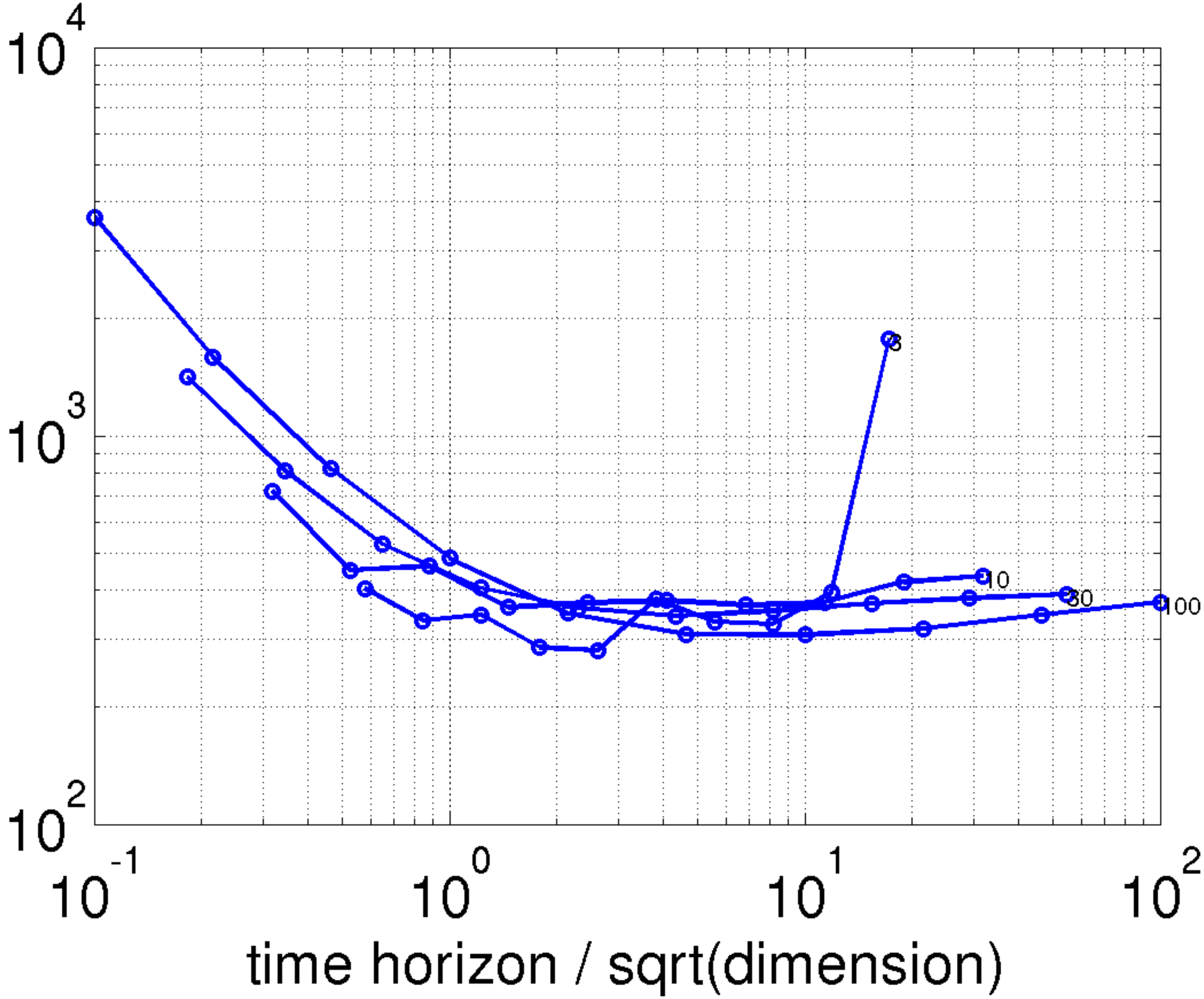


$$f(x) = x_1 + 10^6 \sum_{i=2}^{n} x_i^2$$

The cigar function shows the most