# Investigating the Impact of Adaptation Sampling in Natural Evolution Strategies on Black-box Optimization Testbeds

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## ABSTRACT

Natural Evolution Strategies (NES) are a recent member of the class of real-valued optimization algorithms that are based on adapting search distributions. Exponential NES (xNES) are the most common instantiation of NES, and particularly appropriate for the BBOB 2012 benchmarks, given that many are non-separable, and their relatively small problem dimensions. The technique of *adaptation sampling*, which adapts learning rates online further improves the algorithm's performance. This report provides an extensive empirical comparison to study the impact of adaptation sampling in xNES, both on the noise-free and noisy BBOB testbeds.

## Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

## General Terms

Algorithms

## Keywords

Evolution Strategies, Natural Gradient, Benchmarking

## 1. INTRODUCTION

Evolution strategies (ES), in contrast to traditional evolutionary algorithms, aim at repeating the type of mutation that led to those good individuals. We can characterize those mutations by an explicitly parameterized *search distribution* from which new candidate samples are drawn, akin to estimation of distribution algorithms (EDA). Covariance matrix adaptation ES (CMA-ES [10]) innovated the field by introducing a parameterization that includes the full covariance matrix, allowing them to solve highly non-separable problems.

A more recent variant, *natural evolution strategies* (NES [20, 6, 18, 19]) aims at a higher level of generality, providing a procedure to update the search distribution's parameters for any type of distribution, by ascending the gradient towards higher expected fitness. Further, it has been shown [15, 12] that following the *natural gradient* to adapt the search distribution is highly beneficial, because it appropriately normalizes the update step with respect to its uncertainty and makes the algorithm scale-invariant.

Exponential NES (xNES), the most common instantiation of NES, used a search distribution parameterized by a mean vector and a full covariance matrix, and is thus most similar to CMA-ES (in fact, the precise relation is described in [4] and [5]). Given the relatively small problem dimensions of the BBOB benchmarks, and the fact that many are non-separable, it is also among the most appropriate NES variants for the task.

*Adaptation sampling* (AS) is a technique for the online adaptation of its learning rate, which is designed to speed up convergence. This may be beneficial to algorithms like xNES, because the optimization traverses qualitatively different phases, during which different learning rates may be optimal.

In this report, we retain the original formulation of xNES (including all parameter settings, except for an added stopping criterion), and compare this base-line algorithm with an AS-augmented variant. We describe the empirical performance on all 54 benchmark functions (both noise-free and noisy) of the BBOB 2012 workshop.

## 2. NATURAL EVOLUTION STRATEGIES

Natural evolution strategies (NES) maintain a search distribution $\pi$ and adapt the distribution parameters $\theta$ by following the *natural gradient* [1] of expected fitness $J$, that is, maximizing

$$J(\theta) = \mathbb{E}_\theta[f(\mathbf{z})] = \int f(\mathbf{z})\ \pi(\mathbf{z}\,|\,\theta)\ d\mathbf{z}$$

Just like their close relative CMA-ES [10], NES algorithms are invariant under monotone transformations of the fitness function and linear transformations of the search space. Each iteration the algorithm produces $n$ samples $\mathbf{z}_i \sim \pi(\mathbf{z}|\theta)$, $i \in \{1, \ldots, n\}$, i.i.d. from its search distribution, which is parameterized by $\theta$. The gradient w.r.t. the parameters $\theta$ can be rewritten (see [20]) as

$$\nabla_\theta J(\theta) = \nabla_\theta \int f(\mathbf{z})\ \pi(\mathbf{z}\,|\,\theta)\ d\mathbf{z} = \mathbb{E}_\theta\left[f(\mathbf{z})\ \nabla_\theta \log \pi(\mathbf{z}\,|\,\theta)\right]$$

from which we obtain a Monte Carlo estimate

$$\nabla_\theta J(\theta) \approx \frac{1}{n} \sum_{i=1}^{n} f(\mathbf{z}_i)\, \nabla_\theta \log \pi(\mathbf{z}_i \,|\, \theta)$$

of the search gradient. The key step then consists in replacing this gradient by the natural gradient defined as $\mathbf{F}^{-1}\nabla_\theta J(\theta)$ where $\mathbf{F} = \mathbb{E}\left[\nabla_\theta \log \pi\left(\mathbf{z}|\theta\right) \nabla_\theta \log \pi\left(\mathbf{z}|\theta\right)^\top\right]$ is the Fisher information matrix. The search distribution is iteratively updated using natural gradient ascent

$$\theta \leftarrow \theta + \eta \mathbf{F}^{-1} \nabla_\theta J(\theta)$$

with learning rate parameter $\eta$.

## 2.1 Exponential NES

While the NES formulation is applicable to arbitrary parameterizable search distributions [20, 12], the most common variant employs multinormal search distributions. For that case, two helpful techniques were introduced in [6], namely an exponential parameterization of the covariance matrix, which guarantees positive-definiteness, and a novel method for changing the coordinate system into a "natural" one, which makes the algorithm computationally efficient. The resulting algorithm, NES with a multivariate Gaussian search distribution and using both these techniques is called *xNES*, and the pseudocode is given in Algorithm 1.

**Algorithm 1:** Exponential NES (xNES)

```
input: $f$, $\boldsymbol{\mu}_{\text{init}}$, $\eta_\sigma$, $\eta_{\mathbf{B}}$, $u_k$
           $\boldsymbol{\mu} \leftarrow \boldsymbol{\mu}_{\text{init}}$
initialize $\sigma \leftarrow 1$
           $\mathbf{B} \leftarrow \mathbb{I}$
repeat
    for $k = 1 \ldots n$ do
        draw sample $\mathbf{s}_k \sim \mathcal{N}(0, \mathbb{I})$
        $\mathbf{z}_k \leftarrow \boldsymbol{\mu} + \sigma \mathbf{B}^\top \mathbf{s}_k$
        evaluate the fitness $f(\mathbf{z}_k)$
    end

    sort $\{(\mathbf{s}_k, \mathbf{z}_k)\}$ with respect to $f(\mathbf{z}_k)$
    and assign utilities $u_k$ to each sample

    compute gradients
     $\nabla_{\boldsymbol{\delta}} J \leftarrow \sum_{k=1}^{n} u_k \cdot \mathbf{s}_k$
     $\nabla_{\mathbf{M}} J \leftarrow \sum_{k=1}^{n} u_k \cdot (\mathbf{s}_k \mathbf{s}_k^\top - \mathbb{I})$
     $\nabla_\sigma J \leftarrow \mathrm{tr}(\nabla_{\mathbf{M}} J)/d$
     $\nabla_{\mathbf{B}} J \leftarrow \nabla_{\mathbf{M}} J - \nabla_\sigma J \cdot \mathbb{I}$

    update parameters
     $\boldsymbol{\mu} \leftarrow \boldsymbol{\mu} + \sigma \mathbf{B} \cdot \nabla_{\boldsymbol{\delta}} J$
     $\sigma \leftarrow \sigma \cdot \exp(\eta_\sigma/2 \cdot \nabla_\sigma J)$
     $\mathbf{B} \leftarrow \mathbf{B} \cdot \exp(\eta_{\mathbf{B}}/2 \cdot \nabla_{\mathbf{B}} J)$
until stopping criterion is met
```

## 2.2 Adaptation Sampling

First introduced in [12] (chapter 2, section 4.4), *adaptation sampling* is a new meta-learning technique [17] that can adapt hyper-parameters online, in an economical way that is grounded on a measure statistical improvement.

Here, we apply it to the learning rate of the global step-size $\eta_\sigma$. The idea is to consider whether a larger learning-rate $\eta'_\sigma = \frac{3}{2}\eta_\sigma$ would have been more likely to generate the good samples in the current batch. For this we determine the (hypothetical) search distribution that would have resulted from such a larger update $\pi(\cdot|\theta')$. Then we compute importance weights

$$w'_k = \frac{\pi(\mathbf{z}_k|\theta')}{\pi(\mathbf{z}_k|\theta)}$$

for each of the $n$ samples $\mathbf{z}_k$ in our current population, generated from the actual search distribution $\pi(\cdot|\theta)$. We then conduct a *weighted* Mann-Whitney test [12] (appendix A) to determine if the set $\{\mathrm{rank}(\mathbf{z}_k)\}$ is inferior to its reweighted counterpart $\{w'_k \cdot \mathrm{rank}(\mathbf{z}_k)\}$ (corresponding to the larger learning rate), with statistical significance $\rho$. If so, we increase the learning rate by a factor of $1 + c'$, up to at most $\eta_\sigma = 1$ (where $c' = 0.1$). Otherwise it decays to its initial value:

$$\eta_\sigma \leftarrow (1 - c') \cdot \eta_\sigma + c' \cdot \eta_{\sigma,\text{init}}$$

The procedure is summarized in algorithm 2 (for details and derivations, see [12]). The combination of xNES with adaptation sampling is dubbed *xNES-as*.

