Genetic Programming

A Tutorial Introduction

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Agenda

CONTEXT: Evolutionary computation and algorithms
- brief history
- Evolutionary Algorithms: programming the biological evolutionary process
- Genetic programming in relation to other evolutionary algorithms

1. Genetic evolution of executable expressions
2. Example: Block Stacking problem
3. Nuts and Bolts Descriptions of Algorithm Components
4. Finish with more example problems
5. How does it work?
6. Issues and Challenges

Neo-Darwinian Evolution

- Survival and thriving in the environment
- Offspring quantity - based on survival of the fittest
- Offspring variation: genetic crossover and mutation
- Population-based adaptation over generations

Problem Types and Applications

- Generating complex solutions - evolution is a process that gives rise to complexity
  - a continually evolving, adapting process, potentially with changing environment from which emerges modularity, hierarchy, complex behavior and complex system relationships
- GA: discrete variables - Combinatorial optimization
  - NP-complete and/or poorly scaling solutions via LP or convex optimization
  - unyielding to approximations (SQP, GEO-P)
  - eg. TSP, graph coloring, bin-packing, flows
  - for: logistics, planning, scheduling, networks, bio gene knockouts

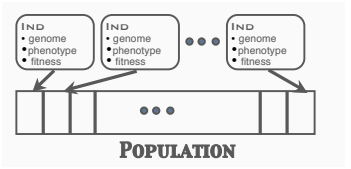
Problem Types and Applications

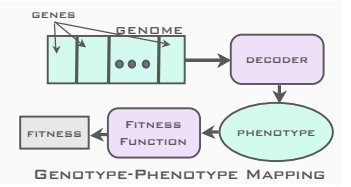
- ES: continuous variables: Continuous Optimization
  - non-differentiable, discontinuous, multi-modal, large scale objective functions
  - for: engineering, mechanical, material, physics
- Genetic Programming
  - system identification: chemical processes, financial strategies
  - design: creative blueprints, generative designs - antennae, Genr8, chairs, lens
  - automatic programming: compiler heuristics
  - AI ODEs, invariants, knowledge discovery

Key EA Components

POPULATION
- array of struct ind with fields genome, phenotype fitness
- random initialization

- GENOME is an array of gene(s)
- GENOME is input parameter to decoder procedure that returns PHENOME
- PHENOME is input parameter to fitness-evaluation routine that returns a numeric variable called FITNESS





EA Examples

Problem	Gene	Genome	Phenotype	Fitness Function
TSP	110	sequence of cities	tour	tour length
Function optimization	3.21	variables x_of function	f(x)	|min-f(x)|
graph k-coloring	permutation element	sequence for greedy coloring	coloring	# of uncolored nodes
investment strategy	rule	agent rule set	trading strategy	portfolio change

EA Generation Loop

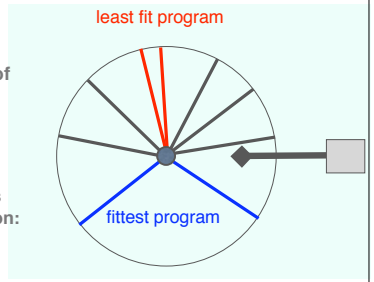
- generations
  - select
  - breed
  - replace

```
population = random_pop_init()
generation = 0
while needToStop == false
generation++
phenotypes =decoder(genotypes)
calculateFitness(phenotypes)
parents = select (phenotypes)
offspring = breed(genotypes)
population = replace(parents, offspring)
solution = bestOf(population)
```

EA Selection

Principles:

- everyone has non-zero probability of being an ancestor
- individual fitness relative to population mean fitness or rank of fitness is important
- Sometimes the best of a population is always bred directly into next generation: "elitism"

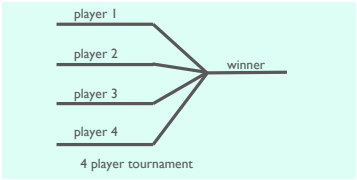


Some standard methods:

- Roulette wheel
- Tournament Selection
  - n tournments of size k

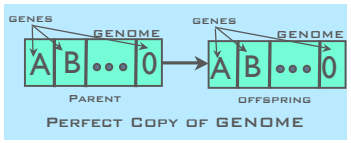
*We give the algorithm a "seed" for its RNG.

