Probabilistically Interpolated Rational Hypercube Landscape Evolutionary Algorithm

David Andrew Cape
7201 Ridgewood Avenue, Unit 12
Cape Canaveral, Florida 32920
dcape15@cfl.rr.com

Daniel R. Tauritz
Missouri University of Science and Technology
Natural Computation Laboratory
Department of Computer Science
Rolla, Missouri 65409
dtauritz@acm.org

ABSTRACT

Evolutionary Algorithms are powerful function optimizers, but suffer from premature convergence. Quantum-Inspired Evolutionary Algorithm (QEA) has been shown to be less prone to this on an important class of binary encoded problems. QEA uses Q-bits in place of ordinary bits, introducing a rational parameter into an otherwise binary search space. The essential feature of QEA is that the fitness of individuals in the population is defined stochastically by sampling from discrete points in the landscape. The probability of a particular point being sampled is based on the proximity of an individual to that point, where the individual represents a point in the solid hypercube spanned by the possible discrete solutions. This paper presents Probabilistically Interpolated Rational Hypercube Landscape Evolutionary Algorithm (PIRHLEA), which generalizes QEA by relaxing its two vestigial quantum mechanical attributes: quadratic and angular parameterization of probabilities and using single samples to determine fitness estimates of individuals. This is accomplished by replacing each Q-bit with a rational parameter between zero and one. Compared to QEA, PIRHLEA is simpler to code, more computationally efficient, and easier to visualize. PIRHLEA also permits multiple samples from points in the landscape to determine individuals' fitness.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search; F.2 [**Theory of Computation**]: Analysis of Algorithms and Problem Complexity

General Terms

Algorithms, Theory

Keywords

Quantum-Inspired Evolutionary Algorithm, QEA, landscape sampling, hypercube landscape, quantum computing

1. INTRODUCTION

Evolutionary Algorithms (EAs) are powerful function optimizers, but suffer from premature convergence on multimodal problems due to loss of genetic diversity caused by excessive selective pressure. While this may be in part due to inadequate parameter tuning, on problems with binary representations improved performance may be attained by transforming the search space by sampling from an interpolated version of the landscape.

Quantum-Inspired Evolutionary Algorithm (QEA) is less prone to premature convergence on the binary knapsack problem compared to various traditional EAs [1]. This is perhaps due to it introducing a rational parameter, in the form of a Q-bit, into an otherwise binary search space, to enable probabilistic sampling of discrete points in a hypercube during fitness evaluation. A geometric interpretation is possible, if Q-bit vector individuals are identified with points in the solid hypercube spanned by the possible discrete solutions. The probability of a particular point being sampled is based on the proximity of an individual to that point. The quantum mechanical inspiration for QEA has caused the formulation of the algorithm to be unnecessarily complicated. In fact, it is possible to replace each Q-bit by a rational parameter between zero and one to encode probabilities.

This paper presents Probabilistically Interpolated Rational Hypercube Landscape Evolutionary Algorithm (PIRHLEA), which generalizes QEA by relaxing its two vestigial quantum mechanical attributes: quadratic and angular parameterization of probabilities and using single samples to determine fitness estimates of individuals. This is accomplished by replacing each Q-bit with a rational parameter between zero and one. Compared to QEA, PIRHLEA is simpler to code, more computationally efficient, and easier to visualize. This is due to it using a linear parameter for the probability rather than a quadratic or angular one. PIRHLEA also generalizes QEA by permitting multiple samples from points in the landscape to determine the fitness of an individual, potentially resulting in better on-the-fly fitnesss estimation.

2. BACKGROUND

The main topic of this paper, common to all of the algorithms discussed here, is the use of interpolation to transform a potentially rugged binary encoded discrete search space into a smoother one. QEA does so by letting individuals, parameterized by probability, sample the points of the landscape in order to allow an individual's fitness to depend on its distribution. In this way, an individual does not "live" at a single vertex of the original hypercube domain, but instead has a general location, within the solid hypercube that the landscape points span, that depends on its distribution.

GECCO'11, July 12–16, 2011, Dublin, Ireland
ACM 978-1-4503-0690-4/11/07.

It is hypothesized that the superposition of points of the landscape inside the distribution of an individual is the key to QEA's success; yet, it is also fundamentally important that individuals' distributions may vary smoothly within the transformed search space of a solid hypercube. For when the search space was parameterized more coarsely, the observed performance was unimpressive in some cases. In several applications [1, 2], however, QEA demonstrated its superiority over traditional EAs with respect to finding better solutions in less time. As noted, with QEA and PIRHLEA, an individual may be located in the solid hypercube instead of only at a vertex. In Figure 1, the individual shown has a Q-bit representation with $\alpha_1 = 1/2, \beta_1 = \sqrt{3}/2, \alpha_2 = \sqrt{2}/2, \beta_2 = \sqrt{2}/2$. Its angular representation has $\theta_1 = \pi/3$ and $\theta_2 = \pi/4$, but for PIRHLEA the linear representation is simply $(3/4, 1/2)$.

