Adaptive Parameter Selection for Strategy Adaptation in Differential Evolution

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ABSTRACT

In order to automatically select the most suitable strategy for a specific problem without any prior knowledge, in this paper, we present an adaptive parameter selection technique for strategy adaptation in differential evolution (DE). First, a simple strategy adaptation mechanism is employed to implement the adaptive strategy selection in DE. Then, the probability- matching-based adaptive parameter selection method is proposed to select the best parameter of the strategy adaptation mechanism; in this way, it can accelerate the strategy adaptation mechanism to choose the most suitable strategy while solving a problem. Experimental results indicate that our method obtains better results in terms of the quality of the final solutions and the convergence speed, compared with the classical DE algorithms with single strategy.

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Differential evolution, adaptive parameter selection, strategy adaptation, global optimization

1. INTRODUCTION

Differential evolution (DE), proposed by Storn & Price in 1995 [4], is an efficient and effective population-based evolutionary algorithm for the global optimization. Although augmenting the robustness of the underlying algorithm, these many available strategies led the user to the problem of defining which of them would be most suitable for the problem at hand – a difficult and crucial task for the performance of DE [3].

In [1], we proposed several simple strategy adaptation mechanisms (SaMs) for strategy adaptation in DE. However, the final performance of SaJADE with the first SaM may be influenced by the order of the strategies in the strategy pool [1]. To remedy this drawback, in this paper, the probability-matching-based adaptive parameter selection (APS) method is proposed to select the best parameter of the strategy adaptation mechanism; in this way, it can

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accelerate the strategy adaptation mechanism to choose the most suitable strategy while solving a problem. To verify the performance of our approach, 13 benchmark functions are selected as the test suit; and the performance of our approach is compared with the classical DE algorithm and some state-of-the-art DE variants.

2. OUR APPROACH

Although there are some DE variants with strategy adaptation, the study on adaptive strategy selection in DE is still scarce. In addition, in [1], we proposed several simple strategy adaptation mechanisms (SaMs) for strategy adaptation in DE. However, the final performance of SaJADE with the first SaM may be influenced by the order of the strategies in the strategy pool [1]. Based on these considerations, in this work, we propose an adaptive parameter selection (APS) method to choose the most suitable parameter for the first method proposed in [1]. Our proposed approach is referred to as APS-SADE, *i.e.*, APS for the Strategy Adaptation in DE.

2.1 Strategy Adaptation Mechanism

In this work, the strategy adaptation mechanism proposed in [1] is employed. Due to the tight space limitation, interested reader can be found more details in [1].

2.2 Adaptive Parameter Selection

The performance of SaM presented in [1] may be influenced by the strategy order in the pool. That is, its performance could be influenced by the initial value of μ_s . To remedy this drawback, in this section, we propose an probability-matching (PM) based APS method to adaptively choose the most suitable μ_s for a specific problem. The PM-based APS method is inspired by our previous work for the strategy adaptation in DE [2].

Let A_{μ_s} be a parameter pool to store the initial μ_s values. Suppose we have K > 1 values in the pool $A_{\mu_s} = \{a_1, \dots, a_K\}$ and a probability vector $\mathbf{P}(t) = \{p_1(t), \dots, p_K(t)\} \ (\forall t : 0 \le p_i(t) \le 1; \sum_{i=1}^K p_i(t) = 1)$. In this work, the PM technique is used to adaptively update the probability $p_a(t)$ of each value a based on its reward. Denote $r_a(t)$ as the reward that a value areceives after its application at time t. $q_a(t)$ is the known quality of a value a, that is updated as follows [5]:

$$q_a(t+1) = q_a(t) + \alpha [r_a(t) - q_a(t)], \tag{1}$$

where $\alpha \in (0, 1]$ is the adaptation rate. The PM method updates the probability $p_a(t)$ as follows [5]:

$$p_a(t+1) = p_{min} + (1 - K \cdot p_{min}) \frac{q_a(t+1)}{\sum_{i=1}^{K} q_i(t+1)}.$$
 (2)

where $p_{min} \in (0, 1)$ is the minimal probability value of each value, used to ensure that no operator gets lost [5]. In this work, $A_{\mu_s} = \{0.1, 0.5, 0.9\}$, and K = 3.