One interpretation of why adaptation sampling is helpful is that half-way into the search, (after a local attractor has been found, e.g., towards the end of the valley on the Rosenbrock benchmarks $f_8$ or $f_9$), the convergence speed can be boosted by an increased learning rate. For such situations, an online adaptation of hyper-parameters is inherently well-suited.

**Algorithm 2:** Adaptation sampling

```
input : $\eta_{\sigma,t}$,$\eta_{\sigma,\text{init}}$, $\theta_t$, $\theta_{t-1}$, $\{(\mathbf{z}_k, f(\mathbf{z}_k))\}$, $c'$, $\rho$
output: $\eta_{\sigma,t+1}$
compute hypothetical $\theta'$, given $\theta_{t-1}$ and using $3/2\eta_{\sigma,t}$
for $k = 1 \ldots n$ do
    $w'_k = \frac{\pi(\mathbf{z}_k|\theta')}{\pi(\mathbf{z}_k|\theta)}$
end
$S \leftarrow \{\mathrm{rank}(\mathbf{z}_k)\}$
$S' \leftarrow \{w'_k \cdot \mathrm{rank}(\mathbf{z}_k)\}$
if weighted-Mann-Whitney$(S, S') < \rho$ then
    return $(1 - c') \cdot \eta_\sigma + c' \cdot \eta_{\sigma,\text{init}}$
else
    return $\min((1 + c') \cdot \eta_\sigma, 1)$
end
```

## 3. EXPERIMENTAL SETTINGS

We use identical default hyper-parameter values for all benchmarks (both noisy and noise-free functions), which are taken from [6, 12]. Table 1 summarizes all the hyper-parameters used.

In addition, we make use of the provided target fitness $f_{\text{opt}}$ to trigger *independent* algorithm restarts[1], using a simple ad-hoc procedure: If the log-progress during the past $1000d$

[1]It turns out that this use of $f_{\text{opt}}$ is technically not permitted by the BBOB guidelines, so strictly speaking a different restart strategy should be employed, for example the one described in [12].

**Table 1: Default parameter values for xNES (including the utility function and adaptation sampling) as a function of problem dimension $d$.**

| parameter | default value |
|---|---|
| $n$ | $4 + \lfloor 3\log(d) \rfloor$ |
| $\eta_\sigma = \eta_{\mathbf{B}}$ | $\dfrac{3(3+\log(d))}{5d\sqrt{d}}$ |
| $u_k$ | $\dfrac{\max\left(0, \log(\frac{n}{2}+1) - \log(k)\right)}{\sum_{j=1}^{n} \max\left(0, \log(\frac{n}{2}+1) - \log(j)\right)} - \dfrac{1}{n}$ |
| $\rho$ | $\dfrac{1}{2} - \dfrac{1}{3(d+1)}$ |
| $c'$ | $\dfrac{1}{10}$ |

evaluations is too small, i.e., if

$$\log_{10}\left|\frac{f_{\text{opt}} - f_t}{f_{\text{opt}} - f_{t-1000d}}\right| < (r+2)^2 \cdot m^{3/2} \cdot [\log_{10}|f_{\text{opt}} - f_t| + 8]$$

where $m$ is the remaining budget of evaluations divided by $1000d$, $f_t$ is the best fitness encountered until evaluation $t$ and $r$ is the number of restarts so far. The total budget is $10^5 d^{3/2}$ evaluations.

Implementations of this and other NES algorithm variants are available in Python through the PyBrain machine learning library [16], as well as in other languages at `www.idsia.ch/~tom/nes.html`.

## 4. RESULTS

Results from experiments according to [7] on the benchmark functions given in [2, 8, 3, 9] are presented in Figures 1, 2 and 3, and in Tables 2, 3 and 4. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value, $f_\text{t} = f_\text{opt} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach $f_\text{t}$, summed over all trials and divided by the number of trials that actually reached $f_\text{t}$ [7, 11]. **Statistical significance** is tested with the rank-sum test for a given target $\Delta f_\text{t}$ ($10^{-8}$ using, for each trial, either the number of needed function evaluations to reach $\Delta f_\text{t}$ (inverted and multiplied by $-1$), or, if the target was not reached, the best $\Delta f$-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

Some of the result plots (like performance scaling with dimension), as well as the CPU-timing results were omitted here but are available in stand-alone benchmarking reports [14, 13].

## 5. DISCUSSION

Naturally, the core algorithm (xNES) being identical in both variants presented here, we do not expect drastically different results. In fact, the direct comparison Figures 1 and 2 show remarkably similar results on the majority of functions, which we see from the points clearly scattered along the diagonal. This indicates that on most benchmarks, adaptation sampling is not used (much) and xNES-as operates with the same conservative learning rates as xNES.

The functions where we observe differences are telling, however. Most notably on the sphere function ($f_1$, and its noisy siblings), adaptation sampling drastically improves performance, by up to a factor 4. Adaptation sampling still improves performance significantly on many other functions (see Table 2), but the speed-ups are less striking. There is a single function where adaptation sampling significantly hurts performance, namely on $f_{13}$ in dimension 20 (the sharp ridge): apparently the learning rates are increased to an overly aggressive level.

In conclusion, we can recommend using adaptation sampling for xNES in general, and particularly so if the fitness landscapes are smooth.

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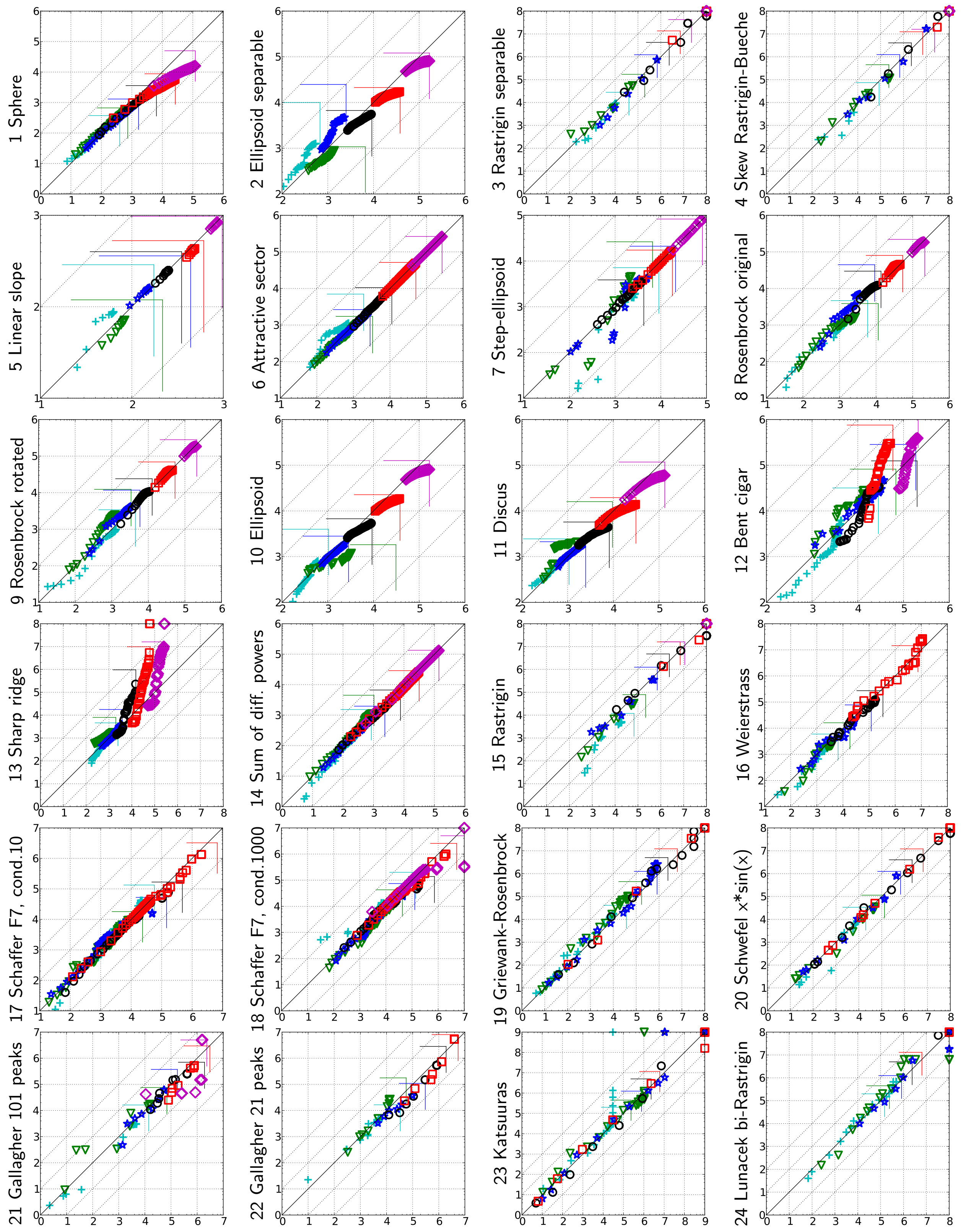