EA Selection

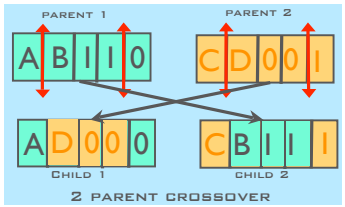


EA Breeding

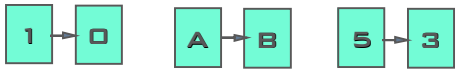
- Replication of parent [inheritance]



- crossover - [sexual recombination]



- mutation - [imperfect copy]



Choose crossover points and apply mutation randomly
Use a random number generator

EA Replacement

Deterministic

- use best of parents and offspring to replace parents
- replace parents with offspring

Stochastic

- some sort of tournament or fitness proportional choice
- hold a tournament with old pop and offspring
- run a tournament with parents and offspring

EA Pseudocode

```
population.genotypes = random_pop_init()                    birth
population.phenotypes =decoder(population.genotypes)        development
population.fitness= calculate_fitness(population.phenotypes) fitness for breeding

generation = 0                                               generations
while needToStop == false
    generation++
    parents.genotypes = select (population.fitness)          select
    offspring.genotypes = crossover_mutation(parents.genotypes) breed
    offspring.phenotypes =decoder(offspring.genotypes)       development
    offspring.fitness= calculate_fitness(offspring.phenotypes) fitness for breeding
    population = replace(parents.fitness, offspring.fitness) replace

solution = bestOf(population)
```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1. Genetic evolution of executable expressions
   - as first introduced in 1988 by John R. Koza
   - Executable expressions
   - Operators and operands
2. Example: Block Stacking problem
   - Definition
   - Operators and operands
   - What random solutions look like Block stacking
   - Test cases and fitness function
   - Evolved solutions

Agenda

3. Nuts and Bolts Descriptions of Algorithm Components
   - Initialization of population of random expressions
     - Selection of operators and operands
     - Closure and sufficiency
   - Fitness of an expression
   - Genetic crossover and mutation
   - Selection
   - Prepartory Steps of GP
   - Control parameters of a GP-tree "run"
4. Finish with more example problems
   - Symbolic regression, (simple, with constants)
5. How does it work?
6. Issues and Challenges
   - Problems and solutions for tree overhead
   - Alternate representations for expression genome
     - Linear and graph-based genomes

GP: Evolution of executable expressions

As introduced in 1988 by John R. Koza
- Operators and operands were derived from lisp built-in functions, problem oriented high-level "language"
- Expression genomes represented as trees,
  - » What are expressions and expression trees?
  - » What are GP operators and operands?

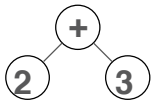
Executable Expressions

- Context of an interpreter or compiler
  - 3+2
  - (+ 2 3) ; same as above, different syntax
  - 3 + square(a)
  - myFunction(arg1 arg2) - could have side effect!
- Note that expressions have a universal way of being described via a tree
  - Tree traversal order creates syntax and control flow

Common terminology and my terms today

Components		What is executable	Genotype
Operators	Operands	expression	structure
Functions	Terminals	structure	structure

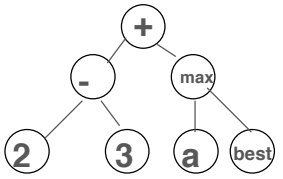
Expressions as Trees



Inorder: 2+3

preorder: + 2 3

Post-order: 2 3 +



Inorder: (2-3) + (max a best)

preorder: (+ (-2 3) (max a best))

Post-order: (2 3 -) (a best max) +)

- Whether parsed preorder (node, left-child, right-child) or postorder (left-child, right-child, node) or inorder (left, node, right) the expression evaluates to the same result
- (tree)GP uses an expression tree as its genotype structure