pronounced effect on cumulation

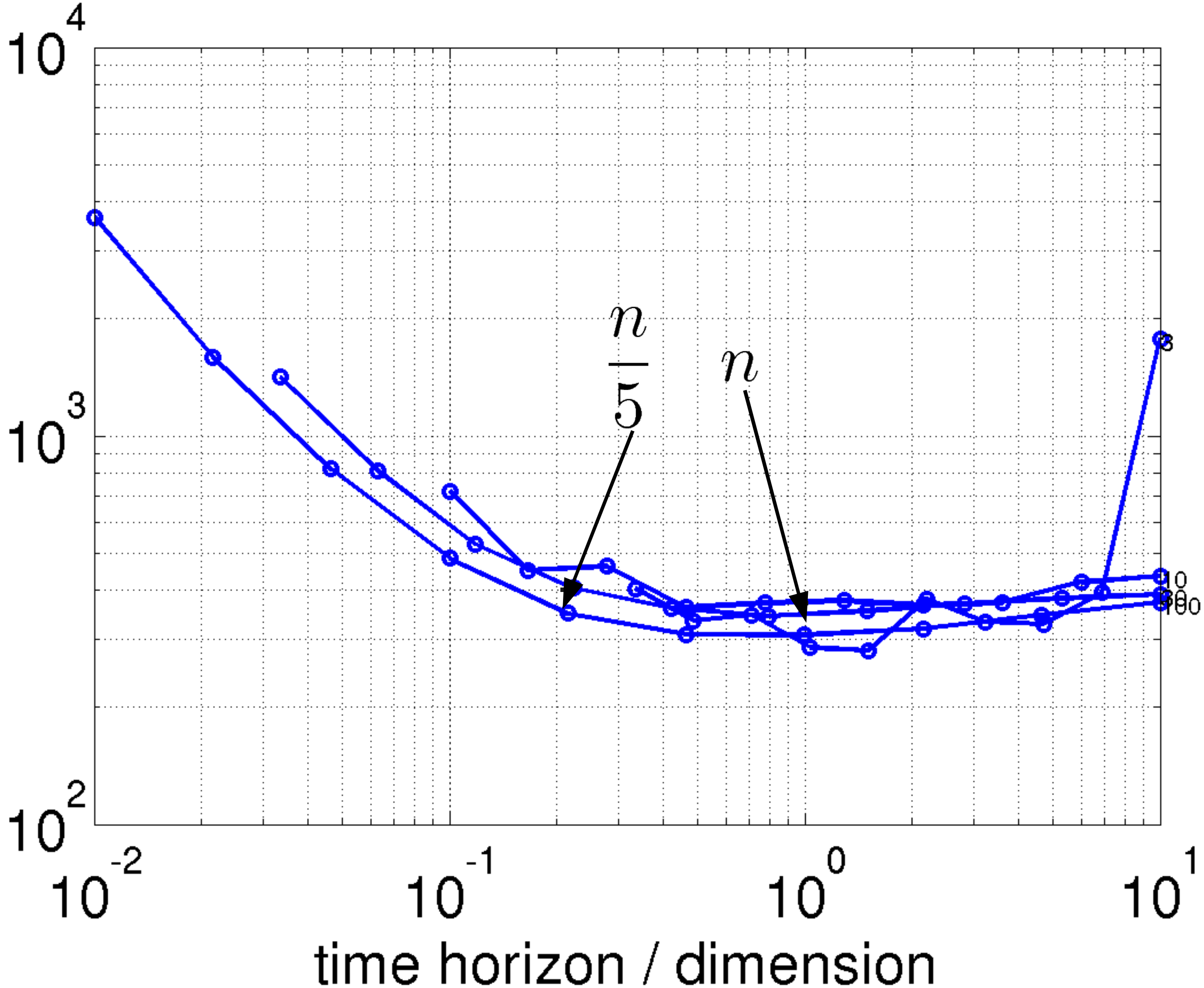
#Fevals divided by dimension in 3,10,30,100-D

