Benchmarking the Differential Evolution with Adaptive Encoding on Noiseless Functions

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ABSTRACT

The differential evolution (DE) algorithm is equipped with the recently proposed adaptive encoding (AE) which makes the algorithm rotationally invariant. The resulting algorithm, DEAE, should exhibit better performance on non-separable functions. The aim of this article is to assess what benefits the AE has, and what effect it has for other function groups. DEAE is compared against pure DE, an adaptive version of DE (JADE), and an evolutionary strategy with covariance matrix adaptation (CMA-ES). The results suggest that AE indeed improves the performance of DE, particularly on the group of unimodal non-separable functions, but the adaptation of parameters used in JADE is more profitable on average. The use of AE inside JADE is envisioned.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Differential evolution, Evolution strategy, Covariance matrix adaptation, Adaptive encoding

1. INTRODUCTION

Differential evolution (DE) [9] is a population-based optimization algorithm, popular thanks to its simplicity and good results on many practical problems. To create an offspring individual, it uses a mutation operator followed by a crossover. The mutation operators are usually rotationally invariant, however, the crossover is not. On separable functions, the crossover helps to properly mix the good values of solution components in the population. On non-separable functions, however, it mostly only destroys the potentially good combinations of values generated by the mutation.

There are several possibilities how to overcome the crossover issue for non-separable functions. (1) Turn off the crossover operator completely. The DE then relies on the mutation operator only and may have worse performance on (partially) separable functions. (2) Choose the suitable operators adaptively. There are several algorithms [8, 1, 10] able to choose suitable DE operators and their parameters during the optimization run. For non-separable functions, they may actually find that the use of crossover is not profitable at all and may switch it off effectively. (3) Use adaptive encoding. If we were able to perform the crossover in a suitable coordinate system, we may enjoy the benefits of crossover even for the non-separable functions.

In this article, the last listed possibility is explored. We chose the recently proposed adaptive encoding (AE) procedure [2] which adapts the coordinate system in a step-wise manner during the search. The goal of this paper is to assess how AE affects the DE algorithm, what benefits and what downsides it has, and also to compare the potential of parameter adaptation as used in JADE on the one hand, and encoding adaptation brought by AE on the other hand.

The rest of this article is organized as follows. Section 2 reviews the DE algorithm, and describes the use of AE inside DE, i.e. the proposed DEAE algorithm. Section 3 describes the experiment carried out, together with the COCO benchmarking framework. The results are presented in Sec. 4 and discussed in Sec. 5. Sec. 6 concludes the paper and points out some directions for future work.

2. ALGORITHMS

The following paragraphs review the DE algorithm and the AE procedure, introduce the DEAE algorithm and shortly describe the reference algorithms used in this paper.

Differential evolution (DE) [9] is a simple and easy-to-implement optimization algorithm (see the unshaded lines in Alg. 1). DE mutation operators create the donor individuals $\mathbf{v}_i$ as a linear combination of several individuals randomly chosen from the current population.

$$\mathbf{v}_i = \mathbf{x}_{\text{best}} + F \cdot (\mathbf{x}_{r1} - \mathbf{x}_{r2}), \tag{1}$$

Eq. 1 describes the so called “best/1” mutation operator, a highly exploitative mutation variant, where F is the mutation factor (a positive number typically chosen from $[0.5, 1]$).

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The crossover creates the offspring $\mathbf{u}_i$ by taking some solution components from the parent $\mathbf{x}_i$ and other components from the donor $\mathbf{v}_i$. Eq. (2) describes the binomial crossover. It creates the offspring individual $\mathbf{u}_i = (u_{i,1}, \ldots, u_{i,D})$ as follows:

$$u_{i,j} = \begin{cases} v_{i,j} & \text{if } r_j \leq CR_i \text{ or } j = j_{i,\text{rand}}, \\ x_{i,j} & \text{otherwise,} \end{cases} \tag{2}$$

where r_j is a random number uniformly distributed in $[0,1]$, $CR_i \in [0,1]$ is the crossover probability representing the average proportion of components the offspring gets from its donor, and $j_{i,\text{rand}}$ is the randomly chosen index of the solution component surely donated from the donor.

Due to the crossover, DE is biased towards separable functions, and is not rotationally invariant. This bias, however, can be controlled with the parameter CR. The tuning of CR is part of many adaptive DE variants which try to find the right operators and/or parameter values [8, 1, 10] to make the resulting algorithm more robust.

DE and adaptive encoding. The adaptive encoding (AE) framework [2] is a general method that makes an optimization algorithm rotationally invariant. It maintains a linear transformation of the coordinate system—the candidate solutions are evaluated in the original space, but the offspring creation takes place in a different space given by the linear transformation. Alg. 1 shows a simple combination of the basic DE algorithm with AE, i.e. the DEAE algorithm, first proposed in [6]. The shaded lines are the modifications needed for AE.

Algorithm 1: DE with Adaptive Encoding

Initialize the population $P \leftarrow \{\mathbf{x}_i\}_{i=1}^{NP}$.
Initialize the transformation matrix $\mathbf{B} \in \mathbb{R}^{D \times D}$
while *stopping criteria not met* **do**
  Transform P: $P' \leftarrow \{\mathbf{x}'_i | \mathbf{x}'_i \leftarrow \mathbf{B}^{-1}\mathbf{x}_i\}$.
  for $i \leftarrow 1$ *to* NP **do**
    $\mathbf{v}'_i \leftarrow$ `mutate`$(i,\ P')$ (Eq. 1)
    $\mathbf{u}'_i \leftarrow$ `crossover`$(\mathbf{x}'_i,\ \mathbf{v}'_i)$ (Eq. 2)
    Transform offspring back: $\mathbf{u}_i \leftarrow \mathbf{B}\mathbf{u}'_i$.
    if $f(\mathbf{u}_i) < f(\mathbf{x}_i)$ **then**
      $\mathbf{x}_i \leftarrow \mathbf{u}_i$
    end
  end
  $\mathbf{B} \leftarrow$ `update`$(\mathbf{B},\ \mathbf{x}_{(1)}, \ldots, \mathbf{x}_{(\mu)})$
end

The forward and backward linear transformations are implemented by matrix multiplication (using the transformation matrix $\mathbf{B}$). The procedure for updating $\mathbf{B}$ is crucial for the algorithm success. We adopted the method derived from the CMA-ES algorithm (we refer the reader to [2] for more details).

Reference algorithms. JADE [10] serves as a reference adaptive DE algorithm. It was chosen because it was reported [10] to have a better performance than other adaptive DE variants. JADE uses a special mutation strategy called "current-to-*p*best", but most importantly it adapts the crossover probability CR and mutation factor F to values which turned out to be beneficial in recent generations. This algorithm thus does not adapt the coordinate system, does not adaptively select the operators it uses, but thanks to the adaptation of CR, it can effectively turn off the crossover.