Operators and operands in GP

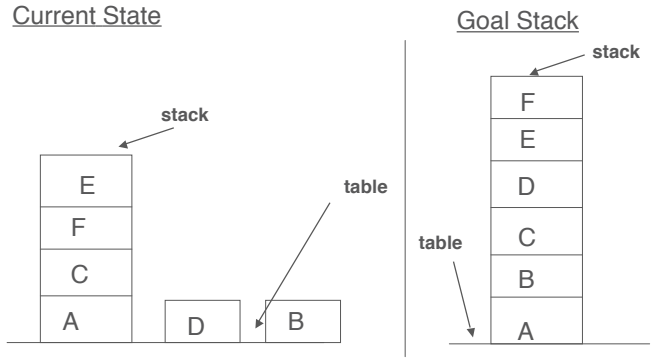
GP uses operators and operands as the genetic material of its expressions

Possible Operators

- Arithmetic: +, - , div, mult
- Transcendental: log, exp,
- Trigonometric: cos, sine,
- Variable assignment
  - (setq a 10)
  - (seta 10)
- Register read and write
- Index memory r/w
- Conditionals
  - If <pred> <then>
  - Ifpred <then>
- Iteration
  - Do-until action predicate
  - Over X
  - Reverse-Over X
- Specialized Subroutines, Procedures, or functions from the problem domain

The Block Stacking Problem

Koza-92



Goal: a plan to rearrange the current state of stack and table into the goal stack

Block Stacking Problem: Operators and Operands

- State (updated via side-effects)
  - *currentStack*
  - *currentTable*
- The operands
  - Each block by label
- Operators returning a block based on current stack
  - top-block
  - next-needed
  - top-correct
- Block Move Operators return boolean
  - Return nil if they do nothing, t otherwise
  - Update *currentTable* and *currentStack*
  - to-stack(block)
  - to-table(block)
- Sequence Operator returns boolean
  - Do-until(action, test)
    - » Macro, iteration timeouts
    - » Returns t if test satisified, nil if timed out
- Boolean operators
  - NOT(a), EQ(a b)

Random Block Stacking Expressions

- eq(to-table(top-block) next-needed)
  - Moves top block to table and returns nil
- to-stack(top-block)
  - Does nothing
- eq(to-stack(next-needed)
  eq (to-stack(next-needed) to-stack(next-needed)
  - Moves next-needed block from table to stack 3 times
- do-until(to-stack(next-needed)
  (not(next-needed))
  - completes existing stack correctly (but existing stack could be wrong)

Fitness Cases

- different initial stack and table configurations (Koza - 166)
  - stack is correct but not complete
  - top of stack is incorrect and stack is incomplete
  - Stack is correct and partial then has blocks on top that are wrong
- Each correct stack at end of expression evaluation scores 1 "hit"
- fitness is number of hits (out of 166)

Evolved Solutions to Block Stacking

eq(do-until(to-table(top-block) (not top-block))
do-until(to-stack(next-needed) (not next-needed)

- first do-until removes all blocks from stack until it is empty and top-block returns nil
- second do-until puts blocks on stacks correctly until stack is correct and next-needed returns nil
- eq is irrelevant boolean test but acts as connective
- wasteful in movements whenever stack is correct

- Add a fitness factor for number of block movements

do-until(eq (do-until (to-table(top-block)
(eq top-block top-correct))
(do-until (to-stack(next-needed) (not next-needed))
(not next-needed)

- Moves top block of stack to table until stack is correct
- Moves next needed block from table to stack
- Eq is again a connective, outer do-until is harmless, no-op

Agenda Checkpoint

- Introduced to evolutionary algorithm
- GP is an EA that evolves executable expressions composed of operators and operands
  - Expressions and their parse trees
- The block stacking problem
  - Definition
  - Operators and operands
  - Fitness of a block stacking expression
  - GP-evolved solutions to block stacking problem
- Next,
  - How can we create random GP expressions?
  - How can we create a diverse population of expressions?
  - What is general procedure for fitness function design?
  - How do we mutate and crossover expressions?
  - Selection?

