# Improving the Performance of MAX-MIN Ant System on the TSP Using Stubborn Ants


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## ABSTRACT

In ant colony optimization (ACO) methods, including Ant System and $\mathcal{MAX}$-$\mathcal{MIN}$ Ant System, each ant stochastically generates its candidate solution, in a given iteration, based on the same pheromone $\tau$ and heuristic $\eta$ information as every other ant. Stubborn ants is an ACO variation in which if an ant generates a particular candidate solution in a given iteration, then the components of that solution will have a higher probability of being selected in the candidate solution generated by that ant in the next iteration. We evaluate this variation in the context of $\mathcal{MAX}$-$\mathcal{MIN}$ Ant System using 41 instances of the Traveling Salesman Problem (TSP), and find that it improves solution quality to a statistically-significant extent.



## Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Heuristic methods*

## Keywords

Ant colony optimization, traveling salesman problem.


## 1. STUBBORN ANTS

Stubborn ants [1] are an ACO [2] variation in which the familiar probabilistic action choice equation:

$$p_{ij}^{k} = \frac{[\tau_{ij}]^{\alpha} \cdot [\eta_{ij}]^{\beta}}{\sum_{\ell \in \mathcal{N}_i^k} [\tau_{i\ell}]^{\alpha} \cdot [\eta_{i\ell}]^{\beta}} , \tag{1}$$

is replaced by:

$$p_{ij}^{k} = \frac{[\tau_{ij}]^{\alpha} \cdot [\eta_{ij}]^{\beta} \cdot \delta_{i,j}^{k}(t-1)}{\sum_{\ell \in \mathcal{N}_i^k} [\tau_{i\ell}]^{\alpha} \cdot [\eta_{i\ell}]^{\beta} \cdot \delta_{i,\ell}^{k}(t-1)} , \tag{2}$$

where

$$\begin{aligned} \delta_{a,b}^{k}(t-1) &= \gamma \quad \text{if } (a,b) \in \mathcal{S}_k^{t-1} , \\ &= 1 \quad \text{otherwise} , \end{aligned} \tag{3}$$

where $\mathcal{S}_k^{t-1}$ denotes the solution constructed by ant $k$ in the previous iteration, and $\gamma$ is a stubbornness parameter that determines the degree to which an ant is biased towards its past solution. Of course, if $\gamma = 1$, then our model reduces to the standard model.



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## 2. EXPERIMENTAL RESULTS

We evaluate stubborn ants in the context of $\mathcal{MAX}$-$\mathcal{MIN}$ Ant System ($\mathcal{MM}$AS) [4] and the Traveling Salesman Problem (TSP), using problem instances from TSPLIB. The TSPLIB naming convention is that the number of cities is included in the instance name, e.g. `rl5934` has 5,934 cities.

Our implementation is based on Thomas Stützle's `acotsp` implementation [3], which is the companion open-source software to the Dorigo and Stützle book [2]. In order to implement stubborn ants, we did not need to modify or add more than 30 lines of code to `acotsp`. Our modified code is available at `http://aucegypt.edu/faculty/abdelbar/acotsp.v1.02.tar.gz`. With `acotsp`, we use the default values (based on the recommendations in [2]) for most of the parameters ($\alpha = 1$, $\beta = 2$, $\rho = 0.2$). We set the number of ants to 50, and use `2-opt` local search.

**Experiment A:** We use seven problem instances of gradually increasing size, as shown in Figure 1. We ran each instance for 30 trials for each value of $\gamma$ in the set $\{1, 2, 5, 10, 50, 100, 200, 300, 400, 500, 600, 700, 800\}$, running each trial for 200 iterations.

Figure 1 shows, for each problem instance, a plot of $\gamma$ ($x$-axis) versus mean solution cost ($y$-axis), where throughout the paper solution cost is expressed as percentage in excess of the optimal tour. We can see from the figure that $\gamma = 300$ is a good working value, and we use this as the default value in Experiments B and C. In addition, we note that for all 7 instances, the worst mean solution cost is obtained without stubbornness (i.e. $\gamma = 1$).

**Experiment B:** The purpose of this experiment is to evaluate the performance of stubborn ants, using the default value of $\gamma = 300$ identified in Experiment A, against standard $\mathcal{MM}$AS, in the first 200 iterations of the computation. In this experiment, we use almost all instances at TSPLIB with number of cities between 300 and 6,000, ex-

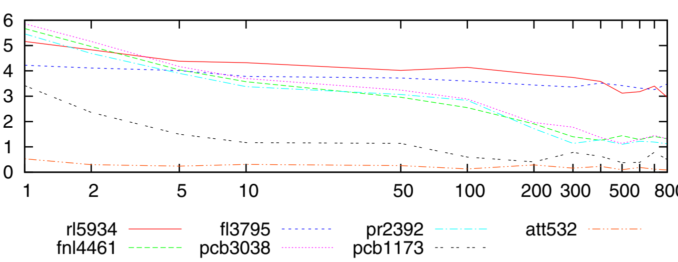


**Figure 1: Experiment A: Plot of mean solution cost ($y$-axis) versus $\gamma$ ($x$-axis, logscale), for several problem instances.**