CMA-ES, evolution strategy with covariance matrix adaptation [5] was chosen for the comparison because the AE procedure is largely based on this algorithm. The algorithm samples new candidate solutions from a multivariate Gaussian distribution and adapts its mean and covariance matrix (i.e. it actually uses the adaptation of the coordinate system). The algorithm CMA-ES used in this paper is a conventional multistart version.

3. EXPERIMENT DESIGN

In the experiments, we compare DE, DEAE, JADE, and CMA-ES. By comparing DE to DEAE, we can assess the performance boost the DE algorithm can gain using AE. By comparing DEAE with CMA-ES, we can get some insight if the sampling process of CMA-ES (drawing points from normal distribution) is more suitable than the sampling process of DE (using mutation and crossover). The comparison of DEAE with JADE shall reveal which of the two different types of adaptation is more suitable for which kinds of functions.

Each of the algorithms was run on 15 instances of all the 24 functions in dimensions 2, 3, 5, 10, 20, and 40. The evaluations budget was set to $5 \cdot 10^4 D$ for each run. All algorithms were restarted when they stagnate for more than 30 generations and the population diversity measure $\frac{1}{D}\sum_{i=1}^{D} Var(X_i) < 10^{-10}$.

The multistart CMA-ES algorithm was benchmarked anew with its default settings using the BBOB 2012 procedure.

For most parameters of DE and JADE, default values from the literature were used. For DE: the binomial crossover with $CR = 0.5$, the "best" mutation strategy with $F \sim U(0.5, 1)$ (sampled anew each generation). For JADE: initial $\mu_{CR} = 0.5$, initial $\mu_F = 0.5$, the parameter of the "current-to-*p*best" mutation is $p = 0.1$, the archive size $|A| = 0.1NP$. The population size was set to $NP = 5D$ for both algorithms after a small systematic study performed on JADE and DE using the values $(3, 4, 5, 6, 8, 10, 15, 20) \cdot D$. Values of NP lower than $5D$ gave erratic behavior even on uni-modal functions, values larger than $5D$ wasted evaluations on uni-modal functions and did not bring significant advantages on multi-modal functions.

The DEAE algorithm inherited the parameters of DE. The AE part of DEAE uses a learning rate parameter $\alpha_c = 8$ chosen after testing the values 1, 4, 8, 10, 15, and 20 (increasing the learning rate from 1 to 8 brought significant speedups, further increase provided questionable advantage only).

4. RESULTS

Results from experiments according to [3] on the benchmark functions given in [4] are presented in Figures 1, 2 and 3 and in Tables 1 and 2. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value, $f_t = f_{\text{opt}} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t, summed over all trials and divided by the number of trials that actually reached f_t [3, 7]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t (10^{-8} as in Figure 1) using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

4.1 CPU Timing Experiments

The timing experiments were carried out with f_8 on a machine with Intel Core 2 Duo processor, 2.4 Ghz, with 4 GB RAM, on Windows 7 64bit in MATLAB R2009b 64bit. The average time per function evaluation in 2, 3, 5, 10, 20, 40 dimensions was about 52, 35, 21, 12, 8, and 7×10^{-6} s for DE, about 70, 45, 28, 16, 9, 10×10^{-6} s for JADE, and 68, 45, 27, 15, 9, 10 for DEAE, i.e. the cost of AE updates is negligible.

5. DISCUSSION

Considering the comparison of DEAE and DE, it can be stated that the application of AE to DE generally helps the DE algorithm to solve a higher percentage of problems, i.e. to find more precise optima of the functions, and to solve them faster, especially in the group of non-separable unimodal functions (for the ill-conditioned functions, speedup factors of 10 are observed in 5D, the percentage of solved problems arose from about 20 % to 100 % in 20-D), which is an expected result. In case of multi-modal functions, the difference is not that large, but DEAE is only seldom worse than the pure DE. The only exception in this comparison is the group of separable functions (namely f_3 and f_4), where the application of AE actually destroys the initially ideal coordinate system and prevents the DEAE algorithm from solving these functions.

The comparison of DEAE to CMA-ES reveals that on the group of unimodal functions, the multistart CMA-ES is usually faster than DEAE (about 2 to 5 times faster, depending on dimensionality), probably thanks to its much smaller population. The exception are the functions f_3 and f_4 (where neither of the 2 algorithms is competitive), and f_7 and f_{13} (where the DEAE profits from its larger population size). On the group of multi-modal functions with adequate structure, DEAE performs better (larger population), while on the group of weakly structured functions, CMA-ES is comparable or better (thanks to larger number of restarts).

Comparing DEAE to JADE, the first observation is that JADE has an advantage in case of separable functions. For non-separable unimodal functions, DEAE is (up to 5 times) faster. For multimodal functions, the results are quite mixed. In general (and especially in higher dimensions), JADE is expected to solve a larger proportion of functions than DEAE.

6. SUMMARY AND CONCLUSIONS

The search space representation is a key issue when designing a well performing optimization algorithm. In this work, the AE procedure was applied to the DE algorithm. The resulting DEAE algorithm was compared with a conventional DE algorithm, JADE, an adaptive version of DE, and with CMA-ES.

The application of AE significantly improved the performance of the DE algorithm for moderate and ill-conditioned unimodal functions, as expected, but also had a positive (although less pronounced) effect on multimodal functions.

JADE (with a different kind of adaptation than DEAE) also showed quite competitive results. The two forms of adaptation are based on different principles and are in fact complementary. Implementing the AE procedure inside JADE may be very profitable: JADE may adapt the probability of applying AE in a similar way it adapts the CR and F parameters. The evaluation of such approach remains a topic for the future work.

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7. REFERENCES

[1] J. Brest, S. Greiner, B. Boskovic, M. Mernik, and V. Zumer. Self-Adapting control parameters in differential evolution: A comparative study on numerical benchmark problems. *Evolutionary Computation, IEEE Transactions on*, 10(6):646–657, Dec. 2006.

[2] N. Hansen. Adaptive encoding: How to render search coordinate system invariant. In G. Rudolph, editor, *Parallel Problem Solving from Nature – PPSN X*, volume 5199 of *LNCS*, pages 205–214. Springer, 2008.

[3] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2012: Experimental setup. Technical report, INRIA, 2012.

[4] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noiseless functions definitions. Technical Report RR-6829, INRIA, 2009. Updated February 2010.

[5] N. Hansen and A. Ostermeier. Completely derandomized self-adaptation in evolution strategies. *Evolutionary Computation*, 9(2):159–195, 2001.

[6] V. Klemš. Differential evolution with adaptive encoding. Master's thesis, Czech Technical University in Prague, 2011. Available online, `http://cyber.felk.cvut.cz/research/theses/papers/177.pdf`.

[7] K. Price. Differential evolution vs. the functions of the second ICEO. In *Proceedings of the IEEE International Congress on Evolutionary Computation*, pages 153–157, 1997.

[8] A. K. Qin and P. N. Suganthan. Self-adaptive differential evolution algorithm for numerical optimization. In *Evolutionary Computation, 2005. The 2005 IEEE Congress on*, volume 2, pages 1785–1791 Vol. 2. IEEE, 2005.

