A Memetic Algorithm for Solving Multiperiod Vehicle Routing Problem with Profit *

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ABSTRACT

Most literature on variations of vehicle routing problem assumes that a vehicle is continuously available within the planning horizon. However, in practice, due to the working time regulation, this assumption may not be valid in some applications. In this paper, we study a multiperiod vehicle routing problem with profit (mVRPP), where the goal is to determine a set of routes within the planning horizon that maximizes the collected reward from nodes visited. The vehicles can only travel during working hours within each period in the planning horizon. An effective memetic algorithm with giant-tour representation is proposed to solve the mVRPP. To efficiently evaluate a chromosome, we develop a greedy split procedure to optimally partition a given giant-tour into individual routes. We conduct extensive experiments on a set of modified benchmark instances. The result demonstrates that our approach generates promising solutions which are close to the upper bounds.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Heuristic methods, Scheduling*

General Terms

Algorithm

Keywords

multiperiod, vehicle routing problem, team orienteering problem, memetic algorithm, giant-tour

1. PROBLEM DESCRIPTION

Most studies on routing and scheduling problems usually make an assumption that vehicles or field technicians are continuously available within the planning horizon. However, in practice, due to the working hour regulation, this assumption may not be valid in some applications. In addition, the consideration of working hours in routing and scheduling is important to increase safety in road freight transport and punctuality of service delivery.

In this paper, we investigate a multiperiod vehicle routing problem with profit (mVRPP), which takes regular working time restriction into consideration. The problem is defined on an undirected graph $G = (V, E)$, where $V = \{0, \ldots, n\}$ is the set of nodes, $E = \{(i, j)|i, j \in V\}$ is the set of edges. Each node i associates a reward w_i. The depot node is 0 and $w_0 = 0$. Each edge $(i, j) \in E$ has a nonnegative cost c_{ij}, where c_{ij} is the travel time between i and j. The travel time matrix satisfies the triangle inequality. There are K vehicles which stay at the depot initially. The mVRPP aims to find K routes that each starts and finishes at node 0 in a planning horizon consisting of D periods such that the total collected reward is maximized. Each node can be visited at most once. Note that there is a working time restriction for each vehicle, where the accumulated travel time in each period taken by a vehicle cannot exceed a limit L.

We denote S as a feasible solution, consisting of K routes, i.e. $S = (r_1, r_2, \ldots, r_K)$. Each route starts and ends at the depot. A route $r_k = (r_k^1, r_k^2, \ldots, r_k^D)$ is divided by D trips, where r_k^d is a sequence of nodes representing a trip in period d. Denote $v_s(r_k^d)$ and $v_e(r_k^d)$ to be the starting node and the ending node of the trip r_k^d. After finishing the last visit in period d, the vehicle will stay at the node $v_e(r_k^d)$. Then the vehicle will start a new trip from $v_e(r_k^d)$ in next period, i.e. $v_e(r_k^d) = v_s(r_k^{d+1})$, $d \leq D - 1$. Note that $v_s(r_k^1) = v_e(r_k^D) = 0$. Let $T(r_k^d)$ be the total travel time of route k in period d. A trip r_k^d is feasible if $T(r_k^d) \leq L$. A route r_k is feasible if all trips in route r_k are feasible. A solution S is feasible if all routes are feasible and all nodes are visited at most once. A solution S is optimal if the total reward collected by S is maximum among all the feasible solutions.

We apply a Memetic Algorithm (MA[2]) to resolve the mVRPP. In our proposed MA, the set of routes is represented by a sequence of distinct nodes without delimiters, called a *giant-tour* [3]. An exact *split* procedure is employed to evaluate the chromosomes.