F	DE/rand/1	DE/rand-to-best/2	DE/rand/2	DE/current-to-rand/1	APS-SADE
f_{01}	$6.41E-32 \pm 8.33E-32^{\dagger}$	$1.15\text{E-54}\pm1.59\text{E-54}^\dagger$	$8.21E-01 \pm 2.41E-01^{\dagger}$	$3.08E+00 \pm 3.98E+00^{\dagger}$	$3.74E-99 \pm 4.76E-99$
f_{02}	$6.50E-16 \pm 4.67E-16^{\dagger}$	$\textbf{2.48E-25} \pm \textbf{1.20E-25}^\dagger$	$3.13E+00 \pm 7.29E-01^{\dagger}$	$2.62E-02 \pm 3.01E-02^{\dagger}$	$1.46\text{E-47} \pm 1.02\text{E-47}$
f_{03}	$2.49E-05 \pm 2.07E-05^{\dagger}$	$\textbf{3.11E-10} \pm \textbf{2.58E-10}^\dagger$	$5.09E+03 \pm 1.15E+03^{\dagger}$	$2.84E+01 \pm 1.91E+01^{\dagger}$	$1.74\text{E-}21 \pm 3.41\text{E-}21$
f_{04}	$8.82\text{E-}02 \pm 2.19\text{E-}01^\dagger$	$1.06\text{E-}11\pm6.33\text{E-}12^\ddagger$	$1.57E+01 \pm 2.06E+00^{\dagger}$	$2.94E+00 \pm 1.09E+00^{\dagger}$	$2.04\text{E-}08 \pm 6.64\text{E-}08$
f_{05}	$1.43E+00 \pm 1.01E+00^{\dagger}$	$\mathbf{2.39E} ext{-}01\pm\mathbf{9.56E} ext{-}01^{\dagger}$	$3.82E+02 \pm 1.23E+02^{\dagger}$	$9.97E+01 \pm 6.72E+01^{\dagger}$	$7.97E-02 \pm 5.64E-01$
f_{06}	$0.00E$ +00 \pm 0.00E+00	$\textbf{0.00E+00} \pm \textbf{0.00E+00}$	$2.74E+00 \pm 1.10E+00^{\dagger}$	$3.04E+00 \pm 2.94E+00^{\dagger}$	$0.00E$ +00 \pm 0.00E+00
f_{07}	$4.71E-03 \pm 1.21E-03^{\dagger}$	$3.02\text{E-}03 \pm 8.79\text{E-}04^\dagger$	$7.45\text{E-}02 \pm 1.85\text{E-}02^{\dagger}$	$1.01\text{E-}03 \pm 4.53\text{E-}04$	$\textbf{9.78E-04} \pm \textbf{2.90E-04}$
f_{08}	$6.59E+03 \pm 7.04E+02^{\ddagger}$	$7.44E+03 \pm 3.18E+02^{\dagger}$	$7.42E+03 \pm 2.48E+02^{\dagger}$	$7.88E+03 \pm 2.60E+02^{\dagger}$	$7.22E+03 \pm 3.11E+02$
f_{09}	$1.41E+02 \pm 2.06E+01$	$1.69E+02 \pm 9.16E+00^{\dagger}$	$2.19E+02 \pm 1.22E+01^{\dagger}$	$1.31\text{E+02}\pm8.18\text{E+00}^\ddagger$	$1.40\text{E+02} \pm 1.03\text{E+01}$
f_{10}	$\textbf{4.14E-15} \pm \textbf{0.00E+00}$	$4.57\text{E-}15 \pm 1.17\text{E-}15$	$1.05E+00 \pm 2.61E-01^{\dagger}$	$2.79\text{E-}01 \pm 3.01\text{E-}01^{\dagger}$	$4.07\text{E-}15 \pm 5.02\text{E-}16$
f_{11}	$0.00\text{E+00} \pm 0.00\text{E+00}$	$1.92\text{E-}03 \pm 4.54\text{E-}03^{\dagger}$	$8.95E-01 \pm 4.46E-02^{\dagger}$	$5.98E-01 \pm 3.83E-01^{\dagger}$	$3.45\text{E-04} \pm 1.73\text{E-03}$
f_{12}	$1.93\text{E-}32\pm6.70\text{E-}33^\dagger$	$\textbf{1.57E-32} \pm \textbf{0.00E+00}$	$3.33E+00 \pm 9.20E-01^{\dagger}$	$4.34\text{E-}03 \pm 1.29\text{E-}02^{\dagger}$	$1.57\text{E-}32 \pm 0.00\text{E+}00$
f_{13}	$1.44\text{E-30}\pm1.80\text{E-30}^\dagger$	$1.35\text{E-}32\pm0.00\text{E+}00$	$2.20E+01 \pm 6.89E+00^{\dagger}$	$1.63E-02 \pm 4.14E-02^{\dagger}$	$\textbf{1.35E-32} \pm \textbf{0.00E+00}$
win/tie/lose	8/4/1	8/4/1	13/0/0	11/1/1	—

Table 1: Comparison on the Error Values Between the Classical DE with Single Strategy and APS-SADE for All Functions at D = 30.