**Figure 1: Expected running time (ERT in $\log_{10}$ of number of function evaluations) of xNES ($x$-axis) versus xNES-as ($y$-axis) for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension on functions $f_1$–$f_{24}$. Markers on the upper or right edge indicate that the target value was never reached. Markers represent dimension: 2:$+$, 3:$\triangledown$, 5:$\star$, 10:$\circ$, 20:$\square$, 40:$\diamond$.**

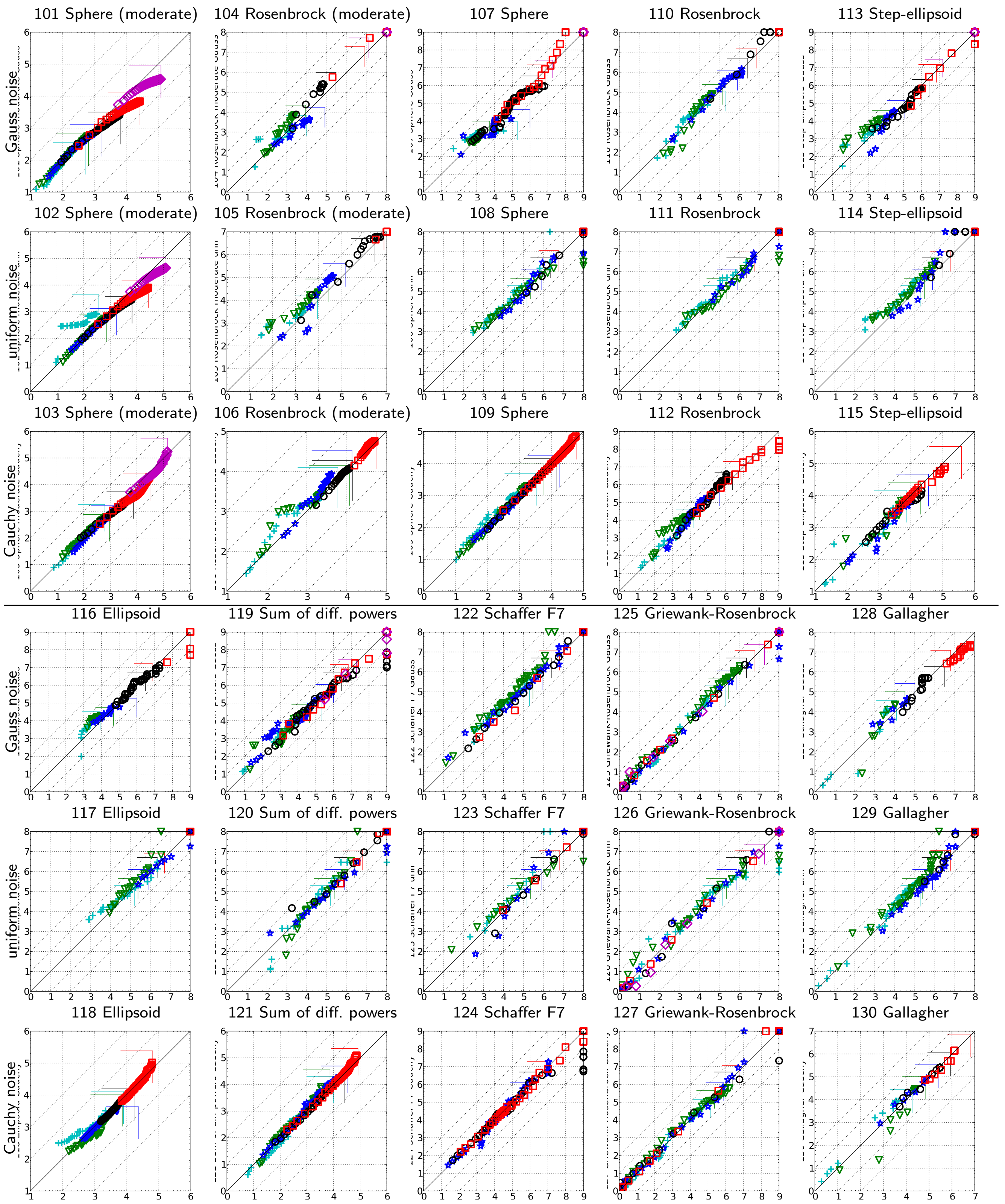

**Figure 2: Expected running time (ERT in $\log_{10}$ of number of function evaluations) of xNES ($x$-axis) versus xNES-as ($y$-axis) for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension on functions $f_{101}$–$f_{130}$. Markers on the upper or right edge indicate that the target value was never reached. Markers represent dimension: 2:+, 3:▽, 5:⋆, 10:○, 20:□, 40:◇.**

