# A Classification of Dynamic Multi-Objective Optimization Problems *

Alexandru-Adrian Tantar, Emilia Tantar, Pascal Bouvry
CSC-FSTC, University of Luxembourg
6 rue Coudenhove-Kalergi, Luxembourg
{alexandru.tantar, emilia.tantar, pascal.bouvry}@uni.lu

## ABSTRACT

A classification of dynamic multi-objective optimization problems is proposed in this article. As compared to previous studies, we focus not on the changes or the effects that are induced in the Pareto optimal front or set but on the components that lead to the observed dynamic behaviour. Four main classes are identified, including parameter and function time-dependent evolution as well as state-dependent parameter and function transforms or environment changes.

## Categories and Subject Descriptors

A.1 INTRODUCTORY AND SURVEY [

## General Terms

]: Design, Theory

## Keywords

Dynamic multi-objective classification, dynamic optimization, multi-objective optimization

## 1. INTRODUCTION

Over the past decade a trend towards modeling optimization problems in full detail can be identified where multi-objective and dynamic aspects are being dealt with. Nonetheless, we still need to understand what *dynamic* means, e.g. where is the limit between models defined as static snapshots at discrete time moments and models subject to continuous evolution. Moreover we need to be able to define sound formal models and to understand what is the connection between dynamic changes in the input variables and the resulting effects in the objective space. Answering the question of what classes can be defined for dynamic multi-objective functions leads to (1) delimiting the basic time-dependent components, e.g. in the decision space, environment, or (2) describing the combination of changes that appear in the Pareto front and set, as in Farina *et al.* [4], Mehnen *et al.* [5]. As no clear formal models were provided for the former part, in this paper we focus on providing a classification of the different dynamic elements that induce dynamism for multi-objective formulations.

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*GECCO'11,* July 12–16, 2011, Dublin, Ireland.
ACM 978-1-4503-0690-4/11/07.

The main classification criteria found in literature can be separated in (a) studies dealing with periodicity, continuity or sparsity characteristics, environment time-dependent terms, behavior of specific algorithms [7], (b) changing morphology aspects, drifting landscapes [3], (c) dynamic components with transforms of the coordinates or fitness rescaling [6] or (d) particular aspects of optimization algorithms and the role of parameters in problem generators [1]. A recent overview of the domain can be found in the work of Bu and Zheng [2].

## 2. DYNAMIC MULTI-OBJECTIVE PROBLEMS: DEFINITION AND CLASSIFICATION

The behavior of a dynamic multi-objective optimization problem (system) can be modeled as $H(F_\sigma, D, x, t)$, with $\sigma$ being an environment derived set of parameters; $x$, the set of variables/parameters; $t$, the time moment; $F_\sigma$, the multi-objective support function, $F_\sigma : X \rightarrow Y$, $F_\sigma(x) = [f_{\sigma,1}(x), \ \ldots \ , f_{\sigma,k}(x)]$ and $D = [d_1, \ \ldots \ , d_k]$, a vector of time dependent functions, modeling the dynamic behavior of the time-changing component of the system. We denote the state of the system at a time $t$ as described by the values of the state variables, given by the parameters of $F_\sigma$ and $D$.

Based on the different types of dynamic transformations that can occur we propose in the following a classification for dynamic multi-objective optimization problems. For previous existing classifications no time dependency was considered at an environment level (static definition). Moreover, the state of a dynamic system (function and environment) at a given moment in time was generally defined as or assumed to be independent of the previous states. Of specific interest for our work, four different types of multi-objective dynamic functions were identified in Mehnen *et al.* [5] using as classification criteria the link between the behavior (static or dynamic) of the Pareto Optimal Set and that of the Pareto Optimal Front. Nonetheless, this classification, although of undisputed importance, does not capture nor describes the elements that turn a static problem into a dynamic one. In order to answer this concern we propose the following classification:

- $1^{st}$ **order** [*parameter evolution*]: dynamic transform of the input parameters;
- $2^{nd}$ **order** [*function evolution*]: dynamic evolution of the objective functions values;
- $3^{rd}$ **order** [*state dependency*]: parameter or function state time-dependency, i.e. the parameters or the func-

tion is defined by taking into account the previous values obtained at previous states;

- $4^{th}$ **order** [*changing environment*]: parts of or the entire environment evolves with time.

