

# The Evolutionary Algorithm SAMOA with Use of Dynamic Constraints

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## ABSTRACT

Common evolutionary algorithms are often inapplicable to technical design problems in the automotive industry. They cannot deal with multi-objective optimization problems, they need too many function evaluations and time and they cannot handle with a priori unknown dynamic constraints. They also fail because of the growing complexity of the optimization tasks and the huge experimental effort.

Therefore, the principle target of this contribution is the improvement of the simulation and optimization techniques on technical design problems. For this task the new developed evolutionary algorithm *SAMOA* is presented, which is a combination of a genetic algorithm and an evolutionary strategy and can handle with a priori unknown dynamic constraints. It works with a number of potential solutions in progress and it is variable, robust and powerful. It can operate parallel and can deal with multi-criteria problems.

## Categories and Subject Descriptors

G.4 [MATHEMATICAL SOFTWARE]: Algorithm design and analysis;

G.1.6 [NUMERICAL ANALYSIS]: Optimization—*Constrained optimization*

## General Terms

Algorithms

## Keywords

Evolutionary Algorithm, SAMOA, dynamic constraints

## 1. INTRODUCTION

The automotive industry is, based on the sales, the most important industry section in Germany and in other countries and a powerful factor of innovation, growth and employment. But with many various framework conditions, like the  $CO_2$  and fuel reduction at equal engine power, the application of the control units becomes more important for the hole system automotive. Traditional application methods fail because of the growing complexity of the optimization tasks, the huge experimental effort and the contradictory optimization aims during the optimization. They also

cannot deal with the dynamic constraints, which are a priori unknown and change with every measurement. Therefore, the new evolutionary algorithm *SAMOA* is developed, which can handle with these a priori unknown dynamic constraints and with multi-criteria problems.

In the following the phases of the new developed algorithm *SAMOA* are presented and the handling of a priori unknown dynamic constraints within the algorithm is further explained.

## 2. STRUCTURE OF SAMOA

In the following the structure of the evolutionary algorithm *SAMOA* (**S**elf-**A**daptive **M**ulti-criteria **O**nline optimization **A**lgorithm) is presented.

At the beginning of *SAMOA* the individuals must be created. This is done by an intelligent Design of Experiments strategy, like the S-optimal Latin-Hypercube design (see [6]). Then the fitness assignment defines how many offsprings every individual produces. A robust solution, which is also implemented in *SAMOA*, is the nonlinear ranking [1] and in use of multi-criteria problems the S-metric or hypervolume measure [2]. The direct selection of the individuals follows in the next step. In *SAMOA* the truncation selection is implemented, because it is faster as other selection methods and is advantageous in parameter optimizations [4]. After that the recombination creates the offsprings of the parents. The information of the parents are combined to each other (the parents are the pairing population). In *SAMOA* the intermediary recombination is used. After the recombination the mutation of the offsprings is performed. Therefore, the variables of the offsprings are changed by little disturbances (mutation step). In *SAMOA* a mutation of real variables with adaption of the step sizes is realized, which is known as covariance matrix adaption [3]. After the production of the offsprings, they must be reinserted in the population. The reinsertion with offspring selection, where only a part of the offsprings is reinserted in the population, is used in *SAMOA*. The best individuals are used for the reinsertion (truncation selection). The evolutionary algorithm *SAMOA* runs as long as the determined termination criterion, which is a combination of the direct termination criterion "Maximum number of generations respectively function evaluations" and of the indirect termination criterion "Running mean", is reached.

## 3. DYNAMIC CONSTRAINTS

At the online optimization of engines the measurement process must be done automatically at the test bench and