Population Initialization

- Fill population with random expressions
  - Create a operator set and a corresponding argument-count set
  - Create an operand set (arg-count = 0)
  - draw from operator set with replacement and recursively enumerate operator's argument list by additional draws from operators U operands.
  - Recursion ends at draw of an operand
  - requires closure and/or typing
- maximum tree height parameter
  - At max-height-1, draw from operands only
- "ramped half-half" method ensures diversity
  - equal quantities of trees of each height
  - half of height's trees are full
    - » For full tree, only draw from operands at max-height-1

Things to Ensure to Evolve Programs

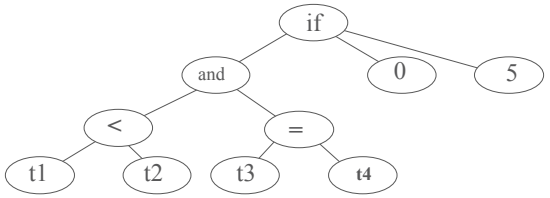
- Sufficiency: the operators and operands that can form executable expressions must be adequate to formulate a solution to the problem
  - I have my students hand code some solution (though not necessarily correct)
  - Operands are usually problem's decision variables
  - Operators must be wisely chosen but not too complex
    - » primitives like arithmetic, boolean, condition, iteration, assignment
    - » Problem specific (eg next-needed)
- Closure: all functions must be coded so that they can accept parameters of any type
  - In block stacking, we can handle boolean or block
- Programs of varying length and structure must compose the search space
- Crossover of the genotype must preserve syntactic correctness so the program can be directly executed

Determining a Expression's Fitness

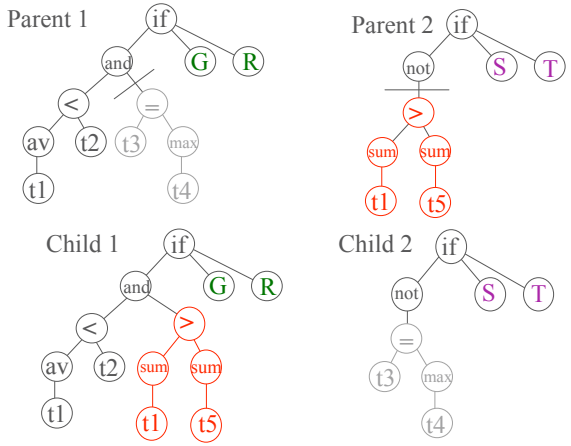
- One test case:
  - Execute the expression with the problem decision variables (ie operands) bound to some test value and with side effect values initialized
  - Designate the "result" of the expression
- Measure the error between the correct output values for the inputs and the final outputs of the expression
  - Final output may be side effect variables, or return value of expression
  - Eg. Examine currentStack vs goalstack for block stacking
  - Eg. the heuristic in a compilation, run the binary with different inputs and measure how fast they ran.
  - EG, Configure a circuit from the genome, test the circuit with an input signal and measure response vs desired response
- Usually have more than one test case but cannot enumerate them all
  - Use rational design to create incrementally more difficult test cases (eg block stacking)
  - Use balanced data for regression

Genotype Representation: Tree

- Required: Crossover of the genotype must preserve syntactic correctness so the program can be directly executed
- Given: expression are created from provided "language" of operators and operands (aka primitives, functions & terminals)
- Solution: Genetically manipulate program in an expression tree representation
- convenient for LISP or Scheme where this is expression tree
- (if (and (< t1 t2) (= t3 t4) 0 5)



Tree Crossover



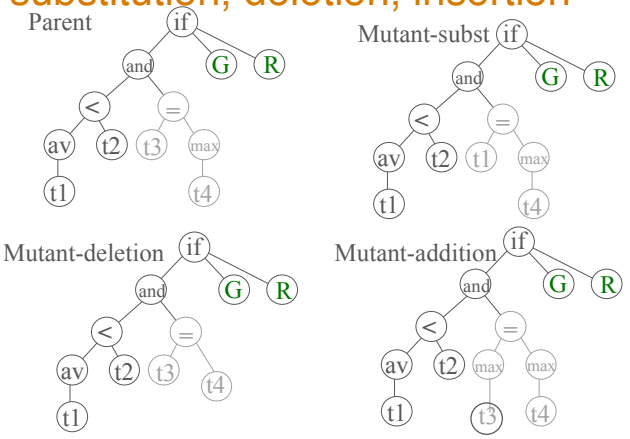
Tree Crossover Details

- Crossover point in each parent is picked at random
- Conventional practices
  - All nodes with equal probability
  - leaf nodes chosen with 0.1 probility and non-leaf with 0.9 probability
- Probability of crossover
  - Typically 0.9
- Maximum depth of child is a run parameter
  - Typically ~ 15
  - Can be size instead
- Two identical parents rarely produce offspring that are identical to them
- Tree-crossover produces great variations in offspring with respect to parents
- Crossover, in addition to preserving syntax, allows expressions to vary in length and structure (sub-expression nesting)
- "raw power"
-

