Surrogate-Assisted Evolutionary Programming for High Dimensional Constrained Black-Box Optimization

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ABSTRACT

This paper presents a novel surrogate-assisted evolutionary programming (EP) method for high dimensional constrained black-box optimization with many black-box inequality constraints. A cubic radial basis function (RBF) surrogate is used and the resulting RBF-assisted EP outperforms a standard EP, an RBF-assisted penalty-based EP, Stochastic Ranking Evolution Strategy and Scatter Search on a 124-D automotive problem with 68 black-box constraints.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*constrained optimization, global optimization*

General Terms

Algorithms, Experimentation

Keywords

Evolutionary programming, constrained black-box optimization, surrogate-assisted algorithms, radial basis functions

1. INTRODUCTION

This paper develops a novel surrogate-assisted evolutionary programming (EP) algorithm for constrained black-box optimization problems of the form: min $f(x)$ s.t. $x \in \mathbb{R}^d$, $a \leq x \leq b$, and $g_i(x) \leq 0$ for $i = 1, 2, \ldots, m$, where f, $g_1, \ldots, g_m$ are black-box functions that are outcomes of time-consuming but deterministic simulations and $[a, b]$ is a hypercube in $\mathbb{R}^d$. Here, one simulation yields the values of the objective and constraint functions at a given point. This problem is important because it arises in many applications.

Evolutionary algorithms (EAs) have been used to solve constrained optimization problems (e.g., [1], [4], [5]). When the objective and constraint functions are expensive, EAs and other optimization methods are sometimes assisted by surrogate models. A common constraint handling approach is to use a penalty function (e.g., [9]). However, this might not be effective for expensive black-box constraints since information about individual constraint violations are lost. Few surrogate-based methods treat the constraints individually (e.g., [3], [6], [7]) and even fewer have been applied to problems with over a hundred decision variables (e.g., [7]).

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2. SURROGATE-ASSISTED EP FOR CONSTRAINED OPTIMIZATION

The proposed surrogate-assisted EP for constrained black-box optimization does not use a penalty function. It builds surrogates for the objective and constraint functions using all previously sampled points (feasible or infeasible). As in a standard EP, there is no recombination and the mutations are normally distributed and uncorrelated. In every generation, each of the μ parents generates ν trial offspring, where μ and ν are parameters. Moreover, each trial offspring is generated as in a standard EP (e.g., [4]) except that it allows for only a fraction of the components of the parent to be perturbed. The initial standard deviation of the Gaussian mutations is given by the parameter σ_{init}. The probability of perturbing a coordinate of a parent is given by the parameter p_{mut}. This idea is similar to the BCS strategy that was helpful for ConstrLMSRBF [7] on a 124-D problem. Next, the surrogates are used to select the most promising of the ν trial offspring from each parent, and this becomes the actual offspring where the objective and constraint functions are evaluated. The trial offspring selected is the one with the best *predicted* objective value among those with the minimum number of *predicted* constraint violations. Then the parents for the next generation are the best μ individuals from the μ parents and μ actual offspring of the current generation. The method then proceeds as in a standard EP.

3. RESULTS AND DISCUSSION

The above surrogate-assisted EP is applied to a 124-D automotive problem with 68 black-box constraints called MOPTA08 [2]. This problem is much larger than those typically used in surrogate-based optimization. The cubic RBF model in [7] is used to construct surrogates for the objective and each of the constraint functions in every generation. The standard EP and RBF-assisted EP are labeled $(\mu+\mu)$-CEP and $(\mu+\mu)$-CEP-RBF, respectively. Moreover, variants of these methods that use the BCS strategy are labeled $(\mu+\mu)$-CEP-BCS and $(\mu+\mu)$-CEP-RBF-BCS. The parameter settings are: $\mu = 2$ or 10, $\nu = \min(1000d, 10000)$, $p_{\text{mut}} = 0.1$ and $\sigma_{\text{init}} = 0.2\ell([a, b])$, where $\ell([a, b])$ is the length of one side of the hypercube $[a, b]$. The alternative methods are RBF-assisted penalty-based EPs ($(2+2)$-PenCEP-RBF and $(2+2)$-PenCEP-RBF-BCS), Stochastic Ranking Evolution Strategy (SRES) [8], and Scatter Search (eSS) [1].