[9] R. Storn and K. Price. Differential evolution — a simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization*, 11(4):341–359, Dec. 1997.

[10] J. Zhang and A. C. Sanderson. JADE: Adaptive differential evolution with optional external archive. *Evolutionary Computation, IEEE Transactions on*, 13(5):945–958, Oct. 2009.

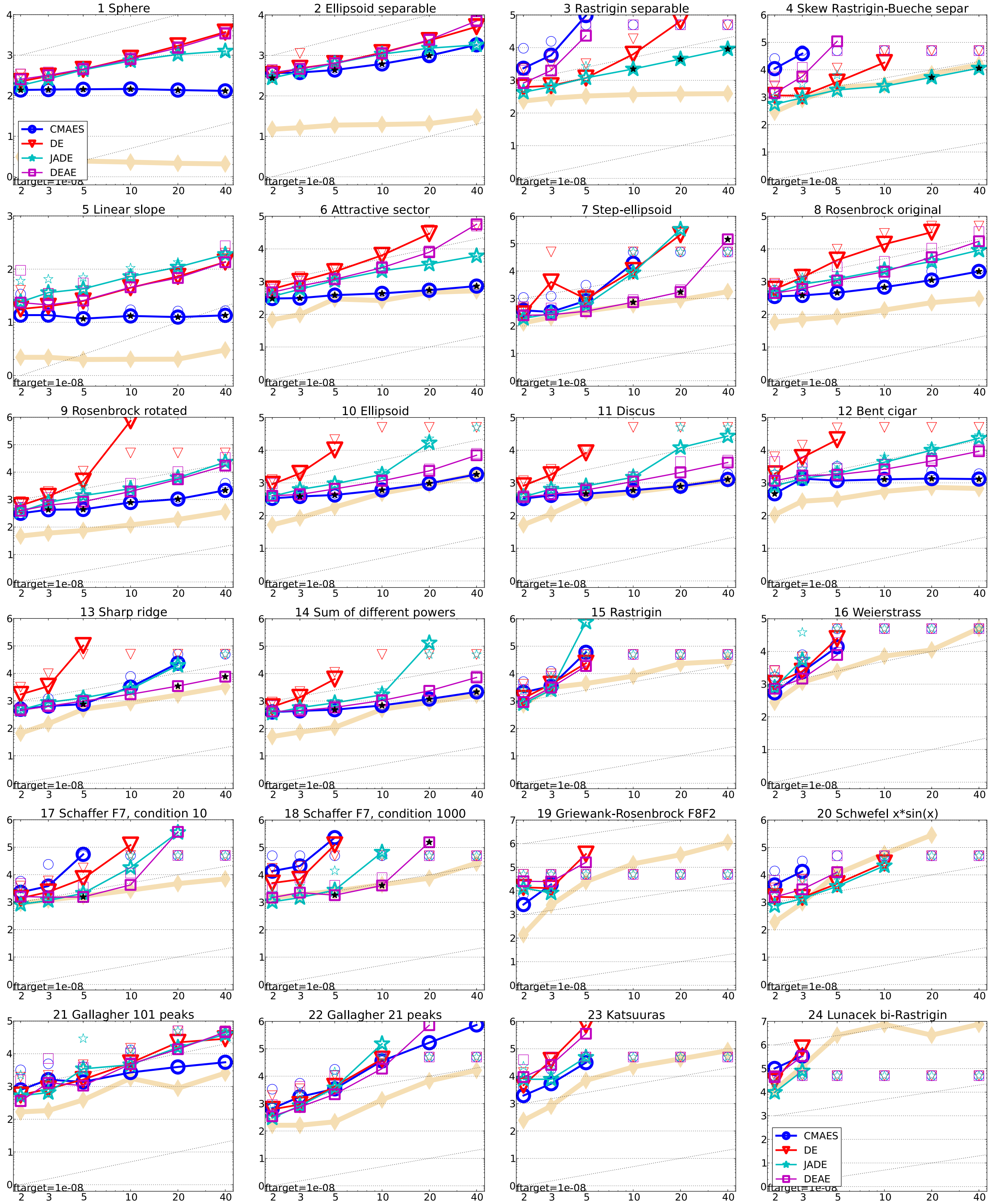

Figure 1: Expected running time (ERT in number of f-evaluations) divided by dimension for target function value 10^{-8} as $\log_{10}$ values versus dimension. Different symbols correspond to different algorithms given in the legend of f_1 and f_{24}. Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Horizontal lines give linear scaling, slanted dotted lines give quadratic scaling. Black stars indicate statistically better result compared to all other algorithms with $p < 0.01$ and Bonferroni correction number of dimensions (six). Legend: ○: CMAES, ▽: DE, ⋆: JADE, □: DEAE.