Tree Mutation

- Often only crossover is used
- But crossover behaves often like macro-mutation
- Mutation can be better tuned to control the size of the change
- A few different versions

HVL-Mutation: substitution, deletion, insertion



Other sorts of mutation

- **Koza:**
  - Randomly remove a sub-tree and replace it
  - Permute: mix up order of args to operator
  - Edit: + 1 3 -> 4, and(t t) -> t
  - Encapsulate: name a sub-tree, make it one node and allow re-use by others (protection from crossover)
    - » Developed into advanced GP concept known as
      - Automatic module definition
      - Automatically defined functions (ADFs)
- **Make your own**
  - Could even be problem dependent (what does a subtree do? Change according to its behavior)

Selection in Genetic Programming

- **Proceeds in same manner as evolutionary algorithm**
  - Same set of methods
  - Conventionally use tournament selection
  - Also see fitness proportional selection
  - Cartesian genetic programming:
    - » One parent: generate 5 children by mutation
    - » Keep best of parents and children and repeat
      - If parent fitness = child fitness, keep child

Top Level GP Algorithm

Begin

pop = random programs from a set of operators and operands

repeat

execute each program in pop with each set of inputs

assign each program's fitness

repeat

select 2 parents

copy 2 offspring from parents

crossover

mutate

add to new-pop

until pop-size

pop = new-pop

until max-generation

or

adequate program found

End

Grow or Full | Ramped-half-half

Max-init-tree-height

·Tournament selection
·Fitness proportional selection
·Your favorite selection

Tournament size

Prepare input data
Designate solution
Define error between actual and expected

·HVL-mutate
·Subtree subst
·Permute
·Edit
·Your own

Mutation probs

Sub-tree crossover

Prob to crossover

Max-tree-height

GP Preparatory Steps

1. Decide upon operators and operands
2. Set up the fitness function
3. Decide upon run parameters
4. Determine settings for the parameters

GP Parameters

- Population size
- Number of generations
- Max-height of trees on random initialization
  - Typically 6
- Probability of crossover
  - Higher than mutation
  - 0.9
  - Rest of offspring are copied
- Probability of mutation
  - Probabilities of addition, deletion and insertion
- Population initialization method
  - Ramped-half-half
  - All full
  - All non-full
- Selection method
  - Elitism?
- Termination criteria
- Fitness function
- what is used as "solution" of expression

Simple Symbolic Regression

- Given a set of independent decision variables and corresponding values for a dependent variable
- Want: a model that predicts the dependent variable
  - Eg: linear model with numerical coefficients
    - » Y= aX1 + bX2 + c(X1X2)
  - Eg: non-linear model
    - » $y= a\,x1^2 + bx2^3$
  - Prediction accuracy: minimum error between model prediction and actual samples
- Usually: designer provides a model and a regression (ordinary least squares, Fourier series) determines coefficients
- With genetic programming, the model (structure) and the coefficients can be learned
- Example: y=f(x)
- Domain of x [-1.0,+1.0]
- Choose the operands
  - X
- Choose the operators
  - +, - , *, / (protected)
  - Maybe also sin, cos, exp, log (protected)
- Fitness function: sum of absolute error between yi, and expression's return values
- Prepare 20 points for test cases
- Test problem:
  - Y=x4 + x3 + x2 + x
  - GP can create coefficeints (x/x div x+x = 1/2) but…

Symbolic Regression with Numeric Coefficients:Ephemeral Random Constants

- New Test problem:
  - $Y=3x^4 + 10x^3 + 2x^2 + 3x$
- requires constant creation
- Ephemeral random constants provide GP with numerical solution components
- Provide ERC set
  $R = \{-10,-9,-8,..0...8,9,10\}$
- Include R among the operands. When individual is to be randomly created and R is drawn, one of the elements in R becomes the new operand.
- GP only has the constants that are randomly drawn in the initial population
- Constants could be lost through the selection process (no expression with a constant survives reproduction)
- But, GP has more primitive material to work with
- It works…partially
- Issue with size of constants, coordination of model and coefficient search, as a "clump" of numbers grows, it is more vulnerable to crossover disruption