**Table 1: Results of Experiment B**

| | Standard | | Stubborn | | |
|---|---|---|---|---|---|
| | $\mu$ | $\sigma$ | $\mu$ | $\sigma$ | $p$-val |
| `rl5934` | 5.24 | 0.24 | **3.61** | 0.31 | **7E-31** |
| `rl5915` | 5.13 | 0.22 | **3.47** | 0.26 | **3E-34** |
| `fnl4461` | 5.67 | 0.13 | **1.55** | 0.54 | **2E-44** |
| `fl3795` | 4.37 | 0.39 | **3.43** | 0.49 | **3E-11** |
| `pcb3038` | 5.80 | 0.23 | **1.50** | 0.69 | **8E-39** |
| `pr2392` | 5.46 | 0.22 | **1.20** | 0.80 | **2E-35** |
| `u2319` | 2.28 | 0.08 | **0.54** | 0.19 | **2E-47** |
| `u2152` | 4.17 | 0.54 | **0.58** | 0.45 | **1E-35** |
| `d2103` | 1.59 | 0.36 | **0.55** | 0.25 | **7E-19** |
| `rl1889` | 3.85 | 0.38 | **2.42** | 0.36 | **1E-21** |
| `u1817` | 3.33 | 0.55 | **0.50** | 0.25 | **3E-33** |
| `vm1748` | 3.93 | 0.40 | **1.62** | 0.45 | **9E-29** |
| `d1655` | 3.76 | 0.39 | **1.22** | 0.30 | **1E-35** |
| `fl1577` | 2.44 | 0.28 | **1.73** | 0.24 | **2E-15** |
| `u1432` | 4.10 | 0.24 | **0.79** | 0.28 | **4E-49** |
| `fl1400` | 1.94 | 0.36 | **1.33** | 0.45 | **3E-07** |
| `nrw1379` | 3.86 | 0.24 | **0.64** | 0.21 | **1E-51** |
| `rl1323` | 1.91 | 0.34 | **0.41** | 0.06 | **1E-31** |
| `rl1304` | 2.80 | 0.62 | **1.97** | 0.20 | **4E-09** |
| `d1291` | 0.94 | 0.42 | **0.25** | 0.11 | **3E-12** |
| `pcb1173` | 3.41 | 0.33 | **0.36** | 0.21 | **1E-45** |
| `vm1084` | 2.37 | 0.37 | **0.39** | 0.22 | **5E-33** |
| `u1060` | 4.62 | 0.25 | **1.61** | 0.53 | **2E-35** |
| `pr1002` | 4.12 | 0.41 | **0.87** | 0.42 | **4E-37** |
| `dsj1000` | 3.99 | 0.28 | **1.00** | 0.60 | **2E-32** |
| `rat783` | 1.93 | 0.51 | **0.14** | 0.03 | **7E-27** |
| `u724` | 1.04 | 0.35 | **0.34** | 0.13 | **1E-14** |
| `gr666` | 1.71 | 0.37 | **0.66** | 0.14 | **6E-21** |
| `d657` | 1.59 | 0.53 | **0.22** | 0.10 | **2E-20** |
| `p654` | 0.89 | 0.17 | **0.72** | 0.21 | **1E-03** |
| `rat575` | 1.44 | 0.58 | **0.19** | 0.11 | **8E-17** |
| `u574` | 1.92 | 0.65 | **0.69** | 0.26 | **2E-13** |
| `ali535` | 0.32 | 0.17 | **0.21** | 0.04 | **2E-03** |
| `att532` | 0.54 | 0.36 | **0.30** | 0.03 | **5E-04** |
| `d493` | 1.47 | 0.36 | **0.36** | 0.11 | **7E-23** |
| `pcb442` | **0.45** | 0.19 | 0.48 | 0.16 | 0.586 |
| `pr439` | 0.16 | 0.09 | **0.05** | 0.03 | **5E-09** |
| `gr431` | 0.48 | 0.26 | **0.16** | 0.04 | **6E-09** |
| `fl417` | 0.27 | 0.09 | **0.14** | 0.04 | **4E-09** |
| `rd400` | 0.22 | 0.15 | **0.03** | 0.06 | **1E-07** |
| `lin318` | 0.29 | 0.21 | **0.15** | 0.02 | **1E-03** |