[†] indicates APS-SADE is significantly better than its competitor by the Wilcoxon signed-rank test at $\alpha = 0.05$.

[‡] means that APS-SADE is significantly worse than its competitor by the Wilcoxon signed-rank test at $\alpha = 0.05$.

2.3 Strategy Pool

In order to select different mutation strategies to form the strategy pool, in this work, we have chosen four strategies that were also used in SaDE [3]. The four strategies are "DE/rand/1", "DE/randto-best/2", "DE/rand/2", and "DE/current-to-rand/1".

3. EXPERIMENTAL RESULTS AND ANAL-YSIS

In order to evaluate the performance of our approach, 13 benchmark functions are chosen from the literature as the test suite. These functions $(f_{01} - f_{13})$ are selected from [6]. For all experiments, the parameters are set as: D = 30; NP = 100; CR = 0.9, F = 0.5, K = 3, $p_{min} = 0.05$, and $\alpha = 0.3$; and Max_NFFEs=300, 000¹.

3.1 On the Adaptation of APS-SADE

To analyze the adaptation of our proposed APS-SADE, in this section, APS-SADE is compared with the original DE algorithm with single strategy in the strategy pool as mentioned in Section 2.3. The results are shown in Table 1. All results are averaged over 50 independent runs. The best and the second best results are highlighted in **grey boldface** and **boldface**, respectively. In Table 1, the paired Wilcoxon signed-rank test at $\alpha = 0.05$ is adopted to compare the significance between two algorithms.

According to results shown in Table 1, w.r.t. the error values, it can be seen that APS-SADE obtains the best overall results compared with the original DE algorithm with single strategy. APS-SADE significantly outperforms DE with "DE/rand/1", "DE/rand-to-best/2", "DE/rand/2", and "DE/current-to-rand/1" on 8, 8, 13, and 11 functions, respectively. On 9 functions, APS-SADE is able to provide the best error values. On the rest 4 functions (f_{04} , f_{08} , f_{09} , and f_{11}), our approach is the second best one.

4. CONCLUSIONS AND FUTURE WORK

In this paper, we proposed a probability-matching-based adaptive parameter selection (APS) technique for the strategy adaptation in DE. The APS technique is used to alleviate the drawback in our previous proposed strategy adaptation mechanism [1]. The main contribution of this paper is the APS technique for adaptively choose the most suitable parameter in the parameter pool while solving a problem. By integrating APS and SaM into DE, APS-SADE is proposed to adaptively select the most suitable strategy for a specific problem. APS-SADE is able enhance the original DE algorithm with single strategy.

Although the APS method is used for the strategy adaptation in this work, it is a general method for parameter adaptation. This method could also be used for other parameter adaptation in evolutionary algorithms, such as the adaptation of CR and F in DE, the adaptation of μ_{CR} and μ_F in JADE [7], etc. In our future work, we will verify this expectation.

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5. REFERENCES

- W. Gong, Z. Cai, C. X. Ling, and H. Li. Enhanced differential evolution with adaptive strategies for numerical optimization. *IEEE Transactions on Systems, Man, and Cybernetics: Part B* - *Cybernetics*, 41(2):397–413, 2011.
- [2] W. Gong, A. Fialho, and Z. Cai. Adaptive strategy selection in differential evolution. In J. Branke, editor, *Genetic and Evolutionary Computation Conference (GECCO 2010)*, pages 409–416. ACM Press, July 2010.
- [3] A. K. Qin, V. L. Huang, and P. N. Suganthan. Differential evolution algorithm with strategy adaptation for global numerical optimization. *IEEE Trans. on Evol. Comput.*, 13(2):398–417, Apr 2009.
- [4] R. Storn and K. Price. Differential evolution–A simple and efficient heuristic for global optimization over continuous spaces. J. of Global Optim., 11(4):341–359, Dec 1997.
- [5] D. Thierens. An adaptive pursuit strategy for allocating operator probabilities. In *Proc. Genetic Evol. Comput. Conf.*, pages 1539–1546, 2005.
- [6] X. Yao, Y. Liu, and G. Lin. Evolutionary programming made faster. *IEEE Trans. on Evol. Comput.*, 3(2):82–102, Jul 1999.
- [7] J. Zhang and A. C. Sanderson. JADE: Adaptive differential evolution with optional external archive. *IEEE Trans. on Evol. Comput.*, 13(5):945–958, Oct 2009.

¹All experimental results can be obtained in the full version of this paper online at: http://cs.cug.edu.cn/teacherweb/gwy/pubs.htm