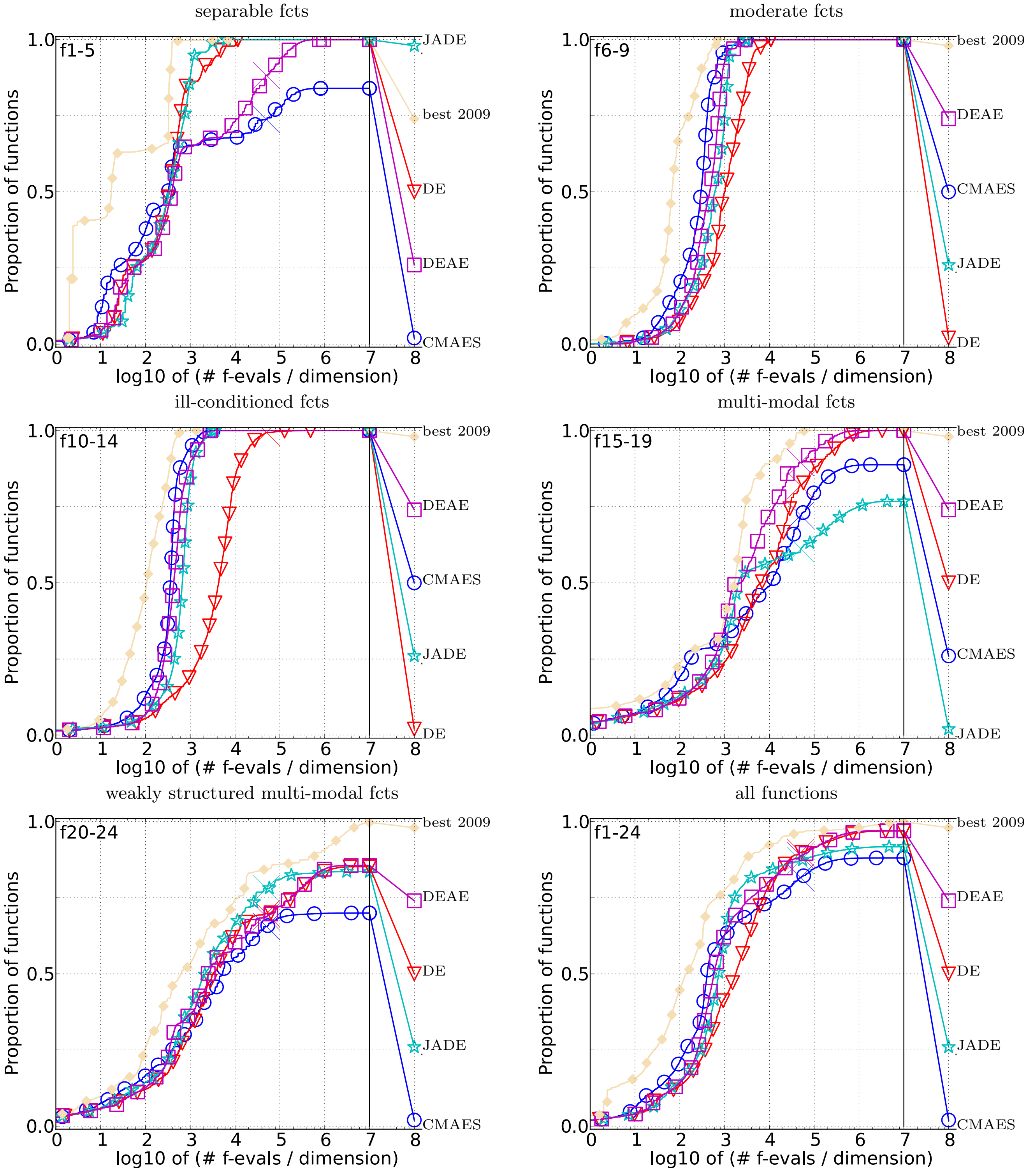

Figure 2: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

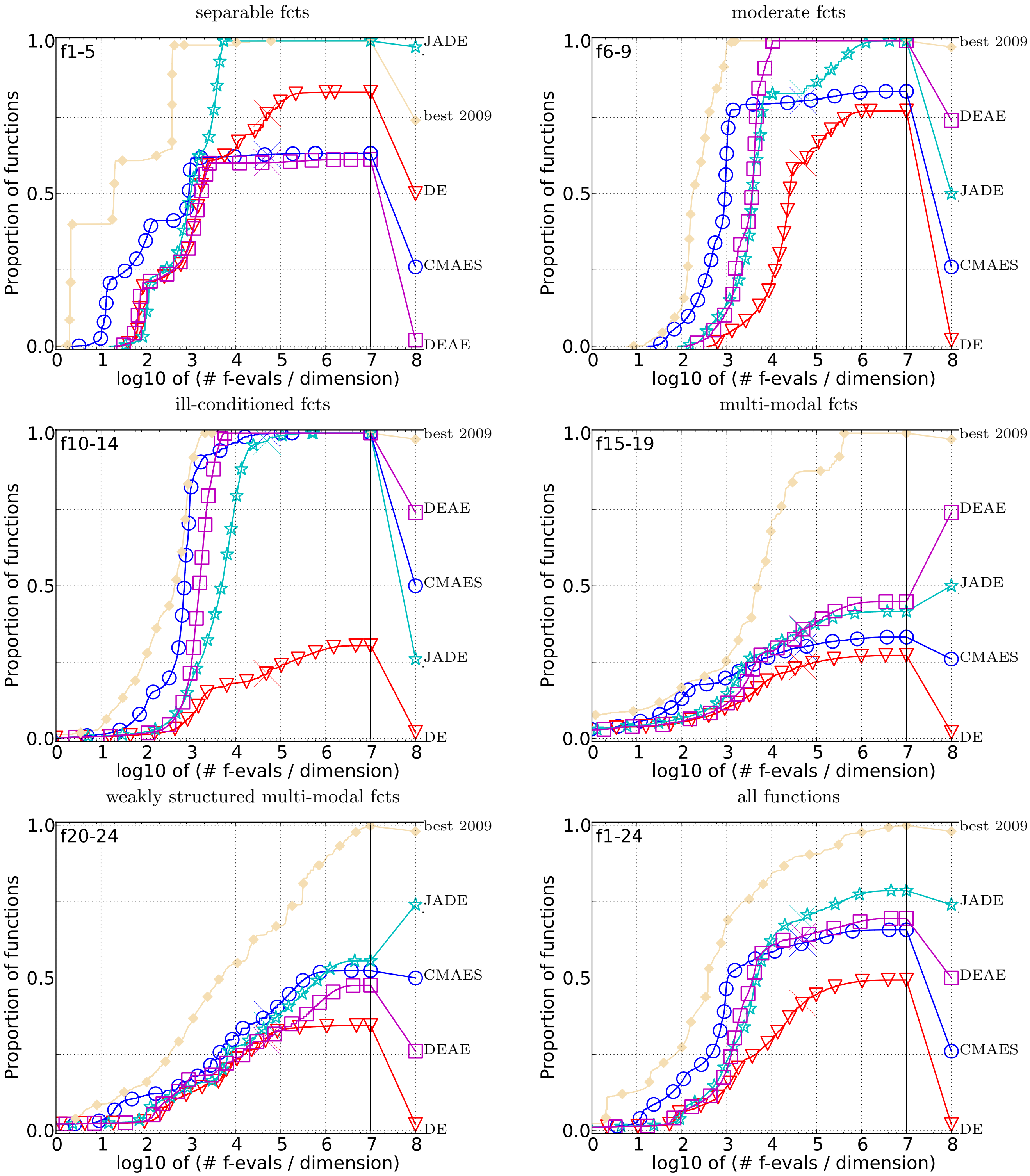

Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f1	11	12	12	12	12	12	15/15
CMAES	**2.3**(2)	**8.6**(3)$^{\star 3}$	**15**(5)$^{\star 3}$	**27**(5)$^{\star 4}$	**41**(4)$^{\star 4}$	**53**(6)$^{\star 4}$	15/15
DE	5.0(4)	21(8)	39(8)	82(8)	122(9)	164(7)	15/15
JADE	4.1(3)	18(6)	36(7)	77(7)	115(10)	154(10)	15/15
DEAE	5.1(6)	23(10)	42(10)	82(6)	124(8)	166(8)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f2	83	87	88	90	92	94	15/15
CMAES	16(4)	17(3)	18(3)	**20**(2)	**21**(2)$^{\star 3}$	**22**(2)$^{\star 4}$	15/15
DE	11(1)	13(2)	16(2)	21(2)	27(2)	31(2)	15/15
JADE	**10**(1)	**12**(2)	**15**(2)	20(2)	26(2)	31(2)	15/15
DEAE	12(3)	14(2)	17(3)	22(3)	27(3)	32(3)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f3	716	1622	1637	1646	1650	1654	15/15
CMAES	1.7(2)	23(26)	293(319)	292(304)	291(324)	290(321)	6/15
DE	1.1(0.4)	**1.4**(0.2)*	2.5(2)	2.8(2)	3.1(2)	**3.4**(2)	15/15
JADE	**1.1**(0.5)	1.6(0.3)	**2.2**(0.4)	**2.7**(0.3)	**3.0**(0.3)	3.4(0.3)	15/15
DEAE	2.4(1)	11(11)	70(81)	70(78)	70(86)	70(80)	13/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f4	809	1633	1688	1817	1886	1903	15/15
CMAES	2.2(1)	216(189)	∞	∞	∞	∞ *3e5*	0/15
DE	**1.2**(0.3)	**1.7**(0.3)$^{\star 2}$	9.4(14)	9.0(13)	9.0(13)	9.2(13)	15/15
JADE	1.4(0.4)	2.0(0.5)	**3.9**(3)	**4.1**(3)	**4.4**(3)	**4.7**(3)	15/15
DEAE	4.1(3)	47(26)	323(304)	300(275)	290(294)	287(291)	6/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f5	10	10	10	10	10	10	15/15
CMAES	**4.3**(2)*	**5.8**(2)$^{\star 2}$	**5.8**(2)$^{\star 2}$	**5.9**(2)$^{\star 2}$	**5.9**(2)$^{\star 2}$	**5.9**(2)$^{\star 2}$	15/15
DE	8.3(3)	12(4)	12(3)	13(3)	13(3)	13(3)	15/15
JADE	11(6)	20(8)	21(7)	21(7)	21(7)	21(7)	15/15
DEAE	8.0(4)	12(7)	13(7)	13(7)	13(7)	13(7)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f6	114	214	281	580	1038	1332	15/15
CMAES	**2.4**(0.7)	**2.0**(0.5)$^{\star 4}$	**2.2**(0.5)$^{\star 4}$	**1.7**(0.4)$^{\star 4}$	**1.3**(0.3)$^{\star 4}$	**1.3**(0.2)$^{\star 4}$	15/15
DE	5.4(1)	6.6(2)	8.4(2)	8.0(2)	6.3(1)	6.7(2)	15/15
JADE	4.1(2)	4.5(1.0)	5.4(0.9)	4.4(0.5)	3.6(0.4)	3.7(0.5)	15/15
DEAE	3.7(2)	4.8(1)	5.7(0.9)	4.6(0.5)	3.8(0.4)	3.9(0.3)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f7	24	324	1171	1572	1572	1597	15/15
CMAES	**5.1**(3)	**1.1**(0.9)	2.0(2)	2.6(3)	2.6(3)	2.6(3)	15/15
DE	13(11)	3.4(2)	1.9(0.6)	2.5(0.7)	2.5(0.7)	2.7(0.8)	15/15
JADE	8.4(5)	2.0(0.7)	1.3(0.4)	1.4(0.3)	1.4(0.3)	1.6(0.3)	15/15
DEAE	7.9(5)	1.4(0.4)	**0.68**(0.1)	**0.87**(0.2)	**0.87**(0.2)	**0.94**(0.2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f8	73	273	336	391	410	422	15/15
CMAES	**4.3**(4)*	**3.6**(1)$^{\star 2}$	**4.4**(1)$^{\star 3}$	**4.7**(1)$^{\star 3}$	**5.0**(0.9)$^{\star 4}$	**5.2**(1)$^{\star 4}$	15/15
