

# A Surrogate Multiobjective Evolutionary Strategy with Local Search and Pre-Selection

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## ABSTRACT

In this paper we present an evolutionary strategy for multi-objective optimization. This evolution strategy is based on a surrogate memetic operator and a surrogate preselection model which provides several individuals in each generation. Thus, the optimization may be easily parallelized. The proposed algorithm is compared to some of existing evolutionary algorithms from the literature.

## Categories and Subject Descriptors

I.2.8 [Artificial Intelligence]: Problem Solving, Control Methods, and Search—*heuristic methods*; I.6.3 [Simulation and Modeling]: Applications

## General Terms

Algorithms

## Keywords

Multiobjective optimization, meta-model, evolutionary algorithm

## 1. INTRODUCTION

Recently, we proposed a surrogate based evolution strategy SBMO-ES [2], and showed that it greatly reduces the number of objective function evaluations needed to reach a specified quality of the solution expressed in terms of the hypervolume. Its main disadvantage is that it is an  $(\lambda + 1)$  evolution strategy, and thus it provides only one individual for evaluation in each generation. Therefore, it is difficult to parallelize. In this paper we propose an algorithm based on SBMO-ES which is able to provide more individuals per generation while preserving the original algorithm's level of performance.

## 2. ALGORITHM DESCRIPTION

The proposed algorithm is based on SBMO-ES [2]. It uses the same distance based aggregate meta-model to assess the quality of individuals and during the local search (a different model is used during pre-selection). The distance to the current non-dominated front in the archive is modeled. The archive of evaluated individuals – which is used for the meta-model training – is updated during the run of the algorithm.

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In each generation the algorithm first generates new individuals using the SBX crossover and polynomial mutation and then it creates a global meta-model. A quarter of the best individuals (according to the global meta-model) is improved using the local search described in the next paragraph. Then, a meta-model for each of the objectives is constructed, and pre-selection is performed (see below). The pre-selected individuals are then evaluated, and a selection mechanism is used (we use NSGA-II or  $\epsilon$ -IBEA selection) to select the new parents.

In the local search phase we use another evolutionary algorithm to find better points in the surroundings of each individual. The algorithm runs only for a few generations, and it uses only the meta-model evaluations. The newly found individuals are placed back to the population. During the initialization, the individual which should be optimized is inserted into the initial population, and its variables are perturbed to create the rest of the population.

In the pre-selection phase, a model for each of the objectives is trained (it is trained to predict the objective values directly), and only those offspring which are not dominated by any of the parents are selected for the next generation. These are then evaluated and added to the archive.

We have also experimented with other pre-selection options (such as using clustering to partition the offspring set and selecting the best individual from each cluster) but these did not provide such good results.

## 3. TEST SETUP

We tested our approach on the widely used ZDT [3] benchmark problems. These problems are all two dimensional, and we used 15 variables for each of them, except ZDT1 where 30 variables were used. In the local search phase we use linear regression based models, and a support vector regression model is used for pre-selection. All the models use default parameters from the Weka framework (which we used to run the experiments). We also tested other configurations of local search and pre-selection models, however these results are not presented here due to space limitations.

The main multiobjective algorithm (NSGA-II [1], or  $\epsilon$ -IBEA) used a population of 50 individuals and stopped after 50,000 objective function evaluations. SBX crossover (probability 0.8) and polynomial mutation (probability 0.1) were used. During the memetic search we used an evolutionary algorithm which run with 50 individuals in the population for 30 generations, again SBX crossover and polynomial mutation were used.

To compare the results we use a measure we call  $H_{ratio}$ , it

is defined as the ratio of the hypervolume of the dominated space attained by the algorithm to the hypervolume of the global Pareto front.

## 4. RESULTS

Table 1 shows the results of our algorithm compared to original NSGA-II [1], and  $\epsilon$ -IBEA [4], and SBMO-ES [2]. In the table NSGA means the original NSGA-II and IBEA is the  $\epsilon$ -IBEA. "ES" denotes the SMBO-ES. The "LR" stands for the memetic meta-model used – linear regression, and "SVM" stands for the pre-selection model used – support vector regression. The results of the new algorithm are thus suffixed with "LR-SVM". The pre-selection meta-model uses support vector regression in all cases.