5-D

| $\Delta f$ | 1e+1 | 1e-1 | 1e-3 | 1e-5 | 1e-7 | #succ |
|---|---|---|---|---|---|---|
| **f1** | 11 | 12 | 12 | 12 | 12 | 15/15 |
| 1: xNES | 3.0(3) | 21(3) | 50(6) | 81(8) | 110(7) | 15/15 |
| 2: +as | 2.9(2) | 16(5) | **37**(8)$^{\star 2}$ | **60**(12)$^{\star 3}$ | **78**(17)$^{\star 3}$ | 15/15 |
| **f2** | 83 | 88 | 90 | 92 | 94 | 15/15 |
| 1: xNES | 8.7(2) | 12(1) | 16(1) | 20(1) | 23(1) | 15/15 |
| 2: +as | 11(5) | 19(18) | 39(62) | 43(63) | 49(92) | 15/15 |
| **f3** | 716 | 1637 | 1646 | 1650 | 1654 | 15/15 |
| 1: xNES | 3.0(1) | 414(470) | 412(489) | 412(444) | 411(440) | 9/15 |
| 2: +as | 1.5(0.7) | 454(375) | 452(459) | 451(383) | 450(364) | 13/15 |
| **f4** | 809 | 1688 | 1817 | 1886 | 1903 | 15/15 |
| 1: xNES | 4.5(6) | 5871(6803) | 5453(5586) | 5255(5732) | 5208(5693) | 1/15 |
| 2: +as | 3.8(5) | 9998(10379) | 9287(10842) | 8949(9482) | 8868(9550) | 1/15 |
| **f5** | 10 | 10 | 10 | 10 | 10 | 15/15 |
| 1: xNES | 9.5(4) | 15(6) | 16(6) | 16(6) | 16(6) | 15/15 |
| 2: +as | 10(4) | 16(9) | 16(8) | 16(8) | 16(8) | 15/15 |
| **f6** | 114 | 281 | 580 | 1038 | 1332 | 15/15 |
| 1: xNES | 1.6(1) | 2.5(0.3) | 2.2(0.3) | 1.8(0.1) | 1.8(0.1) | 15/15 |
| 2: +as | 1.5(1) | 2.4(0.6) | 2.0(0.2) | **1.5**(0.2)$^{\star}$ | **1.6**(0.2)$^{\star 2}$ | 15/15 |
| **f7** | 24 | 1171 | 1572 | 1572 | 1597 | 15/15 |
| 1: xNES | 4.5(2) | 2.7(4) | 3.0(6) | 3.0(6) | 3.0(6) | 15/15 |
| 2: +as | 4.4(3) | 3.2(4) | 2.6(3) | 2.6(3) | 2.6(3) | 15/15 |
| **f8** | 73 | 336 | 391 | 410 | 422 | 15/15 |
| 1: xNES | 4.1(2) | 5.8(3) | 7.3(4) | 7.7(4) | 8.3(4) | 15/15 |
| 2: +as | 3.4(2) | 8.7(4) | 16(13) | 16(13) | 16(12) | 15/15 |
| **f9** | 35 | 214 | 300 | 335 | 369 | 15/15 |
| 1: xNES | 7.1(2) | 8.7(4) | 8.4(5) | 8.3(4) | 8.4(4) | 15/15 |
| 2: +as | 6.4(2) | 12(3) | 11(6) | 11(6) | 11(6) | 15/15 |
| **f10** | 349 | 574 | 626 | 829 | 880 | 15/15 |
| 1: xNES | 2.1(0.6) | 1.9(0.2) | 2.3(0.2) | 2.2(0.2) | 2.5(0.2) | 15/15 |
| 2: +as | 2.0(0.8) | 1.8(0.6) | 2.0(0.5) | **1.9**(0.4)$^{\star}$ | **2.0**(0.3)$^{\star 2}$ | 15/15 |
| **f11** | 143 | 763 | 1177 | 1467 | 1673 | 15/15 |
| 1: xNES | 4.4(1) | 1.2(0.3) | 1.1(0.2) | 1.1(0.1) | 1.2(0.1) | 15/15 |
| 2: +as | 4.2(3) | 1.3(0.3) | 1.1(0.2) | 1.0(0.1) | 1.1(0.1) | 15/15 |
| **f12** | 108 | 371 | 461 | 1303 | 1494 | 15/15 |
| 1: xNES | 11(3) | 32(42) | 50(106) | 22(39) | 25(35) | 15/15 |
| 2: +as | 16(28) | 36(58) | 51(97) | 21(35) | 31(34) | 15/15 |
| **f13** | 132 | 250 | 1310 | 1752 | 2255 | 15/15 |
| 1: xNES | 3.6(0.8) | 4.7(0.3) | 1.4(0.1) | 1.5(0.1) | 1.5(0.1) | 15/15 |
| 2: +as | 3.3(0.6) | **4.0**(0.5)$^{\star 3}$ | **1.3**(0.2)$^{\star 2}$ | **1.4**(0.2)$^{\star}$ | 1.5(0.2) | 15/15 |
| **f14** | 10 | 58 | 139 | 251 | 476 | 15/15 |
| 1: xNES | 2.3(2) | 4.2(1) | 5.5(0.7) | 5.1(0.7) | 3.9(0.2) | 15/15 |
| 2: +as | 2.0(2) | 3.9(1) | 4.9(1) | 4.6(0.5) | **3.3**(0.3)$^{\star 3}$ | 15/15 |
| **f15** | 511 | 19369 | 20073 | 20769 | 21359 | 14/15 |
| 1: xNES | 1.8(1) | 25(29) | 24(25) | 24(24) | 23(24) | 11/15 |
| 2: +as | 3.6(6) | 18(20) | 18(19) | 17(19) | 16(18) | 14/15 |
| **f16** | 120 | 2662 | 10449 | 11644 | 12095 | 15/15 |
| 1: xNES | 2.0(2) | 4.4(8) | 2.5(3) | 2.3(3) | 2.2(3) | 15/15 |
| 2: +as | 2.3(2) | 1.7(3) | 1.8(2) | 1.6(2) | 1.6(2) | 15/15 |
| **f17** | 5.2 | 899 | 3669 | 6351 | 7934 | 15/15 |
| 1: xNES | 4.5(6) | 0.79(0.2) | 0.47(0.1) | 1.3(1) | 6.1(9) | 15/15 |
| 2: +as | 6.8(7) | 1.1(0.7) | 0.81(0.7) | 1.4(1) | 2.0(3) | 15/15 |
| **f18** | 103 | 3968 | 9280 | 10905 | 12469 | 15/15 |
| 1: xNES | 1.2(1) | 0.43(0.1) | 0.51(0.6) | 2.0(2) | 3.6(4) | 15/15 |
| 2: +as | 0.80(0.5) | 0.25(0.1) | 0.43(0.5)$^{\downarrow}$ | 1.4(0.9) | 2.0(3) | 15/15 |
| **f19** | 1 | 242 | 1.2e5 | 1.2e5 | 1.2e5 | 15/15 |
| 1: xNES | 15(16) | 386(502) | 5.3(5) | 5.3(5) | 5.6(5) | 10/15 |
| 2: +as | 17(18) | 542(792) | 11(10) | 20(23) | 21(22) | 6/15 |
| **f20** | 16 | 38111 | 54470 | 54861 | 55313 | 14/15 |
| 1: xNES | 2.3(2) | 11(10) | 8.0(7) | 8.0(8) | 7.9(7) | 13/15 |
| 2: +as | 3.2(3) | 21(23) | 15(18) | 15(16) | 14(16) | 12/15 |
| **f21** | 41 | 1674 | 1705 | 1729 | 1757 | 14/15 |
| 1: xNES | 34(123) | 32(44) | 32(44) | 31(43) | 31(42) | 15/15 |
| 2: +as | 12(1) | 36(57) | 35(56) | 35(55) | 35(54) | 15/15 |
| **f22** | 71 | 938 | 1008 | 1040 | 1068 | 14/15 |
| 1: xNES | 66(209) | 104(119) | 97(110) | 95(107) | 92(104) | 15/15 |
| 2: +as | 46(61) | 39(42) | 37(39) | 36(37) | 35(37) | 15/15 |
| **f23** | 3.0 | 14249 | 31654 | 33030 | 34256 | 15/15 |
| 1: xNES | 3.4(3) | 755(897) | ∞ | ∞ | ∞ *7.5e5* | 0/15 |
| 2: +as | 2.2(2) | ∞ | ∞ | ∞ | ∞ *1.3e6* | 0/15 |
| **f24** | 1622 | 6.4e6 | 9.6e6 | 1.3e7 | 1.3e7 | 3/15 |
| 1: xNES | 7.5(10) | ∞ | ∞ | ∞ | ∞ *7.9e5* | 0/15 |
| 2: +as | 6.2(8) | ∞ | ∞ | ∞ | ∞ *1.3e6* | 0/15 |