All these classes can be further declined in the four types defined in [4]. For an arbitrary function one may thus consider a more exact description where both the dynamic components and the induced effects at the Pareto Optimal Set/ Pareto Optimal Front are specified, e.g. having a first order type I function. For defining the four main classes we consider the behavior of the optimization problem over the time interval $[t_0, t_{end}]$. A formal description and a more detailed discussion is provided in the following.

## 2.1 Dynamic parameter-time evolution

**Characteristics**: *dynamic transform applied on the input variables (decision space).* The external environment (described by $\sigma$) does not change with time and the support function formulation remains unchanged.
**Formulation**: $H(F_\sigma, D, x, t) = F_\sigma(D(x, t))$.
**Optimization problem**:

$$\min_{\mathbf{x}(t)} \left\{ \left( \int_{t_0}^{t^{\text{end}}} f_i(\ d_i(\mathbf{x}(t), t)\ )\ \mathrm{d}t \right)_{1 \le i \le k} \right\}$$

## 2.2 Dynamic Function evolution

**Characteristics**: *dynamic transform applied on the support function (affecting the behavior in the objective space), e.g. superposed noise evolving with time.* The external environment (described by $\sigma$) does not change with time.
**Formulation**: $H(F_\sigma, D, x, t) = D(F_\sigma, x, t)$, with $D(F_\sigma, x, t) = (\ d_i(f_{\sigma,i}, x, t))_{1 \le i \le k}$.
**Optimization problem**:

$$\min_{\mathbf{x}(t)} \left\{ \left( \int_{t_0}^{t^{\text{end}}} d_i(\ F_\sigma, \mathbf{x}(t), t)\ )\ \mathrm{d}t \right)_{1 \le i \le k} \right\}$$

## 2.3 State-dependency

**Characteristics**: *the dynamic transform at time $t$, takes into account values obtained by the support functions/variables at $j$ previous time moments,* $1 \le j \le t$. The external environment (described by $\sigma$) does not change with time. Let us consider that the analytical form of $H$ at a given time moment $t$ fuly describes the state of the dynamic system.
**Formulation**: $H(F_\sigma, T^{[t-j,t]}, x, t)$
**Optimization problem**:

$$\min_{\mathbf{x}(t)} \int_{t_0}^{t_{end}} H(F_\sigma, T^{[t-j,t]}, x(t), t)\mathrm{d}t$$

The values of $H$ at time $t$ depend on the values/formulations of the parameters or base functions at previous states of the system, given by a trasformation function $T^{[t-j,t]}$, as opposed to the previous two formulations involving static dependency function $D$ at each state (independent states).

### 2.3.1 State-parameter-dependency

**Characteristics**: *The current state of the optimization system is expressed analytically in function of the past states from $t - j$ to $t$. In this first case of state-parameter dependency, the values of the set of parameters $x$ at time $t$ depend of the values of the $x$ parameters at the time $t - j$ to $t$.*
**Formulation**: $H(F_\sigma, T^{[t-j,t]}, x, t) = F_\sigma(T^{[t-j,t]}(x))$
**Optimization problem**:

$$\min_{\mathbf{x}(t)} \left\{ \left( \int_{t_0}^{t_{end}} f_{\sigma,i}(\ T^{[t-j,t]}(x(t)),\ t)\ \mathrm{d}t \right)_{1 \le i \le k} \right\}.$$

### 2.3.2 State-Function-dependency

**Characteristics**: *The value of the objective function depends on the previous values of the base function $F_\sigma$ on a given time interval $[t - j, t]$.The optimal solution is defined on the previous values of the function $F$.*
**Formulation**: $H(F_\sigma, T^{[t-j,t]}, x, t) = T^{[t-j,t]}(F_\sigma(x))$
**Optimization problem**:

$$\min_{\mathbf{x}(t)} \int_{t_0}^{t_{end}} T^{[t-j,t]}(F_\sigma(\mathbf{x}(t))\mathrm{d}t.$$

## 2.4 Online dynamic multi-objective optimization

**Characteristics**: *the environment $\sigma$ changes dynamically with time.* **Formulation**: $H(F_\sigma, D, x, t) = F_{D(\sigma,t)}(x, t)$
**Optimization problem**:

$$\min_{\mathbf{x}(t)} \left\{ \left( \int_{t_0}^{t_{\text{end}}} f_{D(\sigma,t),i}(\mathbf{x}(t), t)\ \mathrm{d}t \right)_{1 \le i \le k} \right\}$$

All the four previous classes ca be also extended to the case of dynamic environments, leading to online dynamic multi-objective formulations.

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