Cumulation time horizon



#Fevals divided by dimension in 3,10,30,100-D

Cumulation time horizon



now the graphs are virtually invariant under the choice of the dimension

final choice:

$$\frac{1}{c} = \frac{n + 4 + 2\mu_{\mathrm{eff}}/n}{4 + \mu_{\mathrm{eff}}/n}$$

#Fevals divided by dimension in 3,10,30,100-D

On-line adaptation

Another name for control

- Controlled variable(s), caveat: parametrization / representation
  - Step-size
  - Population size
  - Covariance matrix with many degrees of freedom
  - Mutation/recombination rate...
- Update rule, “depends” on representation
- Input variable(s) (used information)
  - survival probability for different settings
  - success, success rate, or progress measurement
  - evolution path, length of evolution path...

- What is the difference between a state variable and an adaptive parameter?

Example: CMA-ES

Input: $m \in \mathbb{R}^n$, $\sigma \in \mathbb{R}_+$, $\lambda \in \{2, 3, 4, \dots\}$

Set $c_c \approx 4/n$, $c_\sigma \approx 4/n$, $c_1 \approx 2/n^2$, $c_\mu \approx \mu_w/n^2$, $c_1 + c_\mu \leq 1$, $d_\sigma \approx 1 + \sqrt{\frac{\mu_w}{n}}$, set $w_{i=1,\dots,\lambda}$ such that $\mu_w = \frac{1}{\sum_{i=1}^{\mu} {w_i}^2} \approx 0.3\,\lambda$

Initialize $\mathbf{C} = \mathbf{I}$, and $\mathbf{p}_c = \mathbf{0}$, $\mathbf{p}_\sigma = \mathbf{0}$

While not *terminate*

$$\mathbf{x}_i = m + \sigma\,\mathbf{y}_i, \quad \mathbf{y}_i \sim \mathcal{N}_i(\mathbf{0}, \mathbf{C}), \quad \text{for } i = 1, \dots, \lambda \qquad \text{sampling}$$

$$m \leftarrow \sum_{i=1}^{\mu} w_i\,\mathbf{x}_{i:\lambda} = m + \sigma\mathbf{y}_w \qquad \text{update mean}$$

$$\mathbf{p}_\sigma \leftarrow (1 - c_\sigma)\,\mathbf{p}_\sigma + \sqrt{1 - (1 - c_\sigma)^2}\sqrt{\mu_w}\,\mathbf{C}^{-\frac{1}{2}}\,\mathbf{y}_w \qquad \text{path for } \sigma$$

$$\sigma \leftarrow \sigma \times \exp\left(\frac{c_\sigma}{d_\sigma}\left(\frac{\|\mathbf{p}_\sigma\|}{\mathsf{E}\|\mathcal{N}(\mathbf{0},\mathbf{I})\|} - 1\right)\right) \qquad \text{update of } \sigma$$