**Table 2: Results of Experiment C**

| | Standard | | Stubborn | | |
|---|---|---|---|---|---|
| | $\mu$ | $\sigma$ | $\mu$ | $\sigma$ | $p$-val |
| `rl5934` | 1.88 | 0.34 | **1.55** | 0.27 | **1E-04** |
| `rl5915` | 1.50 | 0.26 | **1.22** | 0.27 | **2E-04** |
| `fnl4461` | 0.80 | 0.10 | **0.65** | 0.09 | **1E-07** |
| `fl3795` | 2.26 | 0.27 | **2.07** | 0.21 | **0.003** |
| `pcb3038` | 0.79 | 0.14 | **0.48** | 0.14 | **4E-12** |
| `pr2392` | 0.75 | 0.18 | **0.54** | 0.19 | **4E-05** |
| `u2319` | 0.41 | 0.04 | **0.29** | 0.04 | **3E-16** |
| `u2152` | 0.44 | 0.16 | **0.29** | 0.03 | **4E-06** |
| `d2103` | 0.32 | 0.15 | **0.29** | 0.04 | 0.223 |
| `rl1889` | 0.95 | 0.38 | **0.68** | 0.16 | **8E-04** |
| `u1817` | 0.48 | 0.17 | **0.43** | 0.02 | 0.126 |
| `vm1748` | 0.57 | 0.17 | **0.47** | 0.22 | 0.060 |
| `d1655` | 0.77 | 0.27 | **0.45** | 0.16 | **6E-07** |
| `fl1577` | 1.33 | 0.27 | **1.02** | 0.19 | **4E-06** |
| `u1432` | 0.58 | 0.11 | **0.47** | 0.15 | **0.002** |
| `fl1400` | 1.20 | 0.20 | **0.90** | 0.19 | **9E-08** |
| `nrw1379` | 0.57 | 0.12 | **0.38** | 0.09 | **4E-09** |
| `rl1323` | 0.67 | 0.26 | **0.30** | 0.06 | **1E-10** |
| `rl1304` | 0.56 | 0.28 | **0.32** | 0.15 | **1E-04** |
| `d1291` | 0.22 | 0.13 | **0.16** | 0.05 | *0.032* |
| `pcb1173` | 0.45 | 0.21 | **0.27** | 0.01 | **1E-05** |
| `vm1084` | 0.22 | 0.10 | **0.20** | 0.02 | 0.345 |
| `u1060` | 1.59 | 0.33 | **0.56** | 0.22 | **2E-20** |
| `pr1002` | 0.69 | 0.22 | **0.59** | 0.11 | *0.033* |
| `dsj1000` | 1.16 | 0.37 | **0.49** | 0.23 | **2E-11** |
| `rat783` | **0.35** | 0.14 | 0.46 | 0.10 | **0.001** |
| `u724` | 0.28 | 0.07 | **0.15** | 0.02 | **2E-15** |
| `gr666` | **0.40** | 0.15 | 0.53 | 0.07 | **1E-04** |
| `d657` | 0.39 | 0.10 | **0.25** | 0.05 | **1E-08** |
| `p654` | **0.47** | 0.13 | 0.47 | 0.15 | 0.932 |
| `rat575` | **0.28** | 0.13 | 0.32 | 0.06 | 0.121 |
| `u574` | 0.30 | 0.19 | **0.05** | 0.05 | **9E-09** |
| `ali535` | 0.10 | 0.07 | **0.00** | 0.02 | **5E-10** |
| `att532` | 0.18 | 0.11 | **0.03** | 0.03 | **7E-09** |
| `d493` | 0.53 | 0.15 | **0.36** | 0.11 | **7E-06** |
| `pcb442` | 0.40 | 0.13 | **0.33** | 0.06 | **0.008** |
| `pr439` | 0.07 | 0.05 | **0.04** | 0.01 | **0.007** |
| `gr431` | 0.17 | 0.15 | **0.14** | 0.01 | 0.297 |
| `fl417` | 0.16 | 0.07 | **0.09** | 0.05 | **2E-05** |
| `rd400` | 0.11 | 0.16 | **0.03** | 0.06 | **0.009** |
| `lin318` | 0.24 | 0.14 | **0.01** | 0.04 | **2E-12** |

cluding only three instances whose format is not supported by `acotsp`, and running for 100 trials in each case.

The results are shown in Table 1: for each instance, the smaller solution cost is shown in bold. For each instance, we applied an unpaired two-tailed $t$-test to the solution costs obtained with and without stubbornness over the course of the trials. The $p$-values resulting from these $t$-tests are given in the last column of Table 1, and are shown in bold if $p < 0.01$ and in italics if $0.01 \leq p < 0.05$. As the table indicates, the difference in solution cost is significant for all but one instance (`pcb442`): of the instances with a significant difference, stubbornness improves solution cost in all cases.

**Experiment C:** The purpose of this experiment is to evaluate the performance of stubborn ants against standard $\mathcal{MM}$AS beyond the first 200 iterations. In this experiment, we let the number of iterations be equal to the number of cities, and run for 30 trials in each case. The results are shown in Table 2, which follows a similar format to Table 1.

These results indicate that the difference in performance is significant at the 0.05 level for 34 out of the 41 instances, and stubbornness improves performance for 32 of these 34 instances. Even at the 0.01 significance level, the difference is significant for 32 out of the 41 instances, and stubbornness improves performance for 30 out of these 32 instances.

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