# Integration of NSGA-II and MOEA/D in Multimethod Search Approach

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## ABSTRACT

Integration of single methods into their hybrids are researched scarcely in the recent few years. This paper presents the feasibility study for integration of two methods: MOEA/D [7] and NSGA-II [4] in the proposed multimethod search approach (MMTD). During implementation of MMTD, we borrows some concepts from the specialized literature of EMO. In MMTD, the synergetic combination of MOEA/D and NSGA-II can unleash their full power and strength self-adaptively for tackling two set of problems:1) ZDT test problems [6], 2) cec09 unconstrained test instances [1]. The final best approximated results illustrates the usefulness of MMTD in multiobjective optimization (MO).

## Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and SearchHeuristics

## General Terms

Algorithms

## Keywords

Multiobjective optimization, MOEA/D and NSGA-II, MMTD

## 1. INTRODUCTION

In this paper, we consider the following general multiobjective optimization problem (MOP):

$$\text{minimize } F(x) = (f_1(x), f_2(x) \ldots, f_m(x)) \quad (1)$$

where $x = (x_1, \ldots, x_n)^T \in \Omega \subseteq R^n$ is an $n$-dimensional vector of the decision variables, $\Omega$ is the *decision (variable) space*, and $F$ is the objective vector function that contains $m$ real valued functions.

In the last few years, many efficient evolutionary multiobjective optimization (EMO) approaches are developed for solving (1), since the first seminal's work of David Schaffer, so-called VEGA. Most of EMO techniques are Pareto dominance based. Among them, NSGA-II [4] is a well known approach. While, MOEA/D [7] is a recent novel developed paradigm in evolutionary computing (EC) which bridges a traditional mathematical programming and evolutionary algorithms and converting the problem of approximation of the PF into $N$ number of scalar optimization problems.

In this paper, we study the effect of integration of two methods: MOEA/D [7] and NSGA-II [4] in our multimethod search approach (MMTD) in order to use their strengths and biases in solving the cec09 problems [6] and ZDT test problems [1]. Our suggested approach is technically lies in one of increasingly thriving paradigm a hybrid MOEAs for MO which is developed mainly inspired from some recent publications [5, 3, 2] that have shown some promising performance in multiobjective optimization.

### 1.1 Parameter Setting and Discussion

Section 2 describes the algorithmic steps of proposed approach, so-called, MMTD. MMTD utilized a different biases and strengths of two methods: MOEA/D and NSGA-II, for population evolution selfadaptively and cooperatively at each of its generation $t$. In MMTD, a subpopulation are allocated to each method A and B based on the reproduction of non-dominated solutions of each methods that contributed to next generation of MMTD [1]. For fair comparison and evaluation, we have used DE as a crossover operator and polynomial as mutation operator in methods: 1) MMTD, 2) MOEA/D [7], 3) NSGA-II [4], during their 30 independent runs in solving two types of problems: ZDT test problems [1] and cec09 test problems [6]. we settled the parameters as follow: $CR = F = 0.5$, $p_m = 1/n$, $N = 100$, $F_{EVAL} = 250,00$, $T = 0.1 \times N$ in MOEA/D, $min_{pop} = 30$ in tackling the ZDT test problem. While solving cec'09 test problems, we used the parameter setting as: $CR = 1.0$, $F = 0.5$, $p_m = 1/n$, $N = 600$ for two objectives problem, $F_{EVAL} = 300,000$, $min_{pop} = 50$ and $T = 0.1 \times N$ in MOEA/D where $|P| = N$, $P_m$ denotes mutation rate, $F_{EVAL}$ denotes total function evaluations or stopping criteria, $min_{pop}$ denotes the minimum criteria for the size subpopulation $\gamma_1$ or $\gamma_2$ which we adopted in MMTD to avoid the possibility of inactivating of methods A and B. Initially, we fixed the values of $\alpha_1 = 0.5$, $\alpha_2 = 0.5$ and then update based on the formula in equation 2. To establish a fair comparison of MMTD with 1) pure MOEA/D, 2) pure NSGA-II, we carried out the experiments with same parameter settings and collected statistical results which can be seen in the $Table1$ and the final best approximation of PF w.r.t MMTD in $1st$ column panel, MOEA/D in $2rd$ column panel, NSGA-II in $3rd$ column panel, that illustrated and indicates the usefulness and effectiveness of MMTD on most problems. However, due to the space re-

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9[1] *A can be MOEA/D [7] and B can be NSGA-II [4], vice versa.*

striction, we cannot include all the obtained results and the final best approximation of PF for all problems which used in the performance validation of MMTD in this paper, we have included only the results of three problems for the reader satisfaction.