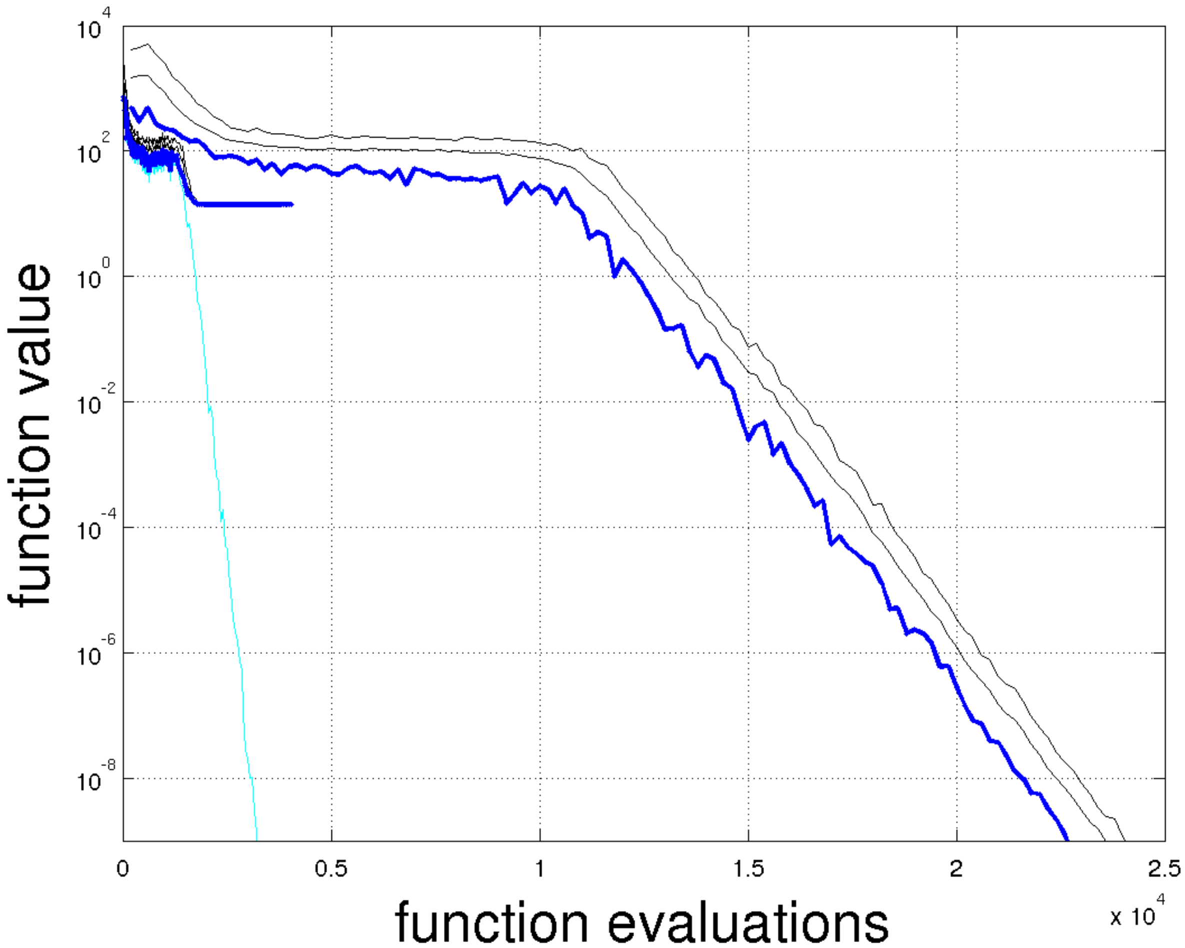
$$\mathbf{p}_c \leftarrow (1 - c_c)\,\mathbf{p}_c + \mathbb{1}_{[0,1.5]}\left(\frac{\|\mathbf{p}_\sigma\|}{\sqrt{n}}\right)\sqrt{1 - (1 - c_c)^2}\sqrt{\mu_w}\,\mathbf{y}_w \qquad \text{path for } \mathbf{C}$$

$$\mathbf{C} \leftarrow (1 - c_1 - c_\mu)\,\mathbf{C} + c_1\,\mathbf{p}_c\mathbf{p}_c^{\mathrm{T}} + c_\mu \sum_{i=1}^{\mu} w_i\,\mathbf{y}_{i:\lambda}\mathbf{y}_{i:\lambda}^{\mathrm{T}} \qquad \text{update } \mathbf{C}$$

Population size adaptation

- Can it work reliably?