More Examples of Genetic Programming

- Evolve priority functions that allow a compiler to heuristically choose between alternatives in hyper-block allocation
- Evolve a model that predicts, based on past market values, whether a stock's value will increase, decrease or stay the same
- Optimal control:
  - the frictionless cart, pole balancing
  - State of system comprises operands set
    - Operators are actions that produce state change
- Artifical Ant
- Boolean Multiplexor
- Image compression

How Does it Manage to Work

- Exploitation and exploration
  - Selection
  - Crossover
- Selection
  - In the valley of the blind, the one-eyed man is king
- Crossover: combining
- Koza's description
  - Identification of sub-trees as sub-solutions
  - Crossover unites sub-solutions
- For simpler problems it does work
- Current theory and empirical research have revealed more complicated dynamics

Why are we still here? Issues and Challenges

- Trees use up a lot of memory
- Trees take a long time to execute
  - Change the language for expressions
    - » C, Java
  - Pre-compile the expressions, PDGP (Poli)
  - Store one big tree and mark each pop member as part of it
    - » Compute subtrees for different inputs, store and reuse
- Bloat: Solutions are full of sub-expressions that may never execute or that execute and make no difference
- Operator and operand sets are so large, population is so big, takes too long to run
- Runs "converge" to a non-changing best fitness
  - No progress in solution improvement before a good enough solution is found

Runs "converge": Evolvability

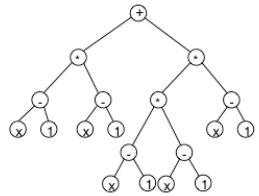
- Is an expression tree ideal for evolvability?
- Trees do not align, not mixing likes with likes as we would do in genetic algorithm
- Biologically this is called "non-homologous"
- One-point crossover
  - By Poli & Langdon
  - Theoretically a bit more tractable
  - Not commonly used
  - Still not same kind of genetic material being swapped

Evolvability: are there building blocks?

- Does a tree or expression have building blocks?
  - Context sensitivity of sub-expressions
  - What is the "gene" or unit of genetic transmission?
  - Building blocks may come and go depending on the context in which they are found
- Where does the Good Stuff Go and Why?
  - Goldberg and O'Reilly
- The semantics of the operators influences the shape of the expressed part of the tree
- A look at two extremes:
  - (iflte x a) -ORDER
    - » Context sensitive
  - (+ a b) - MAJORITY
    - » Aggregation
- Even with this simplification, predicting the dynamics is difficult
- Will an imperative expression language offer better building blocks?
- Will a linear genome provide less complicated genome dynamics?

Evolvability - modularity and reuse

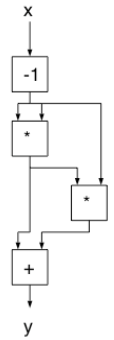
- Expression tree must be big to express reuse and modularity
- Is there a better way to design the genome to allow modularity to more easily evolve?



The representation of $(x-1)^2 + (x-1)^3$ in a tree–based genome

Evolvability: modularity and reuse

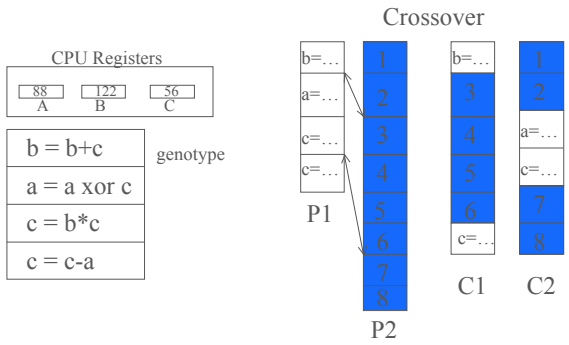
```
(1)   x=x-1
(2)   y=x*x
(3)   x=x*y
(4)   y=x+y
```