20-D

| $\Delta f$ | 1e+1 | 1e-1 | 1e-3 | 1e-5 | 1e-7 | #succ |
|---|---|---|---|---|---|---|
| **f1** | 43 | 43 | 43 | 43 | 43 | 15/15 |
| 1: xNES | 5.8(3) | 137(6) | 274(5) | 410(7) | 546(9) | 15/15 |
| 2: +as | 7.3(2) | **61**(16)$^{\star 3}$ | **88**(23)$^{\star 3}$ | **110**(25)$^{\star 3}$ | **128**(32)$^{\star 3}$ | 15/15 |
| **f2** | 385 | 387 | 390 | 391 | 393 | 15/15 |
| 1: xNES | 29(0.7) | 43(0.9) | 58(1) | 72(0.8) | 87(1) | 15/15 |
| 2: +as | **26**(1)$^{\star 3}$ | **34**(3)$^{\star 3}$ | **38**(4)$^{\star 3}$ | **41**(6)$^{\star 3}$ | **43**(6)$^{\star 3}$ | 15/15 |
| **f3** | 5066 | 7635 | 7643 | 7646 | 7651 | 15/15 |
| 1: xNES | 629(762) | ∞ | ∞ | ∞ | ∞ *7.0e6* | 0/15 |
| 2: +as | 1055(1437) | ∞ | ∞ | ∞ | ∞ *1.3e7* | 0/15 |
| **f4** | 4722 | 7666 | 7700 | 7758 | 1.4e5 | 9/15 |
| 1: xNES | 6323(6520) | ∞ | ∞ | ∞ | ∞ *6.8e6* | 0/15 |
| 2: +as | 4193(4558) | ∞ | ∞ | ∞ | ∞ *1.2e7* | 0/15 |
| **f5** | 41 | 41 | 41 | 41 | 41 | 15/15 |
| 1: xNES | 10(1) | 12(1) | 12(1) | 12(1) | 12(1) | 15/15 |
| 2: +as | 8.6(1) | 11(2) | 11(2) | 11(2) | 11(2) | 15/15 |
| **f6** | 1296 | 3413 | 5220 | 6728 | 8409 | 15/15 |
| 1: xNES | 4.9(0.3) | 4.7(0.2) | 5.0(0.1) | 5.3(0.1) | 5.4(0.1) | 15/15 |
| 2: +as | 4.8(0.2) | 4.5(0.1) | 4.8(0.1) | 5.2(0.1) | 5.3(0.1) | 15/15 |
| **f7** | 1351 | 9503 | 16524 | 16524 | 16969 | 15/15 |
| 1: xNES | 1.8(0.4) | 1.1(0.0) | 0.94(0.1) | 0.94(0.1) | 0.96(0.1) | 15/15 |
| 2: +as | 1.9(0.2) | 1.0(0.1) | 0.89(0.1) | 0.89(0.1) | 0.91(0.1) | 15/15 |
| **f8** | 2039 | 4040 | 4219 | 4371 | 4484 | 15/15 |
| 1: xNES | 7.5(0.8) | 7.5(1) | 7.7(1) | 8.4(1.0) | 10(0.9) | 15/15 |
| 2: +as | 7.2(0.6) | 9.1(3) | 9.4(4) | 10(4) | 10(4) | 15/15 |
| **f9** | 1716 | 3277 | 3455 | 3594 | 3727 | 15/15 |
| 1: xNES | 8.9(1) | 9.3(2) | 9.4(2) | 10(1) | 11(1) | 15/15 |
| 2: +as | 8.1(1) | 10(2) | 10(2) | 11(2) | 11(2) | 15/15 |
| **f10** | 7413 | 10735 | 14920 | 17073 | 17476 | 15/15 |
| 1: xNES | 1.5(0.0) | 1.6(0.0) | 1.5(0.0) | 1.7(0.0) | 2.0(0.0) | 15/15 |
| 2: +as | **1.3**(0.1)$^{\star 3}$ | **1.3**(0.1)$^{\star 3}$ | **1.0**(0.1)$^{\star 3}$ | **0.99**(0.1)$^{\star 3}$ | **1.0**(0.1)$^{\star 3}$ | 15/15 |
| **f11** | 1002 | 6278 | 9762 | 12285 | 14831 | 15/15 |
| 1: xNES | 4.9(0.3) | 1.6(0.1) | 1.6(0.0) | 1.8(0.0) | 1.9(0.0) | 15/15 |
| 2: +as | 4.8(0.3) | **1.4**(0.1)$^{\star 3}$ | **1.1**(0.2)$^{\star 3}$ | **1.00**(0.2)$^{\star 3}$ | **0.91**(0.2)$^{\star 3}$ | 15/15 |
| **f12** | 1042 | 2740 | 4140 | 12407 | 13827 | 15/15 |
| 1: xNES | 16(0.5) | 9.3(1) | 8.3(0.8) | 3.5(0.2) | 3.6(0.2) | 15/15 |
| 2: +as | **6.6**(4)$^{\star 3}$ | 18(18) | 35(38) | 21(21) | 22(21) | 15/15 |
| **f13** | 652 | 2751 | 18749 | 24455 | 30201 | 15/15 |
| 1: xNES | 16(0.5) | 7.9(0.2) | 1.8(0.0) | **1.8**(0.0)$^{\star}$ | **1.9**(0.0)$^{\star 3}$ | 15/15 |
| 2: +as | **7.0**(3)$^{\star 3}$ | 19(28) | 17(21) | 40(33) | 81(83) | 0/15 |
| **f14** | 75 | 304 | 932 | 1648 | 15661 | 15/15 |
| 1: xNES | 2.4(1.0) | 19(0.7) | 14(0.4) | 12(0.3) | 1.7(0.0) | 15/15 |
| 2: +as | 2.6(0.8) | **14**(3)$^{\star 3}$ | **12**(1)$^{\star 3}$ | **10**(1)$^{\star 3}$ | **1.5**(0.1)$^{\star 3}$ | 15/15 |
| **f15** | 30378 | 3.1e5 | 3.2e5 | 4.5e5 | 4.6e5 | 15/15 |
| 1: xNES | 43(41) | ∞ | ∞ | ∞ | ∞ *6.9e6* | 0/15 |
| 2: +as | 44(52) | ∞ | ∞ | ∞ | ∞ *1.6e7* | 0/15 |
| **f16** | 1384 | 77015 | 1.9e5 | 2.0e5 | 2.2e5 | 15/15 |
| 1: xNES | 17(8) | 15(13) | 48(58) | 56(61) | 50(55) | 7/15 |
| 2: +as | 20(10) | 9.2(7) | 108(134) | 133(153) | 119(139) | 6/15 |
| **f17** | 63 | 4005 | 30677 | 56288 | 80472 | 15/15 |
| 1: xNES | 1.9(1) | 3.0(0.1) | 0.94(0.0) | 2.7(3) | 12(14) | 15/15 |
| 2: +as | 2.1(1.0) | 2.9(0.2) | 0.92(0.0) | 1.4(0.7) | 12(9) | 15/15 |
| **f18** | 621 | 19561 | 67569 | 1.3e5 | 1.5e5 | 15/15 |
| 1: xNES | 1.3(0.6) | 0.84(0.0)$^{\downarrow}$ | 0.50(0.0)$^{\downarrow 3}$ | 1.3(2) | 11(14) | 15/15 |
| 2: +as | 1.2(0.5) | 0.81(0.0)$^{\downarrow 3}$ | 0.48(0.0)$^{\downarrow 4}$ | 0.58(0.3)$^{\downarrow 2}$ | 5.6(6) | 15/15 |
| **f19** | 1 | 3.4e5 | 6.2e6 | 6.7e6 | 6.7e6 | 15/15 |
| 1: xNES | 105(59) | ∞ | ∞ | ∞ | ∞ *6.2e6* | 0/15 |
| 2: +as | 108(35) | ∞ | ∞ | ∞ | ∞ *1.2e7* | 0/15 |
| **f20** | 82 | 3.1e6 | 5.5e6 | 5.6e6 | 5.6e6 | 14/15 |
| 1: xNES | 5.6(2) | ∞ | ∞ | ∞ | ∞ *6.4e6* | 0/15 |
| 2: +as | 5.4(3) | ∞ | ∞ | ∞ | ∞ *1.2e7* | 0/15 |
| **f21** | 561 | 14103 | 14643 | 15567 | 17589 | 15/15 |
| 1: xNES | 142(281) | 54(75) | 53(72) | 50(68) | 44(60) | 15/15 |
| 2: +as | 45(71) | 37(44) | 35(55) | 33(51) | 30(45) | 30/30 |
| **f22** | 467 | 23491 | 24948 | 26847 | 1.3e5 | 12/15 |
| 1: xNES | 104(171) | 168(180) | 158(170) | 147(150) | 29(31) | 12/15 |
| 2: +as | 50(86) | 226(236) | 213(247) | 198(225) | 39(45) | 22/30 |
| **f23** | 3.2 | 67457 | 4.9e5 | 8.1e5 | 8.4e5 | 15/15 |
| 1: xNES | 2.0(2) | ∞ | ∞ | ∞ | ∞ *6.0e6* | 0/15 |
| 2: +as | 1.5(2) | ∞ | ∞ | ∞ | ∞ *1.1e7* | 0/30 |
| **f24** | 1.3e6 | 5.2e7 | 5.2e7 | 5.2e7 | 5.2e7 | 3/15 |
| 1: xNES | ∞ | ∞ | ∞ | ∞ | ∞ *6.3e6* | 0/15 |
| 2: +as | ∞ | ∞ | ∞ | ∞ | ∞ *1.2e7* | 0/30 |

**Table 2:** ERT **in number of function evaluations divided by the best** ERT **measured during BBOB-2009 given in the respective first row with the central 80% range divided by two in brackets for different** $\Delta f$ **values. #succ is the number of trials that reached the final target** $f_{\mathrm{opt}} + 10^{-8}$**. 1:xNES is xNES and 2:+as is xNES-as. Bold entries are statistically significantly better compared to the other algorithm, with** $p = 0.05$ **or** $p = 10^{-k}$ **where** $k \in \{2, 3, 4, \dots\}$ **is the number following the** $\star$ **symbol, with Bonferroni correction of 48. A** $\downarrow$ **indicates the same tested against the best BBOB-2009.**