DE	9.3(4)	14(5)	21(8)	30(8)	40(7)	49(8)	15/15
JADE	7.5(3)	7.3(3)	10(3)	12(2)	13(1)	14(1)	15/15
DEAE	8.6(3)	10(12)	11(10)	11(8)	11(8)	12(8)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f9	35	127	214	300	335	369	15/15
CMAES	**5.5**(1)$^{\star 4}$	**7.1**(3)$^{\star 2}$	**6.5**(2)$^{\star 2}$	**5.8**(1)$^{\star 3}$	**5.8**(1.0)$^{\star 4}$	**5.7**(0.9)$^{\star 4}$	15/15
DE	22(8)	37(7)	36(10)	40(11)	50(14)	58(17)	15/15
JADE	14(2)	24(7)	22(5)	19(5)	19(4)	18(4)	15/15
DEAE	17(5)	13(4)	11(4)	10(3)	10(3)	11(3)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f10	349	500	574	626	829	880	15/15
CMAES	3.5(0.7)	2.9(0.5)	2.7(0.4)	**2.8**(0.3)$^{\star 2}$	**2.3**(0.2)$^{\star 4}$	**2.3**(0.2)$^{\star 4}$	15/15
DE	27(8)	28(6)	32(6)	43(8)	42(6)	49(7)	15/15
JADE	6.1(1)	5.1(0.8)	5.0(0.8)	5.4(0.7)	4.8(0.5)	5.1(0.5)	15/15
DEAE	**3.0**(0.4)	**2.5**(0.3)	**2.6**(0.4)	3.2(0.2)	3.0(0.3)	3.4(0.2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f11	143	202	763	1177	1467	1673	15/15
CMAES	9.1(1)	7.7(1)	2.3(0.3)	1.6(0.2)	**1.4**(0.2)	**1.3**(0.1)$^{\star 3}$	15/15
DE	32(23)	42(19)	16(5)	17(4)	19(4)	23(4)	15/15
JADE	8.1(3)	9.0(1)	2.8(0.4)	2.3(0.3)	2.2(0.2)	2.3(0.2)	15/15
DEAE	**4.4**(1)*	**4.9**(1)$^{\star 3}$	**1.7**(0.3)$^{\star 3}$	**1.5**(0.2)	1.5(0.2)	1.6(0.2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f12	108	268	371	461	1303	1494	15/15
CMAES	**8.4**(9)*	**5.9**(7)*	**7.1**(7)	**7.9**(5)	**3.5**(2)	**3.8**(2)	15/15
DE	103(94)	107(85)	117(116)	135(129)	63(51)	66(52)	15/15
JADE	24(3)	13(4)	12(7)	14(7)	6.1(3)	6.2(4)	15/15
DEAE	15(5)	11(4)	11(6)	13(6)	5.6(3)	5.6(3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f13	132	195	250	1310	1752	2255	15/15
CMAES	**4.5**(3)	**4.8**(3)	**5.1**(2)	**1.5**(0.4)$^{\star 3}$	**1.5**(0.4)$^{\star 3}$	**1.4**(0.4)$^{\star 3}$	15/15
DE	14(4)	26(11)	39(11)	19(7)	28(9)	45(23)	6/15
JADE	8.2(1)	12(2)	12(2)	3.1(0.3)	2.9(0.2)	2.6(0.2)	15/15
DEAE	6.0(0.8)	6.4(1.0)	6.7(0.5)	2.0(0.1)	2.0(0.1)	2.0(0.1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f14	10	41	58	139	251	476	15/15
CMAES	1.5(1)	**2.9**(1)$^{\star 2}$	**3.8**(0.8)$^{\star 4}$	**4.2**(1)$^{\star 4}$	**5.4**(1)$^{\star 4}$	**4.4**(0.5)$^{\star 3}$	15/15
DE	1.8(3)	7.2(3)	12(2)	15(4)	37(8)	47(12)	15/15
JADE	**0.95**(0.8)	5.3(2)	8.9(1)	10(2)	12(1)	8.2(0.5)	15/15
DEAE	2.3(3)	6.3(3)	11(2)	9.4(1)	7.9(0.9)	5.6(0.6)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f15	511	9310	19369	20073	20769	21359	14/15
CMAES	**2.1**(2)	5.0(5)	15(14)	15(15)	14(15)	14(13)	9/15
DE	6.1(5)	4.3(4)	6.0(7)	5.9(6)	5.7(6)	5.6(6)	13/15
JADE	3.7(1)	7.5(14)	39(51)	39(42)	59(61)	175(193)	1/15
DEAE	3.9(4)	**4.2**(4)	**4.8**(5)	**4.6**(5)	**4.5**(5)	**4.4**(5)	14/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f16	120	612	2662	10449	11644	12095	15/15
CMAES	**2.0**(2)	**3.6**(4)*	**2.7**(4)$^{\star 2}$	4.5(8)	5.8(7)	5.7(7)	15/15