The dataflow graph of the $(x-1)^2 + (x-1)^3$ polynomial

Register Machine Genotype

- linear genotype, varying length, direct data

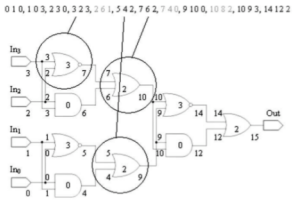


Register Machine Advantages

- Easier on memory and crossover handling
- Supports aligned "homologous" crossover
- Registers can act as poor-man's modules
- The primitive level of expressions allows for
  - Potentially more easily identifiable building blocks
  - Potentially less context dependent building blocks
- The register level instructions can be further represented as machine instructions (bits) and run native on the processor
  - AIM-GP (Auto Induction of Machine Code GP)
  - Intel or PPC or PIC, java byte code,
  - Experience with RISC or CISC architectures
  - Patent number: 5946673, DISCIPLUS system

Cartesian Genetic Programming

- Julian Miller
- operators and operands are nodes and data flow is described by genome
- Fixed length genome but variable length phenome
  - Integers in blocks
  - For each block, integers to name inputs and operator
- Unexpressed genetic material can be turned on later
- No bloat observed (plus nodes are upper bounded



Dealing with Bloat

Examples:

- (not (not x))
- (+ x 0)
- (* x 1)
- (Move left move-right)
- If (2=1) action

No difference to fitness (defn by Banzhaf, Nordin and Keller)

Can be local or global

- Why does it occur?
  - Crossover is destructive
  - Effective fitness is selected for
- Effective fitness
  - Not just my fitness but the fitness of my offspring
- Approaches
  - Avoid - change genome structure
  - Remove: Koza's edit operation
  - Penalize: parsimony pressure
  - Fitness =
    - » A(perf) + (1-a)(complexity

Reference Material

Where to Find It Online

- ACM digital library: http://portal.acm.org/
  - GECCO conferences,
  - GP conferences,
  - Evolutionary Computation Journal (MIT Press)
- IEEE digital library: http://www.computer.org/portal/web/csdl/home
  - Congress on Evolutionary Computation (CEC)
  - IEEE Transactions on Evolutionary Computation
- Springer digital library: http://www.springerlink.com/
  - European Conference on Genetic Programming: "EuroGP"
- Genetic Programming Bibiliography
  - http://www.cs.bham.ac.uk/~wbl/biblio/

Reference Material - Books

- Advances in Genetic Programming
  - 3 years, each in different volume, edited
- Genetic Programming: From Theory to Practice
  - 10 years, annual, on SpringerLink, edited
- John R. Koza
  - Genetic Programming: On the Programming of Computers by Means of Natural Selection, 1992 (MIT Press)
  - Genetic Programming II: Automatic Discovery of Reusable Programs, 1994 (MIT Press)
  - Genetic Programming III: Darwinian Invention and Problem Solving, 1999 with Forrest H Bennett III, David Andre, and Martin A. Keane, (Morgan Kaufmann)
  - Genetic Programming IV: Routine Human-Competitive Machine Intelligence, 2003 with Martin A. Keane, Matthew J. Streeter, William Mydlowec, Jessen Yu, and Guido Lanza
- Genetic Programming: An Introduction, Banzhaf, Nordin, Keller, Francone, 1997 (Morgan Kaufmann)
- Linear genetic programming, Markus Brameier, Wolfgang Banzhaf, Springer (2007)
- A Field Guide to Genetic Programming, Poli, Langdon, McPhee, 2008, Lulu and online digitally

Specific References in Tutorial

Books

- Adaptation in Natural and Artificial Systems, John H Holland, (1992), MIT Press.
- Evolutionsstrategie, Ingo Rechenberg, (1994), Frommann-Holzboog.
- Artificial Intelligence through Simulated Evolution, L.J. Fogel, A.J. Owens, and M.J. Walsh (1966), John Wiley, NY.

Academic Papers

- On the Search Properties of Different Crossover Operators in Genetic Programming, Riccardo Poli and William B. Langdon, Genetic Programming 1998: Proceedings of the Third Annual Conference, pp. 293-301, Morgan Kaufmann, 22-25 July 1998.
- Where does the Good Stuff Go and Why? Goldberg and O'Reilly, Proceedings of the First European Workshop on Genetic Programming, LNCS, Vol. 1391, pp. 16-36, Springer-Verlag, 14-15 April 1998.
- Cartesian genetic programming, GECCO-2008 tutorials, pp. 2701-2726, ACM, 12-16 July 2008.

The End