## 2. INTEGRATION OF NSGA-II AND MOEA/D IN MULTIMETHOD SEARCH APPROACH (MMTD)

**Input:** MOP (1), $F_{EVAL}$, $N$, $T$, $F$, $CR$, $p_m$;
**Output:** $\{x^1, \ldots, x^N\}$ and $\{F(x^1), \ldots, F(x^N)\}$;

**Step 1 Initialization:**

**Step 1.1:** Randomly sample $N$ from the search region to form the initial population, $P = \{x^{(1)}, x^{(2)}, \ldots, x^{(N)}$;
**Step 1.2:** Compute the F-function values of each member of $P$, $F(x^i)$, where $i = 1, 2, \ldots, N\}$;

**Step 2 Execution of methods A and B:**

**Step 2.1:** Select randomly a subpopulation $P^1$ of size $\gamma_1 = \lfloor \alpha_1 \times N \rfloor$, to execute a methods A, to get a new subpopulation $Q^1$ after generation $t$;
**Step 2.2:** Execute a method B on the remaining subpopulation $P^2$ of size $\gamma_2 = N - \gamma_1$ for generation $t$, to get a new subpopulation $Q^2$;

**Step 3 Combination of $Q^1$ and $Q^2$:**

**Step 3.1:** Combine the subpopulation of method A and B $Q = Q^1 \cup Q^2$ at generation $t$;

**Step 4 Update:**

**Step 4.1:** Combine the old population $P$ and the new population $Q$ to form an intermediate population, $C = P \cup Q$, of size $2N$;
**Step 4.2:** Using non-dominating sorting technique, select $N$ best solutions from combined population, $C = P \cup Q$ to form a new population, $\tilde{P}$;
**Step 4.3:** Replace old population $P$ with a new population $\tilde{P}$;
**Step 4.4:** Update $\alpha_1$ and $\alpha_2$ at each generation $t$;

**Step 5 Stoping Criteria:**

If stoping criteria is meet, then **Stop** and give output. **Otherwise**, go to Step2 until stoping criteria is not meet.

### 2.1 Update $\alpha_1$ and $\alpha_2$:

Compute the success ratio of non-dominated solutions, $\frac{\beta_1}{\xi_1}$ and $\frac{\beta_2}{\xi_2}$ which contributes each methods A and B during the process of evolution to next generation of MMTD as follow:

$$\alpha_1 = \frac{\frac{\beta_1}{\xi_1}}{\frac{\beta_1}{\xi_1} + \frac{\beta_2}{\xi_2}}; \qquad \alpha_2 = \frac{\frac{\beta_2}{\xi_2}}{\frac{\beta_1}{\xi_1} + \frac{\beta_2}{\xi_2}}; \tag{2}$$

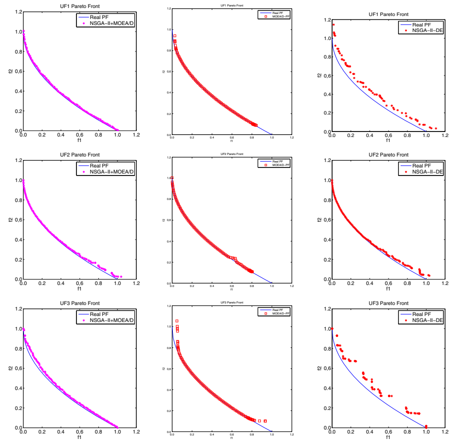

**Figure 1: The best approximation to PF.**

## 3. CONCLUSION

At last, but not least, we conclude this paper with believe that MMTD which merges a strengths and power of MOEA/D and NSGA-II during the population evolution in self-adaptive ways have shown good contribution to MO in solving CEC'09 problems [6] with different characteristics. In future, MMTD would be offer better performance after fine tuning of all its parameter in terms of solving various type optimization and search problems.

**Table 1: The IGD statistics based on 30 independent runs**

| MOEA/D, NSGA-II, MMTD | | | | | | |
|---|---|---|---|---|---|---|
| Prob | min | median | mean | std | max | MOEAs |
| UF1 | 0.008769 | 0.020400 | 0.032928 | 0.032018 | 0.142864 | MOEA/D |
| | 0.051996 | 0.106873 | 0.096076 | 0.024862 | 0.128739 | NSGA-II |
| | **0.007910** | **0.012858** | **0.014228** | **0.004252** | **0.029844** | **MMTD** |
| UF2 | 0.012212 | 0.028806 | 0.040694 | 0.031630 | 0.131926 | MOEA/D |
| | 0.016012 | 0.019849 | 0.020050 | 0.001407 | 0.023589 | NSGA-II |
| | **0.011266** | **0.016915** | **0.017062** | **0.001410** | **0.021270** | **MMTD** |
| UF3 | 0.043906 | 0.251522 | 0.194361 | 0.105401 | 0.327625 | MOEA/D |
| | 0.066353 | 0.098234 | 0.097065 | 0.017958 | 0.134235 | NSGA-II |
| | **0.016901** | **0.061575** | **0.068601** | **0.033416** | **0.139084** | **MMTD** |

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