therefore a robust limit treatment is needed. To avoid limit violations, it is necessary to identify the permissible domain as exactly as possible. Of particular importance for the on-line optimization of combustion engines is, that the nonlinear constraints  $c(x)$ , which define the limits, are known only very inaccurate or not at all before the optimization starts. To arrive at a deeper understanding and to define a limit specific approach it is helpful to divide the engine limits in three categories. The static linear and nonlinear, which are a priori known and unchangeable during the optimization, as well as the dynamic nonlinear engine limits, whereby the last one can be divided into strong and smooth limits. The term dynamic should indicate in this content, that the information over the function profile of this limits increases dynamically during the optimization by the detection of limit violations. Strong or smooth mean a large or relatively low risk regarding occurring engine breakdowns. To include the constraints into the optimization, a model or a mathematical expression is necessary to add this to the objective function with the help of penalty functions. For static limits no model building is necessary, because they can be considered directly at the limitation of the domain by a closed mathematical expression. In contrast the dynamic limits distinguish themselves, that they are dynamical considered during the online optimization at the test bench. For the modeling of the smooth limits regression models [5] can be used. For the strong limits only a few violation points exist and therefore the used models for their modeling must be able to reconstruct these points. Therefore, in SAMOA a hull model with confidence terms is implemented. The hull model interpolates between the  $n$  given violation points  $\epsilon_i$  at one fixed operation point and calculates thereby a closed hull over the permissible domain. A point  $x_h$  on the associated hull surface can be expressed by the equation

$$x_h = x_{zp} + s(x_e) \cdot (x_e - x_{zp}),$$

whereby  $x_{zp}$  is the central point,  $x_e$  is a point on the unit ball around  $x_{zp}$  and  $s(x_e)$  compresses or stretches the vector  $x_e - x_{zp}$ . It is calculated as a weighted sum of all distances  $d_i$  of the violation points  $\epsilon_i$  to  $x_{zp}$ :

$$s(x_e) = \begin{cases} d_i, & \exists i \in \{1, 2, \dots, n\}, \text{ so that } x_e = \epsilon_{i,e} \\ \sum_{i=1}^n \omega_i(x_e) d_i, & \text{otherwise, whereby } \sum_{i=1}^n \omega_i(x_e) = 1. \end{cases}$$

The weights  $\omega_i(x_e)$  are determined for every point  $x_e$  by a function of the inverse arc length  $(b_i(x_e))^{-1}$  between the point  $x_e$  and all violation points  $\epsilon_{i,e}$ , which are projected on the unit ball. One possible form of the weights  $\omega_i(x_e)$  is

$$\omega_i(x_e) = \frac{(b_i(x_e))^{-\gamma}}{\sum_{j=1}^n (b_j(x_e))^{-\gamma}}, \quad \gamma > 0.$$

One can clearly recognize, that the model limits the domain in unknown areas too strong. So in this form it is still too restrictive. To avoid this problem the following confidence term is used as support construct:

$$\begin{aligned} \text{conf}(x) &= 1 - \prod_{i=1}^n \text{conf}_i(x), \quad \text{with} \\ \text{conf}_i(x) &= \begin{cases} 1 - \exp \left( 1 - \frac{1}{1 - \frac{\|x - \epsilon_i\|^2}{r_i^2}} \right), & \text{if } \|x - \epsilon_i\| < r_i \\ 1, & \text{otherwise.} \end{cases} \end{aligned}$$

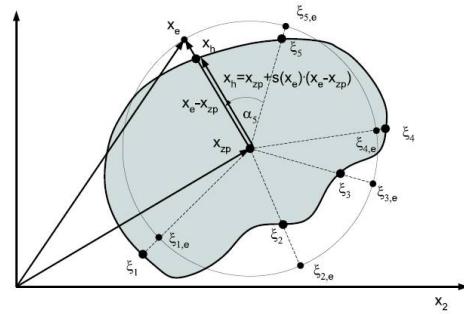
Thus, a hull with controllable restrictivity is created by the correction of the points  $x_n$  on the hull surface with help of the confidence terms defined above and the domain characteristic distance  $d_{max} = \sqrt{d/2}$  ( $d$ : dimension of the domain) to

$$x_h = x_{zp} + (s(x_e) + (1 - \text{conf}(x_e)) \cdot d_{max}) \cdot (x_e - x_{zp}).$$

In this case the confidence terms are calculated by the violation points  $\epsilon_{ie}$ , which are projected on the unit ball around the central point. A limit violation is fulfilled, if the following equation holds for the Euclidean distances between the considered point and the point on the sphere:

$$c(x) := \|x - x_{zp}\| - \|x_h(x)\| > 0.$$

The  $i = 1, \dots, n$  violation points  $\epsilon_i$  are further reflected exactly by the deformed hull model, because it holds that  $\text{conf}(\epsilon_i) = 1$  and therefore the condition  $s(\epsilon_i) = d_i$  is fulfilled.



**Figure 1: Construction of the hull model of the unit ball with the center point  $x_{zp}$  as midpoint.**

## 4. REFERENCES

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