The numbers in the table represent the median number of objective function evaluations needed to reach the specified  $H_{ratio}$  value. Twenty runs for each configuration were made. A “-” symbol means that the particular configuration was not able to attain the specified  $H_{ratio}$ .

The results show that in most cases the new algorithm works at least as well as SBMO-ES, which was our goal. However, there is one exception: on ZDT6 with NSGA-II selection the algorithm suffered greatly with premature convergence and was unable to attain the  $H_{ratio} = 0.95$ . On the same test problem, but with  $\epsilon$ -IBEA selection, the performance was similar to the one of SBMO-ES.

On ZDT1 with  $\epsilon$ -IBEA selection an interesting behavior can be observed. The new algorithm uses almost the same number of evaluation as SBMO-ES to reach the  $H_{ratio} = 0.99$ , however for the other values of  $H_{ratio}$  the performance is very different. Where SBMO-ES needs a lot of evaluations to get from  $H_{ratio} = 0.95$  to  $H_{ratio} = 0.99$ , the new algorithm needs a lot of evaluations to improve from  $H_{ratio} = 0.5$  to  $H_{ratio} = 0.75$ . As such behavior was observed only in this single test instance, it is difficult to make any generalizations, however there might be a local optima between the two values of  $H_{ratio}$  which SMBO-ES can escape easily, while the new algorithm can get stuck in it for a much greater number of generations. This aspect of the algorithm requires further research (e.g. the convergence rate could be monitored to choose a proper selection during the optimization run).

In the remaining cases, i.e., on ZDT2 and ZDT3 test problems the new algorithm outperforms the SBMO-ES in all configurations and the number of required objective evaluations is decreased almost to a half.

## 5. CONCLUSIONS

We presented a new variant of SBMO-ES, which is capable of producing multiple individuals in each generation. This is an important aspect in practice, as it allows for parallelization of the computation, or real-life experiments. We have shown that the new variant performs at least as well as the original SBMO-ES.

In the future some of the aspects of the algorithm should be further evaluated, namely the behavior on ZDT1 with  $\epsilon$ -IBEA selection, and whether it can be used to find a rule, which could be used to change the selection type (or other parameters) adaptively. Experiments with real-life problems are also left as a future work.

**Table 1: Median number of function evaluations needed to reach the specified  $H_{ratio}$**

$H_{ratio}$	0.5	0.75	0.9	0.95	0.99
<b>ZDT1</b>					
NSGA	5600	18600	19850	20750	21850
NSGA-LR-ES	949	<b>1293</b>	<b>1692</b>	<b>1985</b>	<b>5097</b>
NSGA-LR-SVM	<b>500</b>	1420	1904	2053	5285
IBEA	7400	13750	18200	20000	25550
IBEA-LR-ES	798	<b>1197</b>	<b>1456</b>	<b>1759</b>	<b>5639</b>
IBEA-LR-SVM	<b>453</b>	3815	4367	4625	5694
<b>ZDT2</b>					
NSGA	650	1650	3550	5050	7900
NSGA-LR-ES	<b>156</b>	257	367	719	916
NSGA-LR-SVM	162	<b>217</b>	<b>274</b>	<b>320</b>	<b>517</b>
IBEA	750	2050	5150	7800	13000
IBEA-LR-ES	<b>150</b>	246	380	486	788
IBEA-LR-SVM	153	<b>211</b>	<b>267</b>	<b>312</b>	<b>522</b>
<b>ZDT3</b>					
NSGA	600	1250	4150	7250	-
NSGA-LR-ES	166	295	631	831	831
NSGA-LR-SVM	<b>151</b>	<b>223</b>	<b>317</b>	<b>379</b>	<b>699</b>
IBEA	650	1550	5400	8150	33350
IBEA-LR-ES	187	272	450	553	901
IBEA-LR-SVM	<b>156</b>	<b>209</b>	<b>318</b>	<b>367</b>	<b>488</b>
<b>ZDT6</b>					
NSGA	7950	10200	13950	17700	28650
NSGA-LR-ES	<b>1348</b>	<b>3096</b>	<b>6558</b>	<b>9623</b>	<b>19581</b>
NSGA-LR-SVM	2299	4843	8798	13723	-
IBEA	10300	13650	18400	23150	34050
IBEA-LR-ES	<b>1629</b>	<b>4059</b>	<b>8468</b>	<b>12170</b>	<b>21816</b>
IBEA-LR-SVM	2298	4606	8961	11745	21844

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