A precursor of PIRHLEA, Explicit Schema Sampled EA (ESS-EA), used schemata *explicitly* as individuals in a way similar to how QEA uses Q-bit vectors. Considering that the fitness of a schema is defined to be the average of the fitnesses of its constituents, and, geometrically, schemata correspond to midpoints of those constituents inside the hypercube, there is a natural interpretation of the collection of schemata as the product of a finite number (the length of the bit-string in the domain) of copies of the set {0,0.5,1}. It is not necessary to evaluate all members of each individual schema to *estimate* the schematic fitness, so ESS-EA uses the method of sampling the members of schemata a fixed finite number of times. One effect of the addition of midpoints and estimating their schematic fitness is a slight smoothing of the landscape because there will be interpolated midpoints through which an individual may pass on its way from a local maximum to the global maximum. Another effect of interpolation is that the search space becomes exponentially larger, and this is more pronounced for QEA than for the coarser interpolation given by ESS-EA.

3. PIRHLEA

Given a bit-string function to be optimized, one replaces the domain by strings of values from [0,1]. This transformation can be interpreted as enlarging the domain of hypercube vertices by replacing them with a solid hypercube, and one must define the fitness of individuals that have coordinates other than zero or one.

For each point in the solid hypercube domain, a sample vertex is randomly chosen by the following method. For each dimension, choose one with probability equal to the rational value originally in that position, and choose zero otherwise. Thus, the closer the point is to a face of the hypercube, the higher the probability of a vertex on that face being chosen as in Figure 1. The number of sample vertices chosen can be a fixed parameter of the algorithm, and the average over that number of samples is used for the fitness of individuals in the solid hypercube domain. Reproduction in a PIRHLEA can be done as one would for any EA with a solid hypercube domain: mutation and recombination by crossover may be used, for example.

To see that no new local extrema arise in the enlarged search space, observe that the expected value of the fitness for an individual inside the solid hypercube is just a linear weighted average of the expected value of the fitness for individuals on opposite faces of the hypercube that are projections of the individual in the interior.

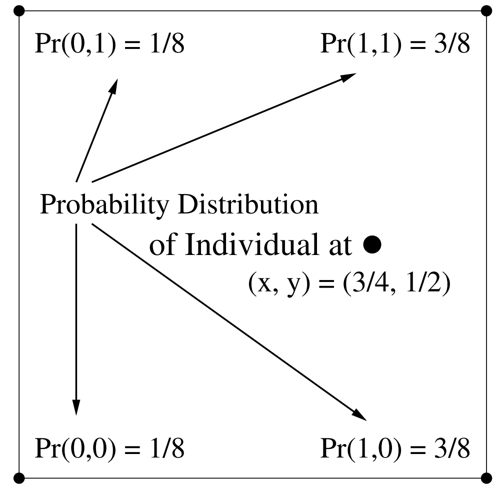

Figure 1: With QEA and PIRHLEA, an individual may be located in the solid hypercube.

4. CONCLUSIONS

This paper introduces PIRHLEA, a novel generalization and simplification of the QEA. The impetus for this research is to improve EA performance on rugged binary fitness landscapes by employing interpolation to smooth the landscape. The first attempt resulted in the ESS-EA, which employs schemata to interpolate the fitness on a hypercube landscape, and this interpolation should make migration away from local extrema at the hypercube vertices easier for individuals in the population.

The strategy for ESS-EA finally merged with that of QEA, producing PIRHLEA, which uses arbitrarily dense interpolation. However, the synthesis of these two approaches may improve upon both earlier algorithms. Empirical studies should be carried out to substantiate this potential. At worst, PIRHLEA does simplify and generalize QEA by relaxing the connection to quantum mechanics while preserving enough of the framework to capture its essential features. As a result, one can expect easier and more efficient implementations as well as better visualization of the algorithm performance on the enlarged domain of a solid hypercube; the potential benefit of PIRHLEA's more general sampling method also needs to be empirically investigated.

5. ACKNOWLEDGMENTS

The first author would like to thank R. Paul Wiegand and Annie S. Wu of the University of Central Florida for valuable suggestions and feedback and for inviting participation in research group meetings.

6. REFERENCES

[1] K. Han and J. Kim. Quantum-Inspired Evolutionary Algorithm for a Class of Combinatorial Optimization. *IEEE Transactions on Evolutionary Computation*, 6(6):580–593, December 2002.
[2] S. Yang, M. Wang, and L. Jiao. A Novel Quantum Evolutionary Algorithm and Its Application. In *Congress on Evolutionary Computation*, volume 1, pages 820–826, 2004.