The numerical experiments are performed on Matlab 7.12. Each algorithm is run for 5 trials on MOPTA08 and uses the same feasible starting point provided by Jones [2]. Each trial of an EP (with or without RBF surrogates) begins with a

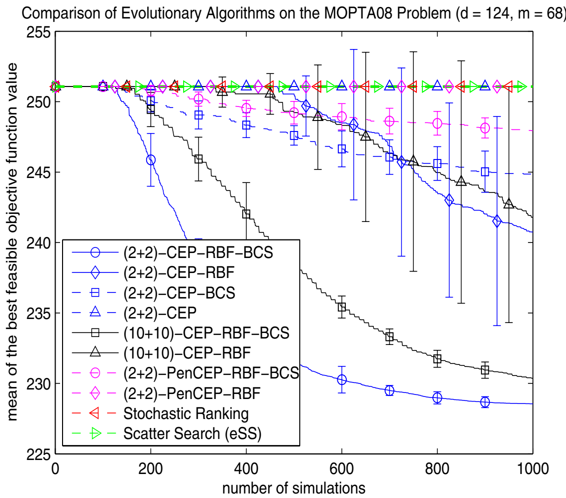

Figure 1: Comparison of Evolutionary Algorithms on the MOPTA08 Problem.

randomly generated Latin hypercube design (LHD) consisting of $d+1$ affinely independent points. The initial parents are the best μ points from the LHD and starting point. For SRES, $\mu = 20$ and $\lambda = 140$ and default values are used for the other parameters. Moreover, SRES and all EP algorithms use the same LHD in a given trial. For eSS, some of the default parameters are modified to reduce the time spent on the initialization phase.

Figure 1 shows the mean of the best feasible objective value found by $(2+2)$-CEP with or without RBF surrogates or BCS on MOPTA08 for a maximum of 1000 simulations. The error bars represent 95% t confidence intervals about the mean. Note that using only RBF surrogates improves the performance of $(2+2)$-CEP, and using only the BCS strategy also improves the performance of $(2+2)$-CEP. However, using RBF surrogates in combination with BCS yields a much better algorithm. Moreover, the figure also shows that $(10+10)$-RBF-BCS is much better than $(10+10)$-RBF.

Figure 1 also shows the results for alternative methods on MOPTA08. The $(2+2)$-CEP-RBF-BCS is the best among the algorithms tested. Its mean best feasible objective function value after 1000 simulations is 228.54. Moreover, it is competitive with ConstrLMSRBF-BCS on MOPTA08 and it outperforms other methods reported in [2] and [7]. Both SRES and eSS did not improve the initial solution after 1000 simulations. However, their performance might improve if they employ surrogates and the BCS strategy. The $(2+2)$-PenCEP-RBF also did not improve the initial solution. This suggests that using a surrogate on a penalty-based EP is not necessarily helpful. However, applying the BCS strategy on $(2+2)$-PenCEP-RBF again makes a substantial difference.

The $(\mu+\mu)$-CEP-RBF is also compared with $(\mu+\mu)$-CEP (for $\mu = 2$ and 10) and with alternatives on the lower dimensional test problems in [7]. The results show that the CEP-RBF algorithms are the best on these problems and are much better than the alternatives, including SRES and eSS. Again, using RBF surrogates to assist an EP makes a big difference. The $(\mu+\mu)$-PenCEP-RBF algorithms performed better than SRES and eSS, but again, they are not as good as the $(\mu+\mu)$-CEP-RBF algorithms.

Finally, the average running time of $(2+2)$-CEP-RBF-BCS for 1000 simulations on the MOPTA08 problem (excluding time spent on the simulations) using an Intel(R) Core(TM) i7 CPU 860 2.8 Ghz desktop machine is about 3.80 hours, which is small compared to the total time spent on the simulations if they are truly expensive.

4. CONCLUSIONS

This paper developed a surrogate-assisted EP that can handle high dimensional problems with many black-box inequality constraints without using a penalty function. The results on a 124-D problem with 68 black-box constraints, and on test problems suggest that RBF surrogates can substantially improve the performance of an EP. Moreover, the BCS strategy from [7] appears to be very promising for high dimensional constrained black-box optimization. In addition, the proposed RBF-assisted EPs are substantially much better than Stochastic Ranking Evolutionary Strategy (SRES) and Scatter Search (eSS) on the problems in this study when the algorithms are given a very limited computational budget. Finally, using a penalty approach might not be the best way to handle constrained black-box problems even if it is combined with a surrogate.

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