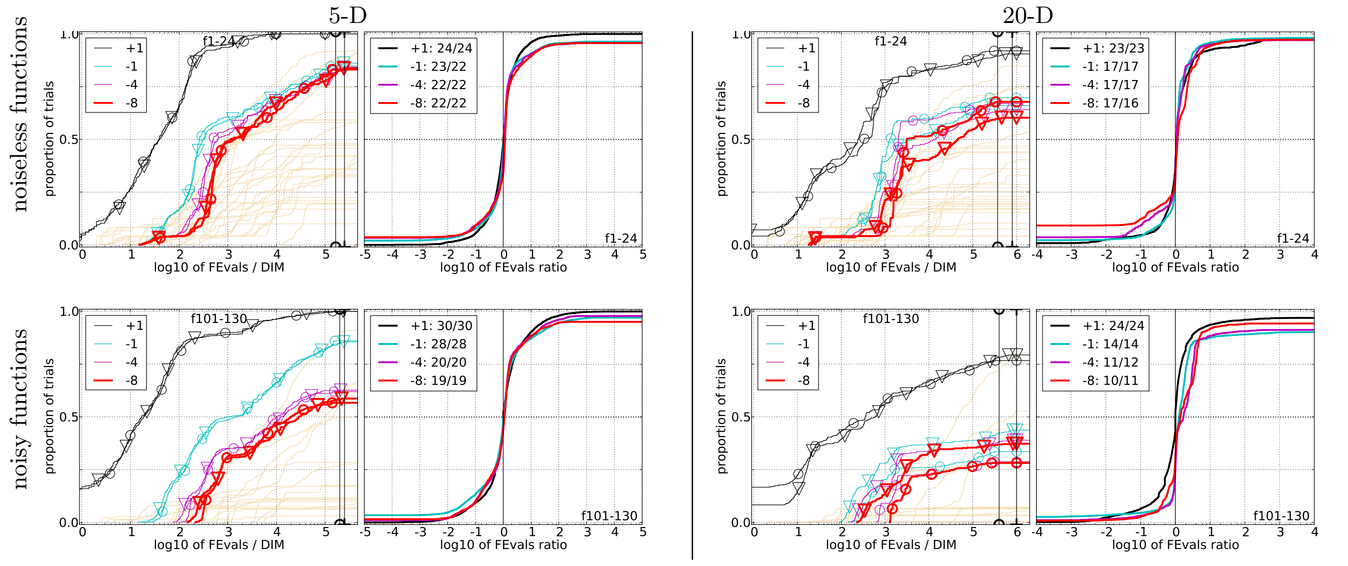

**Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension $D$ (FEvals/D) to reach a target value $f_{\rm opt}+\Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for xNES (○) and xNES-as (▽). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of all algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of xNES divided by xNES-as, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being $> 0$ or $< 1$. The legends indicate the number of functions that were solved in at least one trial (xNES first).**

5-D

| $\Delta f$ | 1e+1 | 1e-1 | 1e-3 | 1e-5 | 1e-7 | #succ |
|---|---|---|---|---|---|---|
| **f$_{101}$** | 11 | 44 | 62 | 69 | 75 | 15/15 |
| 1: xNES | 3.4(1) | 5.4(0.9) | 10(1) | 14(1) | 18(0.6) | 15/15 |
| 2: +as | 2.8(2) | 5.2(2) | **7.9**(2)$^{\star}$ | **10**(2)$^{\star 3}$ | **13**(3)$^{\star 4}$ | 15/15 |
| **f$_{102}$** | 11 | 50 | 72 | 86 | 99 | 15/15 |
| 1: xNES | 3.1(4) | 4.9(1) | 8.8(1) | 12(1) | 14(1) | 15/15 |
| 2: +as | 3.3(3) | 4.7(1) | **7.3**(1)$^{\star 2}$ | **9.2**(1)$^{\star 3}$ | **10**(2)$^{\star 4}$ | 15/15 |
| **f$_{103}$** | 11 | 30 | 31 | 35 | 115 | 15/15 |
| 1: xNES | 3.7(2) | 8.2(2) | 20(3) | 31(3) | 13(1) | 15/15 |
| 2: +as | 2.7(2) | 7.2(2) | **16**(3)$^{\star 2}$ | **24**(5)$^{\star 2}$ | **11**(2)$^{\star 2}$ | 15/15 |
| **f$_{104}$** | 173 | 1287 | 1768 | 2040 | 2284 | 15/15 |
| 1: xNES | 1.4(0.7) | 5.6(10) | 7.3(11) | 6.4(10) | 5.9(9) | 15/15 |
| 2: +as | 1.4(0.4) | 2.7(6) | 2.3(4) | 2.1(4) | 2.0(3) | 15/15 |
| **f$_{105}$** | 167 | 5174 | 10388 | 10824 | 11202 | 15/15 |
| 1: xNES | 1.3(0.5) | 4.9(7) | 3.6(5) | 4.0(5) | 4.0(5) | 15/15 |
| 2: +as | 1.4(0.5) | 14(13) | 11(7) | 11(7) | 10(7) | 15/15 |
| **f$_{106}$** | 92 | 1050 | 2666 | 2887 | 3087 | 15/15 |
| 1: xNES | 2.9(1.0) | 2.2(0.8) | 1.2(0.9) | 1.2(0.9) | 1.3(1) | 15/15 |
| 2: +as | 2.7(0.8) | 3.7(5) | 2.2(3) | 2.2(3) | 2.7(3) | 15/15 |
| **f$_{107}$** | 40 | 453 | 940 | 1376 | 1850 | 15/15 |
| 1: xNES | 3.2(3) | 20(33) | 10(16) | 7.6(11) | 45(8) | 14/15 |
| 2: +as | 3.2(5) | 4.2(0.5) | 6.9(11) | 7.2(8) | 7.0(10) | 15/15 |
| **f$_{108}$** | 87 | 14469 | 30935 | 58628 | 80667 | 15/15 |
| 1: xNES | 84(117) | 36(27) | ∞ | ∞ | ∞*7.5e5* | 0/15 |
| 2: +as | 69(120) | 56(54) | ∞ | ∞ | ∞*1.3e6* | 0/15 |
| **f$_{109}$** | 11 | 216 | 572 | 873 | 946 | 15/15 |
| 1: xNES | 3.4(2) | 1.4(0.3) | 2.0(0.5) | 2.3(0.2) | 4.1(0.7) | 15/15 |
| 2: +as | 3.6(2) | 1.3(0.5) | 1.8(0.5) | 2.3(0.6) | 4.1(0.8) | 15/15 |
| **f$_{110}$** | 949 | 1.2e5 | 5.9e5 | 6.0e5 | 6.1e5 | 15/15 |
| 1: xNES | 0.49(0.2) | 2.1(3) | 1.7(2) | 2.2(2) | 2.2(2) | 7/15 |
| 2: +as | 0.44(0.2) | 3.6(5) | 1.2(1) | 1.6(2) | 2.3(3) | 8/15 |
| **f$_{111}$** | 6856 | 8.8e6 | 2.3e7 | 3.1e7 | 3.1e7 | 3/15 |
| 1: xNES | 7.2(9) | 0.61(0.7) | ∞ | ∞ | ∞*7.8e5* | 0/15 |
| 2: +as | 5.4(8) | 0.96(1) | ∞ | ∞ | ∞*1.3e6* | 0/15 |
| **f$_{112}$** | 107 | 3421 | 4502 | 5132 | 5596 | 15/15 |
| 1: xNES | 4.6(1) | 3.0(3) | 6.5(8) | 12(15) | 18(28) | 15/15 |
| 2: +as | 2.3(1) | 7.9(10) | 17(18) | 28(23) | 37(38) | 15/15 |
| **f$_{113}$** | 133 | 8081 | 24128 | 24128 | 24402 | 15/15 |
| 1: xNES | 10(6) | 2.1(3) | 1.2(2) | 1.2(2) | 1.2(2) | 15/15 |
| 2: +as | 1.2(1) | 2.5(4) | 1.4(2) | 1.4(2) | 1.5(2) | 15/15 |
| **f$_{114}$** | 767 | 56311 | 83272 | 83272 | 84949 | 15/15 |
| 1: xNES | 10(17) | 33(39) | ∞ | ∞ | ∞*7.5e5* | 0/15 |
| 2: +as | 8.0(13) | 157(161) | ∞ | ∞ | ∞*1.3e6* | 0/15 |
| **f$_{115}$** | 64 | 1829 | 2550 | 2550 | 2970 | 15/15 |
| 1: xNES | 1.7(1) | 2.3(5) | 2.5(5) | 2.5(5) | 2.6(4) | 15/15 |
| 2: +as | 1.3(0.9) | 1.8(4) | 1.8(4) | 1.8(4) | 1.6(3) | 15/15 |