2. MEMETIC METHODOLOGY

In this section, we introduce a memetic algorithm to solve mVRPP, which involves chromosome representation, evaluation, initial population, crossover, mutation, local search

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Algorithm 1: Process of the Memetic Algorithm

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Construct the initial population P_I ;
Evaluate each chromosome in population P_I;
while the termination criteria is not reached do
    The new population P_N is set as empty;
    repeat
        Randomly select two parent chromosomes from
        P_I;
        Produce two offspring from the parent
        chromosomes by crossover operator;
        Put offspring chromosomes into P_N;
    until P_N is full;
    P ← P_I ∪ P_N;
    Perform Mutation on each chromosome in P with
    probability p_m except for the best chromosome;
    Perform Local Search on each chromosome in P;
    Set P_I empty;
    Select the elite chromosomes from P into P_I;
```

and population selection. Algorithm 1 presents the general structure of the proposed MA.

2.1 Chromosome Encoding

Suppose that $S = (r_1, r_2, \ldots, r_K)$ is a feasible solution of mVRPP. A chromosome, called 'giant-tour', is defined as a permutation π by the following procedure.

1. Set π as an empty sequence.
2. Sequentially append the nodes in each route r_k, $k = 1, 2, \ldots, K$ to π. For each trip r_k^d, $d = 1, 2, \ldots, D-1$, append all nodes except $v_s(r_k^d)$ into π. For the trip r_k^D, append all the nodes except the first node $v_s(r_k^D)$ and last node $v_e(r_k^D)$ into π.
3. Append all non-visited nodes to π in arbitrary order.

2.2 Chromosome Decoding and Evaluation

Given a giant-tour π, we can convert it into a solution to the mVRPP by partitioning it into $K+1$ sub-sequences. The first K sub-sequences correspond to the K vehicle routes, while the last sub-sequence contains the set of unvisited nodes. A greedy *split* procedure is introduced to partition the permutation π into K feasible routes. For the sake of brevity, we cannot present the detail descriptions of our split procedure in this paper. Nevertheless, we can show that the split procedure runs in $O(n)$ time. Hence, it enables us to devise an efficient algorithm to convert a given chromosome into a feasible solution. Theorem 1 guarantees the optimality of the solution generated by the greedy split procedure.

THEOREM 1. *For the give giant-tour π, the partition S found by the split procedure is optimal in terms of collected reward.*

2.3 Initial Population

Five members of the initial population in our approach are constructed using best-first heuristic approaches. The remaining members are random permutations in $\{1, \ldots, n\}$.

2.4 Crossover

In each generation, we use an order-based crossover (OX) operator to generate all offspring chromosomes.

2.5 Mutation

Each chromosome in the candidate pool P will be chosen for mutation by probability p_m. We randomly select two nodes in the chosen chromosome and exchange the positions of the two nodes.

2.6 Local Improvement Process

We apply a simulated annealing (SA) approach to improve a single chromosome in the candidate pool. The improvement process begins from a temperature $T = t_0$ and continues to cool down until T reaches the target level. In each iteration, we first apply four classic heuristic operators[1] *exchange*, *2-opt*, *relocate* and *segment-move* on the chromosome in this order *num_ops* times. The new solution S_N by generated operators is always accepted if it is better than the incumbent solution S_I. Otherwise S_N is accepted according to probability function $e^{\Delta/T}$, where $\Delta = f(S_N) - f(S_I)$.

In addition, We use a *TSP* operator attempting to improve the resultant solution. Given a partial trip within a period, the lowest travel cost of the partial trip is a travelling salesman path problem, the dynamic programming algorithm proposed by [4] is able to find the optimal sequence when the number of nodes visited is relatively small. We always accept the chromosome generated by TSP operator.

2.7 Selection

To choose elite chromosomes at next generation, we select p best chromosomes from the pool of incumbent population P_I and new populations P_N.

3. RESULT

We first conduct the instances of the team orienteering problem(TOP), which is a special case of mVRPP and only involves single period planning. The algorithm can generate high quality solutions in which the gaps are 0.08% compared to the best solutions while the average computing time is around 20 seconds for each test instance. Besides, we construct 24 mVRPP instances based on the vehicle routing problem with distance restriction(DVRP). For these instances, the node size varies from 240 to 400, and the planning horizon D is equal to 1,2 or 4. The experiments also show that the memetic algorithm has a great potential in solving large scale problem.

4. REFERENCES

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