DE	5.1(6)	31(13)	17(16)	11(10)	10(9)	10(8)	14/15
JADE	2.9(5)	10(5)	36(47)	∞	∞	∞ *2e5*	0/15
DEAE	3.2(4)	26(15)	12(9)	**3.2**(2)	**3.3**(3)	**3.2**(2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f17	5.2	215	899	3669	6351	7934	15/15
CMAES	4.1(4)	**0.96**(0.4)$^{\star 2}$	**0.74**(0.2)$^{\star 2}$	1.2(2)	8.5(11)	30(32)	9/15
DE	4.1(6)	3.2(1.0)	2.2(0.5)	2.1(0.5)	2.8(2)	3.3(3)	15/15
JADE	3.3(4)	2.5(0.7)	1.9(0.3)	1.2(0.2)	1.2(0.3)	1.2(0.2)	15/15
DEAE	**3.1**(3)	2.7(1)	1.5(0.2)	**0.85**(0.1)	**0.80**(0.1)*	**0.91**(0.1)$^{\star 2}$	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f18	103	378	3968	9280	10905	12469	15/15
CMAES	**1.2**(0.7)	**2.2**(4)*	1.5(2)	11(11)	76(81)	92(103)	3/15
DE	2.4(2)	4.4(2)	1.4(0.7)	2.8(3)	7.7(9)	40(46)	5/15
JADE	1.9(1)	3.3(0.7)	0.72(0.1)	0.60(0.1)$_{\downarrow 2}$	1.1(0.2)	1.1(0.2)	15/15
DEAE	2.4(1)	2.8(0.8)	**0.50**(0.1)	**0.43**(0.1)$^{\star 3}_{\downarrow 4}$	**0.53**(0.1)$^{\star 2}_{\downarrow}$	**0.69**(0.5)*	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f19	1	1	242	1.2e5	1.2e5	1.2e5	15/15
CMAES	**21**(13)	2230(1651)	327(335)	∞	∞	∞ *3e5*	0/15
DE	32(28)	4210(2571)	1106(988)	15(16)	15(16)	15(15)	2/15
JADE	35(26)	**2139**(2314)	**276**(160)	∞	∞	∞ *2e5*	0/15
DEAE	32(38)	3555(2186)	388(229)	**6.7**(7)	**6.7**(7)	**6.6**(7)	4/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f20	16	851	38111	54470	54861	55313	14/15
CMAES	**3.7**(3)	27(28)	∞	∞	∞	∞ *3e5*	0/15
DE	8.8(6)	**1.9**(0.7)	0.57(0.8)	0.41(0.6)	0.42(0.6)	0.42(0.5)	15/15
JADE	4.9(3)	2.3(2)	**0.25**(0.2)$_{\downarrow 3}$	**0.25**(0.2)	**0.29**(0.2)	**0.33**(0.2)	15/15
DEAE	10(7)	3.0(2)	1.6(1)	1.1(1)	1.1(1)	1.1(1)	14/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f21	41	1157	1674	1705	1729	1757	14/15
CMAES	1.8(1)	3.1(3)	3.9(5)	3.9(5)	3.9(5)	3.9(5)	15/15
DE	2.7(3)	3.9(6)	3.7(4)	4.0(4)	4.3(4)	4.5(4)	15/15
JADE	**1.7**(2)	**1.1**(1)	**1.2**(1)	2.8(4)	5.0(9)	8.2(13)	15/15
DEAE	3.1(3)	1.2(0.7)	2.5(3)	**2.7**(3)	**2.9**(3)	**2.9**(3)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f22	71	386	938	1008	1040	1068	14/15
CMAES	9.3(10)	21(28)	18(31)	17(28)	16(28)	16(27)	15/15
DE	5.0(4)	13(16)	17(25)	17(22)	18(22)	19(21)	15/15
JADE	**2.1**(1)	**3.4**(3)	**8.0**(11)	12(18)	16(27)	17(30)	15/15
DEAE	3.2(2)	6.2(7)	10(14)	**10**(13)	**10**(12)	**10**(12)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f23	3.0	518	14249	31654	33030	34256	15/15
CMAES	2.9(5)	**13**(15)	**6.9**(9)	**4.9**(5)	**4.7**(5)	**4.5**(5)	12/15
DE	2.9(4)	48(59)	57(61)	35(39)	53(56)	105(120)	1/15
JADE	**2.4**(3)	37(32)	8.3(6)	7.6(8)	7.3(7)	7.1(6)	10/15
DEAE	2.8(4)	72(58)	124(132)	56(59)	54(57)	52(58)	2/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f24	1622	2.2e5	6.4e6	9.6e6	1.3e7	1.3e7	3/15
CMAES	**1.6**(2)*	**3.1**(3)	∞	∞	∞	∞ *3e5*	0/15
DE	10(8)	5.4(6)	∞	∞	∞	∞ *2e5*	0/15
JADE	6.5(5)	∞	∞	∞	∞	∞ *2e5*	0/15
DEAE	6.1(5)	3.7(4)	∞	∞	∞	∞ *2e5*	0/15