20-D

| $\Delta f$ | 1e+1 | 1e-1 | 1e-3 | 1e-5 | 1e-7 | #succ |
|---|---|---|---|---|---|---|
| **f$_{101}$** | 59 | 571 | 700 | 739 | 783 | 15/15 |
| 1: xNES | 5.5(2) | 10(0.7) | 17(0.6) | 24(0.4) | 30(0.6) | 15/15 |
| 2: +as | 4.8(2) | **5.7**(2)$^{\star 5}$ | **6.7**(2)$^{\star 5}$ | **7.7**(3)$^{\star 5}$ | **8.4**(3)$^{\star 5}$ | 30/30 |
| **f$_{102}$** | 231 | 579 | 921 | 1157 | 1407 | 15/15 |
| 1: xNES | 1.5(0.7) | 10(0.5) | 13(0.3) | 15(0.2) | 17(0.4) | 15/15 |
| 2: +as | 1.5(0.4) | **6.2**(2)$^{\star 5}$ | **5.6**(2)$^{\star 5}$ | **5.4**(2)$^{\star 5}$ | **5.4**(2)$^{\star 5}$ | 30/30 |
| **f$_{103}$** | 65 | 629 | 1313 | 1893 | 2464 | 14/15 |
| 1: xNES | 6.2(2) | 10(0.4) | 9.4(0.3) | 10(0.4) | 11(0.3) | 15/15 |
| 2: +as | 5.0(2) | **4.9**(2)$^{\star 5}$ | **4.1**(2)$^{\star 5}$ | **5.5**(2)$^{\star 5}$ | **6.8**(2)$^{\star 5}$ | 30/30 |
| **f$_{104}$** | 23690 | 1.7e5 | 1.8e5 | 1.9e5 | 2.0e5 | 15/15 |
| 1: xNES | 8.1(10) | ∞$^{\star}$ | ∞$^{\star}$ | ∞$^{\star}$ | ∞***8.0e6***$^{\star}$ | 0/15 |
| 2: +as | 24(29) | ∞ | ∞ | ∞ | ∞*1.8e7* | 0/30 |
| **f$_{105}$** | 1.9e5 | 6.3e5 | 6.5e5 | 6.6e5 | 6.7e5 | 15/15 |
| 1: xNES | 17(16) | ∞ | ∞ | ∞ | ∞*7.4e6* | 0/15 |
| 2: +as | 24(22) | ∞ | ∞ | ∞ | ∞*1.6e7* | 0/21 |
| **f$_{106}$** | 11480 | 23746 | 25470 | 26492 | 27360 | 15/15 |
| 1: xNES | 1.3(0.1) | 1.3(0.1) | 1.3(0.1) | 1.5(0.1) | 1.7(0.1) | 15/15 |
| 2: +as | **1.2**(0.2)$^{\star 2}$ | 1.5(0.4) | 1.7(0.9) | 1.8(0.7) | 2.0(0.7) | 30/30 |
| **f$_{107}$** | 8571 | 16226 | 27357 | 52486 | 65052 | 15/15 |
| 1: xNES | 1.7(0.4) | 879(889) | ∞ | ∞ | ∞*6.9e6* | 0/15 |
| 2: +as | 1.6(0.9) | 786(927) | ∞ | ∞ | ∞*1.5e7* | 0/15 |
| **f$_{108}$** | 58063 | 2.0e5 | 4.5e5 | 6.3e5 | 9.0e5 | 15/15 |
| 1: xNES | ∞ | ∞ | ∞ | ∞ | ∞*5.9e6* | 0/15 |
| 2: +as | ∞ | ∞ | ∞ | ∞ | ∞*1.1e7* | 0/15 |
| **f$_{109}$** | 333 | 1138 | 2287 | 3583 | 4952 | 15/15 |
| 1: xNES | 0.94(0.3) | 7.7(0.4) | 10(0.5) | 10(0.5) | 10(0.3) | 15/15 |
| 2: +as | 1.0(0.2) | 6.9(0.8) | 9.4(0.7) | 10(0.5) | 13(8) | 15/15 |
| **f$_{110}$** | ∞ | ∞ | ∞ | ∞ | ∞ | 0 |
| 1: xNES | ∞ | ∞ | ∞ | ∞ | ∞ | 0/15 |
| 2: +as | ∞ | ∞ | ∞ | ∞ | ∞ | 0/15 |
| **f$_{111}$** | ∞ | ∞ | ∞ | ∞ | ∞ | 0 |
| 1: xNES | ∞ | ∞ | ∞ | ∞ | ∞ | 0/15 |
| 2: +as | ∞ | ∞ | ∞ | ∞ | ∞ | 0/15 |
| **f$_{112}$** | 25552 | 69621 | 73557 | 76137 | 78238 | 15/15 |
| 1: xNES | 0.93(0.2) | 1454(1717) | ∞ | ∞ | ∞*7.0e6* | 0/15 |
| 2: +as | 0.99(0.2) | 952(988) | 3878(4223) | 3747(3922) | 3646(4035) | 1/15 |
| **f$_{113}$** | 50123 | 5.6e5 | 5.9e5 | 5.9e5 | 5.9e5 | 15/15 |
| 1: xNES | 5.0(5) | ∞ | ∞ | ∞ | ∞*6.8e6* | 0/15 |
| 2: +as | 1.4(1) | ∞ | ∞ | ∞ | ∞*1.5e7* | 0/15 |
| **f$_{114}$** | 2.1e5 | 1.4e6 | 1.6e6 | 1.6e6 | 1.6e6 | 15/15 |
| 1: xNES | ∞ | ∞ | ∞ | ∞ | ∞*5.8e6* | 0/15 |
| 2: +as | ∞ | ∞ | ∞ | ∞ | ∞*1.0e7* | 0/15 |
| **f$_{115}$** | 2405 | 91749 | 1.3e5 | 1.3e5 | 1.3e5 | 15/15 |
| 1: xNES | 1.0(0.2) | 0.14(0.0)$^{\downarrow}$ | 0.62(0.8) | 0.62(0.8) | 0.69(0.8) | 15/15 |
| 2: +as | 1.0(0.1) | 0.14(0.0)$^{\downarrow}$ | 0.37(0.3)$^{\downarrow 3}$ | 0.37(0.3)$^{\downarrow 3}$ | 0.46(0.5)$^{\downarrow 2}$ | 15/15 |

**Table 3: Relative** ERT **in number of $f$-evaluations, see Table 2 for details.**

5-D

| $\Delta f$ | 1e+1 | 1e-1 | 1e-3 | 1e-5 | 1e-7 | #succ |
|---|---|---|---|---|---|---|
| $\mathbf{f_{116}}$ | 5730 | 22311 | 26868 | 30329 | 31661 | 15/15 |
| 1: xNES | 0.62(1) | 1.3(2) | 1.2(1) | 1.2(1) | 1.2(1) | 14/15 |
| 2: +as | 1.4(2) | 1.1(2) | 1.3(1) | 1.6(2) | 1.8(2) | 15/15 |
| $\mathbf{f_{117}}$ | 26686 | 1.1e5 | 1.4e5 | 1.7e5 | 1.9e5 | 15/15 |
| 1: xNES | 9.4(13) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *7.6e5* | 0/15 |
| 2: +as | 8.4(6) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *1.3e6* | 0/15 |
| $\mathbf{f_{118}}$ | 429 | 1555 | 1998 | 2430 | 2913 | 15/15 |
| 1: xNES | 1.0(0.4) | 0.52(0.1)$^{\downarrow 4}$ | 0.73(0.1)$^{\downarrow 2}$ | 1.0(0.2) | 1.6(0.2) | 15/15 |
| 2: +as | 1.00(0.4) | 0.48(0.1)$^{\downarrow 4}$ | 0.69(0.2)$^{\downarrow 2}$ | 0.96(0.2) | 1.1(0.2) | 15/15 |
| $\mathbf{f_{119}}$ | 12 | 1136 | 10372 | 35296 | 49747 | 15/15 |
| 1: xNES | 1.9(2) | 0.66(0.2) | 2.6(3) | 1.5(1) | 12(13) | 9/15 |
| 2: +as | 3.8(4) | 6.3(13) | 1.6(2) | 2.3(3) | 5.9(6) | 15/15 |
| $\mathbf{f_{120}}$ | 16 | 18698 | 72438 | 3.3e5 | 5.5e5 | 15/15 |
| 1: xNES | 9.0(12) | 43(39) | $\infty$ | $\infty$ | $\infty$ *7.6e5* | 0/15 |
| 2: +as | 51(18) | 25(33) | $\infty$ | $\infty$ | $\infty$ *1.3e6* | 0/15 |
| $\mathbf{f_{121}}$ | 8.6 | 273 | 1583 | 3870 | 6195 | 15/15 |
| 1: xNES | 2.7(3) | 1.2(0.2) | 1.3(0.5) | 1.2(1) | 1.7(2) | 15/15 |
| 2: +as | 2.6(2) | 0.96(0.5) | 1.6(0.8) | 1.5(2) | 2.3(3) | 15/15 |
| $\mathbf{f_{122}}$ | 10 | 9190 | 30087 | 53743 | 1.1e5 | 15/15 |
| 1: xNES | 1.7(2) | 6.4(4) | 200(215) | $\infty$ | $\infty$ *8.5e5* | 0/15 |
| 2: +as | 5.0(4) | 8.9(8) | 583(686) | $\infty$ | $\infty$ *1.3e6* | 0/15 |
| $\mathbf{f_{123}}$ | 11 | 81505 | 3.4e5 | 6.7e5 | 2.2e6 | 15/15 |
| 1: xNES | 36(70) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *7.5e5* | 0/15 |
| 2: +as | 6.6(9) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *1.3e6* | 0/15 |
| $\mathbf{f_{124}}$ | 10 | 1040 | 20478 | 45337 | 95200 | 15/15 |
| 1: xNES | 2.5(2) | 3.8(10) | 2.0(2) | 19(20) | 112(127) | 1/15 |
| 2: +as | 2.9(3) | 2.0(0.7) | 1.1(1) | 36(40) | 60(66) | 1/15 |
| $\mathbf{f_{125}}$ | 1 | 1 | 2.4e5 | 2.4e5 | 2.5e5 | 15/15 |
| 1: xNES | 1.1 | 9476(14165) | $\infty$ | $\infty$ | $\infty$ *7.6e5* | 0/15 |
| 2: +as | 1.3(0.5) | 7786(7916) | $\infty$ | $\infty$ | $\infty$ *1.3e6* | 0/15 |
| $\mathbf{f_{126}}$ | 1 | 1 | $\infty$ | $\infty$ | $\infty$ | 0 |
| 1: xNES | 1.1 | 44499(50671) | $\infty$ | $\infty$ | $\infty$ | 0/15 |
| 2: +as | 1.1(0.5) | 41598(46392) | $\infty$ | $\infty$ | $\infty$ | 0/15 |
| $\mathbf{f_{127}}$ | 1 | 1 | 3.4e5 | 3.9e5 | 4.0e5 | 15/15 |
| 1: xNES | 1.1 | 3395(5420) | 31(36) | $\infty$ | $\infty$ *7.4e5* | 0/15 |
| 2: +as | 1.1(0.5) | 3858(5345) | $\infty$ | $\infty$ | $\infty$ *1.3e6* | 0/15 |
| $\mathbf{f_{128}}$ | 111 | 7808 | 12447 | 17217 | 21162 | 15/15 |
| 1: xNES | 7.1(2) | 5.2(6) | 3.3(4) | 2.4(3) | 2.0(2) | 15/15 |
| 2: +as | 22(67) | 5.7(7) | 3.6(4) | 2.6(3) | 2.1(2) | 15/15 |
| $\mathbf{f_{129}}$ | 64 | 59443 | 2.8e5 | 5.1e5 | 5.8e5 | 15/15 |
| 1: xNES | 38(34) | 10(8) | 11(13) | $\infty$ | $\infty$ *7.5e5* | 0/15 |
| 2: +as | 17(16) | 8.6(12) | 14(16) | $\infty$ | $\infty$ *1.3e6* | 0/15 |
| $\mathbf{f_{130}}$ | 55 | 3034 | 32823 | 33889 | 34528 | 10/15 |
| 1: xNES | 13(1) | 25(25) | 2.4(2) | 2.3(2) | 2.3(2) | 15/15 |
| 2: +as | 17(0.9) | 28(48) | 2.6(4) | 2.5(4) | 2.5(4) | 15/15 |