multimodality as counterexample



Rastrigin function in 10-D

Population size 10 and 200

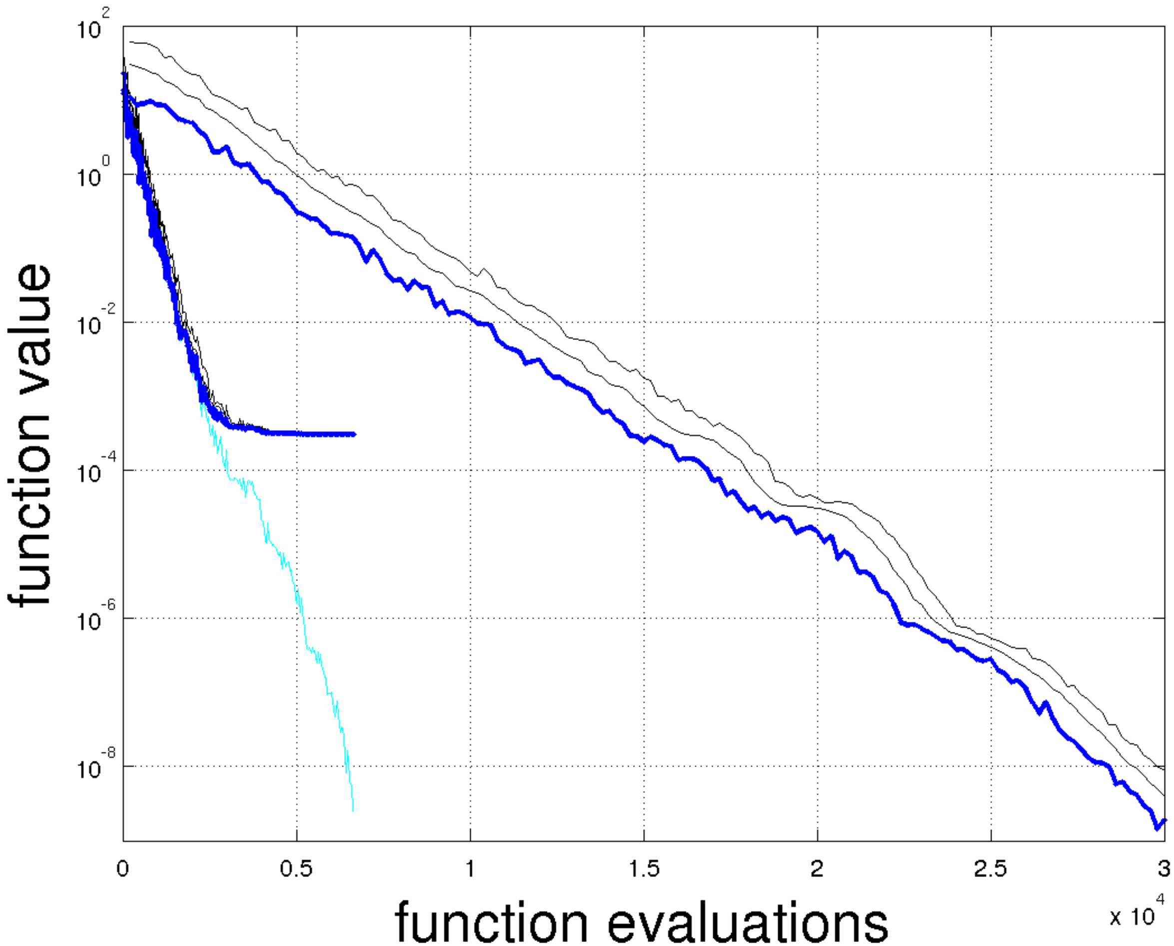
The advantage of the large population becomes visible only way after the run with small population size finished

Is a widely suitable input variable conceivable?

Population size adaptation

- Can it work reliably?

multimodality as counterexample



Schaffer function in 10-D

Population size 10 and 200

The advantage of the large population becomes visible only way after the run with small population size finished

Is a widely suitable input variable conceivable?

It would need to be able to "predict" the late outcome of a run very early

Population size adaptation

A solution

- Conduct several runs with different population sizes
  - Serially with increasing population size [Harik&Lobo 1999], [Auger&Hansen 2005]
  - Parallel (race) with different population sizes [Harik&Lobo 1999], [Hornby 2006]

Summary

1) Remove users trial-and-error procedure from the possibilities (in a scientific context)

the algorithm designer should instead suggest a sequence of settings

2) Parameter identification in the context of algorithm design is difficult and tedious

but it is worth the time

3) Parameter identification can be done successfully on a single function

4) The population size is hard to adapt

Thank you