Table 1: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 5. The central 80% range divided by two is given in braces. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\mathrm{opt}} + 10^{-8}$. Best results are printed in bold.

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f1	43	43	43	43	43	43	15/15
CMAES	**7.5**(2)$^{\star 4}$	**13**(2)$^{\star 4}$	**20**(2)$^{\star 4}$	**33**(2)$^{\star 4}$	**45**(3)$^{\star 4}$	**58**(4)$^{\star 4}$	15/15
DE	89(17)	162(31)	241(28)	400(27)	558(30)	717(39)	15/15
JADE	47(7)	94(8)	143(8)	240(8)	340(10)	437(13)	15/15
DEAE	75(22)	144(28)	214(24)	352(32)	492(42)	635(42)	15/15
f2	385	386	387	390	391	393	15/15
CMAES	37(4)	43(3)	45(2)	**47**(1)$^{\star 2}$	**48**(1)$^{\star 4}$	**50**(1)$^{\star 4}$	15/15
DE	41(3)	50(3)	59(3)	76(5)	93(5)	110(5)	15/15
JADE	**28**(1)$^{\star 3}$	**34**(1)$^{\star 4}$	**39**(2)$^{\star 3}$	50(2)	61(3)	71(4)	15/15
DEAE	53(6)	61(6)	69(7)	84(7)	101(6)	116(7)	15/15
f3	5066	7626	7635	7643	7646	7651	15/15
CMAES	638(691)	∞	∞	∞	∞	∞ *1e6*	0/15
DE	39(10)	67(49)	167(170)	168(170)	168(169)	169(169)	9/15
JADE	**6.4**(0.3)$^{\star 4}$	**6.0**(0.2)$^{\star 4}$	**6.8**(0.2)$^{\star 4}$	**8.3**(0.2)$^{\star 4}$	**10**(0.2)$^{\star 4}$	**11**(0.2)$^{\star 4}$	15/15
DEAE	∞	∞	∞	∞	∞	∞ *1e6*	0/15
f4	4722	7628	7666	7700	7758	1.4e5	9/15
CMAES	∞	∞	∞	∞	∞	∞ *1e6*	0/15
DE	30(9)	∞	∞	∞	∞	∞ *1e6*	0/15
JADE	**8.0**(0.4)$^{\star 4}$	**7.0**(0.3)$^{\star 4}$	**8.0**(0.2)$^{\star 4}$	**10**(0.2)$^{\star 4}$	**11**(0.3)$^{\star 4}$	**0.71**(0.0)$^{\star 4}$	15/15
DEAE	∞	∞	∞	∞	∞	∞ *1e6*	0/15
f5	41	41	41	41	41	41	15/15
CMAES	**5.1**(1.0)$^{\star 4}$	**6.2**(0.9)$^{\star 4}$	**6.2**(0.9)$^{\star 4}$	**6.2**(0.9)$^{\star 4}$	**6.2**(0.9)$^{\star 4}$	**6.2**(0.9)$^{\star 4}$	15/15
DE	27(6)	35(4)	36(6)	36(6)	36(6)	36(6)	15/15
JADE	43(9)	52(7)	54(8)	54(8)	54(8)	54(8)	15/15
DEAE	28(7)	33(6)	34(7)	34(7)	34(7)	34(7)	15/15
f6	1296	2343	3413	5220	6728	8409	15/15
CMAES	**1.7**(0.5)$^{\star 4}$	**1.3**(0.3)$^{\star 4}$	**1.2**(0.3)$^{\star 4}$	**1.2**(0.1)$^{\star 4}$	**1.2**(0.2)$^{\star 4}$	**1.2**(0.1)$^{\star 4}$	15/15
DE	53(9)	46(7)	46(8)	50(9)	57(11)	61(12)	15/15
JADE	9.4(0.7)	7.8(0.8)	7.3(0.7)	7.2(0.9)	7.4(0.9)	7.4(0.9)	15/15
DEAE	21(5)	18(3)	16(2)	16(2)	17(1)	17(1)	15/15
f7	1351	4274	9503	16524	16524	16969	15/15
CMAES	**1.5**(1)$^{\star 3}$	191(210)	754(842)	∞	∞	∞ *1e6*	0/15
DE	24(6)	94(122)	235(265)	256(302)	256(302)	249(296)	3/15
JADE	4.8(0.9)	272(351)	686(842)	402(480)	402(482)	391(442)	2/15
DEAE	4.7(0.4)	**3.5**(0.5)	**2.2**(0.3)$^{\star 2}$	**1.9**(0.3)$^{\star 4}$	**1.9**(0.3)$^{\star 4}$	**1.9**(0.2)$^{\star 4}$	15/15
f8	2039	3871	4040	4219	4371	4484	15/15
CMAES	**3.9**(0.8)$^{\star 4}$	**4.3**(0.7)$^{\star 4}$	**4.7**(0.7)$^{\star 4}$	**4.8**(0.7)$^{\star 4}$	**4.9**(0.7)$^{\star 4}$	**4.9**(0.7)$^{\star 4}$	15/15
DE	53(4)	95(6)	106(9)	120(11)	130(11)	138(12)	14/15
JADE	18(1)	16(0.6)	16(0.6)	17(0.6)	17(0.7)	18(0.7)	15/15
DEAE	19(3)	21(15)	22(14)	23(14)	23(14)	24(13)	15/15
f9	1716	3102	3277	3455	3594	3727	15/15
CMAES	**4.5**(1)$^{\star 4}$	**5.0**(0.7)$^{\star 4}$	**5.4**(0.6)$^{\star 4}$	**5.5**(0.6)$^{\star 4}$	**5.5**(0.6)$^{\star 4}$	**5.5**(0.6)$^{\star 4}$	15/15
DE	∞	∞	∞	∞	∞	∞ *1e6*	0/15
JADE	36(3)	30(3)	32(3)	33(2)	33(2)	33(2)	15/15
DEAE	21(3)	25(17)	26(16)	27(15)	28(15)	29(14)	15/15
f10	7413	8661	10735	14920	17073	17476	15/15
CMAES	**1.7**(0.1)$^{\star 4}$	**1.7**(0.2)$^{\star 4}$	**1.6**(0.1)$^{\star 4}$	**1.2**(0.0)$^{\star 4}$	**1.1**(0.0)$^{\star 4}$	**1.1**(0.0)$^{\star 4}$	15/15
DE	∞	∞	∞	∞	∞	∞ *1e6*	0/15
JADE	12(5)	15(4)	15(4)	15(4)	15(3)	18(4)	15/15
DEAE	2.6(0.3)	2.6(0.2)	2.3(0.2)	2.1(0.1)	2.2(0.2)	2.5(0.2)	15/15
f11	1002	2228	6278	9762	12285	14831	15/15
CMAES	**11**(0.8)	**5.3**(0.2)	**2.0**(0.1)$^{\star 3}$	**1.4**(0.0)$^{\star 4}$	**1.2**(0.0)$^{\star 4}$	**1.0**(0.0)$^{\star 4}$	15/15