20-D

| $\Delta f$ | 1e+1 | 1e-1 | 1e-3 | 1e-5 | 1e-7 | #succ |
|---|---|---|---|---|---|---|
| $\mathbf{f_{116}}$ | 5.0e5 | 8.9e5 | 1.0e6 | 1.1e6 | 1.1e6 | 15/15 |
| 1: xNES | 101(110) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *7.1e6* | 0/15 |
| 2: +as | 39(42) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *1.6e7* | 0/15 |
| $\mathbf{f_{117}}$ | 1.8e6 | 2.6e6 | 2.9e6 | 3.2e6 | 3.6e6 | 15/15 |
| 1: xNES | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ *5.1e6* | 0/15 |
| 2: +as | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ *8.1e6* | 0/30 |
| $\mathbf{f_{118}}$ | 6908 | 17514 | 26342 | 30062 | 32659 | 15/15 |
| 1: xNES | 1.0(0.0) | 0.90(0.0) | 1.1(0.1) | 1.5(0.1) | 1.8(0.0) | 15/15 |
| 2: +as | **0.94**(0.1)$^{\star 3}$ | **0.80**(0.1)$^{\star 3}$ | 1.1(0.1) | 1.7(0.7) | 2.1(0.7) | 15/15 |
| $\mathbf{f_{119}}$ | 2771 | 35930 | 4.1e5 | 1.4e6 | 1.9e6 | 15/15 |
| 1: xNES | 0.54(0.3)$^{\downarrow 2}$ | 2741(2972) | $\infty$ | $\infty$ | $\infty$ *6.7e6* | 0/15 |
| 2: +as | 0.53(0.4)$^{\downarrow}$ | 851(771) | $\infty$ | $\infty$ | $\infty$ *1.4e7* | 0/13 |
| $\mathbf{f_{120}}$ | 36040 | 2.8e5 | 1.6e6 | 6.7e6 | 1.4e7 | 13/15 |
| 1: xNES | 14(23) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *6.0e6* | 0/15 |
| 2: +as | 6.9(7) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *1.1e7* | 0/15 |
| $\mathbf{f_{121}}$ | 249 | 1426 | 9304 | 34434 | 57404 | 15/15 |
| 1: xNES | 0.74(0.2) | 6.7(0.4) | 2.9(0.2) | 1.3(0.1) | 1.1(0.1) | 15/15 |
| 2: +as | 0.81(0.3) | 6.3(0.5) | 2.9(0.2) | 1.7(0.6) | 1.7(1) | 15/15 |
| $\mathbf{f_{122}}$ | 692 | 1.4e5 | 7.9e5 | 2.0e6 | 5.8e6 | 15/15 |
| 1: xNES | 0.91(0.8) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *6.2e6* | 0/15 |
| 2: +as | 0.81(0.9) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *1.2e7* | 0/13 |
| $\mathbf{f_{123}}$ | 1063 | 1.5e6 | 5.3e6 | 2.7e7 | 1.6e8 | 0 |
| 1: xNES | 9.0(9) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *6.1e6* | 0/15 |
| 2: +as | 11(14) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *1.1e7* | 0/15 |
| $\mathbf{f_{124}}$ | 192 | 40840 | 1.3e5 | 3.9e5 | 8.0e5 | 15/15 |
| 1: xNES | 0.63(0.3) | 0.58(0.1) | 6.4(7) | $\infty$ | $\infty$ *6.9e6* | 0/15 |
| 2: +as | 0.84(0.3) | 0.59(0.1) | 4.0(3) | $\infty$ | $\infty$ *1.8e7* | 0/15 |
| $\mathbf{f_{125}}$ | 1 | 1 | 2.5e7 | 8.0e7 | 8.1e7 | 4/15 |
| 1: xNES | 1.5(1) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *6.1e6* | 0/30 |
| 2: +as | 1.7(1) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *1.1e7* | 0/15 |
| $\mathbf{f_{126}}$ | 1 | 1 | $\infty$ | $\infty$ | $\infty$ | 0 |
| 1: xNES | 1.2(0.5) | $\infty$ | $\infty$ | $\infty$ | $\infty$ | 0/30 |
| 2: +as | 1.4(1) | $\infty$ | $\infty$ | $\infty$ | $\infty$ | 0/15 |
| $\mathbf{f_{127}}$ | 1 | 1 | 4.4e6 | 7.3e6 | 7.4e6 | 15/15 |
| 1: xNES | 1.4(1) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *6.1e6* | 0/30 |
| 2: +as | 1.5(0.5) | $\infty$ | $\infty$ | $\infty$ | $\infty$ *1.1e7* | 0/15 |
| $\mathbf{f_{128}}$ | 1.4e5 | 1.7e7 | 1.7e7 | 1.7e7 | 1.7e7 | 9/15 |
| 1: xNES | 28(32) | 1.2(1) | 2.0(2) | 3.4(4) | 3.4(4) | 3/30 |
| 2: +as | 19(23) | 0.78(0.9) | 1.0(1) | 1.0(1.0) | 1.3(1) | 6/15 |
| $\mathbf{f_{129}}$ | 7.8e6 | 4.2e7 | 4.2e7 | 4.2e7 | 4.2e7 | 5/15 |
| 1: xNES | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ *6.0e6* | 0/30 |
| 2: +as | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ *1.1e7* | 0/15 |
| $\mathbf{f_{130}}$ | 4904 | 2.5e5 | 2.5e5 | 2.6e5 | 2.6e5 | 7/15 |
| 1: xNES | 14(20) | 4.9(6) | 4.9(5) | 4.9(5) | 4.9(5) | 29/30 |
| 2: +as | 15(23) | 5.4(9) | 5.4(9) | 5.4(9) | 5.4(9) | 15/15 |

**Table 4: Relative** ERT **in number of** $f$**-evaluations, see Table 2 for details.**

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