DE	7377(7977)	∞	∞	∞	∞	∞ *1e6*	0/15
JADE	92(19)	46(8)	18(4)	15(3)	15(3)	15(3)	14/15
DEAE	13(4)	7.4(2)	3.2(0.7)	2.7(0.5)	2.6(0.4)	2.6(0.3)	15/15
f12	1042	1938	2740	4140	12407	13827	15/15
CMAES	**2.2**(0.2)$^{\star 4}$	**3.2**(3)$^{\star 4}$	**4.0**(3)$^{\star 4}$	**4.0**(2)$^{\star 4}$	**1.8**(0.8)$^{\star 4}$	**1.9**(0.7)$^{\star 4}$	15/15
DE	99(6)	478(538)	1497(1825)	∞	∞	∞ *1e6*	0/15
JADE	19(4)	20(15)	28(15)	28(10)	13(3)	14(3)	15/15
DEAE	22(1)	14(3)	14(4)	14(3)	6.1(1)	6.4(1)	15/15
f13	652	2021	2751	18749	24455	30201	15/15
CMAES	**6.3**(5)$^{\star 4}$	**5.1**(3)$^{\star 2}$	**4.5**(3)$^{\star 2}$	**1.9**(2)	4.6(5)	8.4(9)	12/15
DE	41(7)	214(306)	702(849)	∞	∞	∞ *1e6*	0/15
JADE	17(2)	14(5)	15(4)	3.6(0.6)	4.8(0.8)	9.0(2)	15/15
DEAE	22(2)	10(0.6)	10(0.5)	2.1(0.1)	**2.1**(0.1)	**2.1**(0.1)*	15/15
f14	75	239	304	932	1648	15661	15/15
CMAES	**4.2**(1)$^{\star 4}$	**3.0**(0.5)$^{\star 4}$	**3.7**(0.4)$^{\star 4}$	**4.2**(0.3)$^{\star 4}$	**6.2**(0.5)$^{\star 4}$	**1.2**(0.1)$^{\star 4}$	15/15
DE	55(19)	46(6)	53(6)	502(176)	∞	∞ *1e6*	0/15
JADE	18(6)	18(1)	23(2)	20(1)	38(24)	62(72)	5/15
DEAE	30(4)	27(3)	37(4)	24(2)	19(1)	2.7(0.2)	15/15
f15	30378	1.5e5	3.1e5	3.2e5	4.5e5	4.6e5	15/15
CMAES	**37**(36)	∞	∞	∞	∞	∞ *1e6*	0/15
DE	∞	∞	∞	∞	∞	∞ *1e6*	0/15
JADE	39(35)	∞	∞	∞	∞	∞ *1e6*	0/15
DEAE	∞	∞	∞	∞	∞	∞ *1e6*	0/15
f16	1384	27265	77015	1.9e5	2.0e5	2.2e5	15/15
CMAES	**1.9**(0.8)$^{\star 4}$	**2.7**(2)$^{\star 4}$	∞	∞	∞	∞ *1e6*	0/15
DE	∞	∞	∞	∞	∞	∞ *1e6*	0/15
JADE	24(8)	∞	∞	∞	∞	∞ *1e6*	0/15
DEAE	∞	∞	∞	∞	∞	∞ *1e6*	0/15
f17	63	1030	4005	30677	56288	80472	15/15
CMAES	**2.8**(2)$^{\star 2}$	**0.96**(0.4)$^{\star 4}$	**1.5**(2)*	8.1(8)	∞	∞ *1e6*	0/15
DE	26(19)	16(4)	13(4)	8.8(9)	∞	∞ *1e6*	0/15
JADE	7.8(5)	7.4(1)	4.4(0.7)	**1.7**(0.5)$^{\star 2}$	**7.2**(9)	**23**(25)	2/15
DEAE	19(11)	13(2)	8.4(0.9)	2.3(0.3)	37(37)	90(106)	2/15
f18	621	3972	19561	67569	1.3e5	1.5e5	15/15
CMAES	**1.1**(0.4)$^{\star 4}$	**2.0**(2)	4.2(5)	∞	∞	∞ *1e6*	0/15
DE	17(4)	16(5)	15(7)	∞	∞	∞ *1e6*	0/15
JADE	7.2(2)	4.4(1)	**1.5**(0.4)	19(22)	∞	∞ *1e6*	0/15
DEAE	12(4)	6.3(1)	2.3(0.3)	**2.5**(2)*	**6.7**(7)$^{\star 4}$	**14**(14)$^{\star 4}$	4/15
f19	1	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
CMAES	**162**(70)$^{\star 4}$	**4.8e4**(4e4)$^{\star 4}$	∞	∞	∞	∞ *1e6*	0/15
DE	2739(802)	∞	∞	∞	∞	∞ *1e6*	0/15
JADE	856(206)	7.0e5(6e5)	∞	∞	∞	∞ *1e6*	0/15
DEAE	1434(466)	∞	∞	∞	∞	∞ *1e6*	0/15
f20	82	46150	3.1e6	5.5e6	5.6e6	5.6e6	14/15
CMAES	**5.2**(1)$^{\star 4}$	156(172)	∞	∞	∞	∞ *1e6*	0/15
DE	51(9)	2.4(1)	0.79(0.8)	∞	∞	∞ *1e6*	0/15
JADE	24(3)	**1.2**(0.2)$^{\star 3}$	**0.46**(0.4)	**0.85**(0.9)	**2.7**(3)	∞ *1e6*	0/15
DEAE	39(9)	56(60)	∞	∞	∞	∞ *1e6*	0/15
f21	561	6541	14103	14643	15567	17589	15/15
CMAES	**5.4**(4)	**9.4**(10)	**5.5**(6)	**5.3**(6)	**5.0**(5)	**4.5**(5)	15/15
DE	33(64)	45(68)	30(35)	29(34)	28(34)	25(31)	13/15
JADE	7.2(2)	33(62)	21(38)	20(34)	19(34)	18(31)	13/15
DEAE	28(53)	26(33)	19(35)	18(34)	17(32)	16(28)	13/15
f22	467	5580	23491	24948	26847	1.3e5	12/15
CMAES	**3.8**(5)$^{\star 2}$	**42**(90)	**142**(156)	**133**(153)	**124**(123)	**25**(26)	4/15
DE	23(10)	92(107)	∞	∞	∞	∞ *1e6*	0/15
JADE	25(45)	261(277)	638(681)	601(621)	559(577)	∞ *1e6*	0/15
DEAE	50(121)	123(179)	616(766)	580(671)	539(596)	107(122)	1/15
f23	3.2	1614	67457	4.9e5	8.1e5	8.4e5	15/15
CMAES	2.9(3)	571(707)	**63**(67)	∞	∞	∞ *1e6*	0/15
DE	2.4(2)	∞	∞	∞	∞	∞ *1e6*	0/15
JADE	2.0(2)	**131**(68)*	∞	∞	∞	∞ *1e6*	0/15
DEAE	**1.8**(2)	∞	∞	∞	∞	∞ *1e6*	0/15
f24	1.3e6	7.5e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
CMAES	∞	∞	∞	∞	∞	∞ *1e6*	0/15
DE	∞	∞	∞	∞	∞	∞ *1e6*	0/15
JADE	∞	∞	∞	∞	∞	∞ *1e6*	0/15
DEAE	∞	∞	∞	∞	∞	∞ *1e6*	0/15

Table 2: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 20. The central 80% range divided by two is given in braces. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\mathrm{opt}} + 10^{-8}$. Best results are printed in bold.