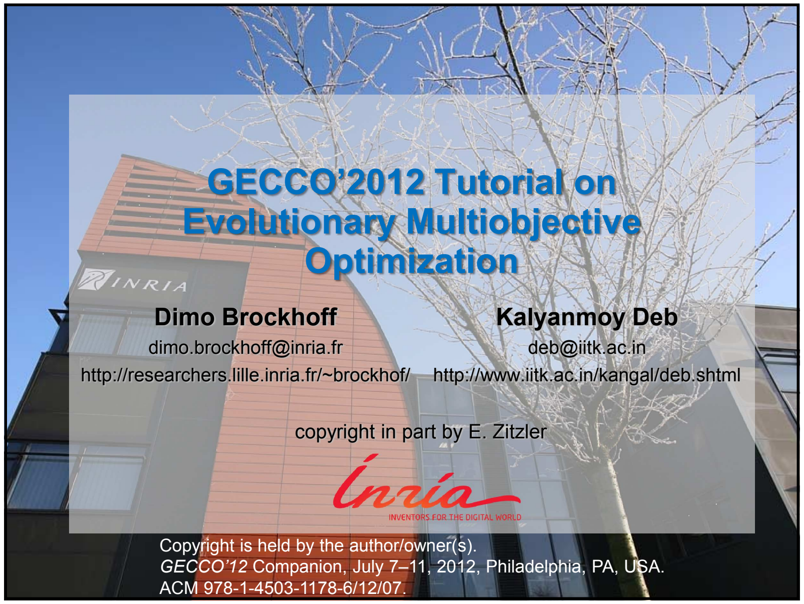

GECCO'2012 Tutorial on Evolutionary Multiobjective Optimization

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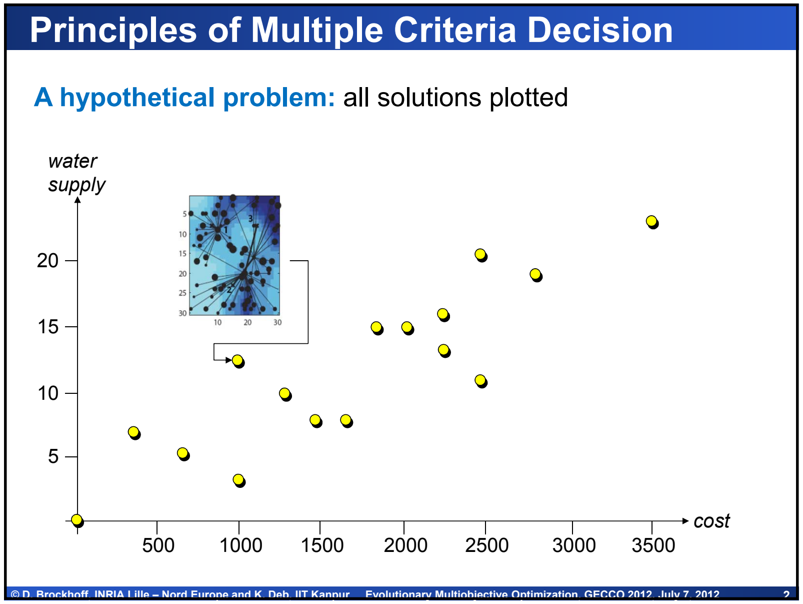
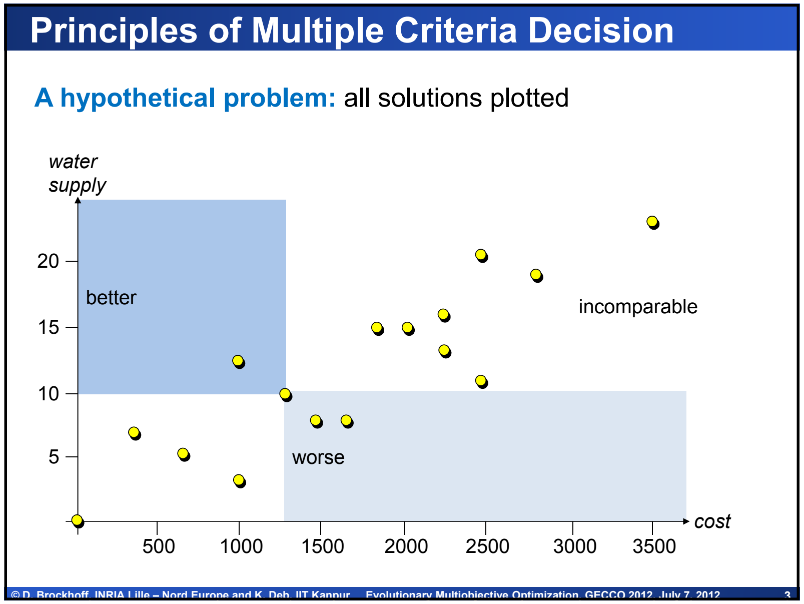


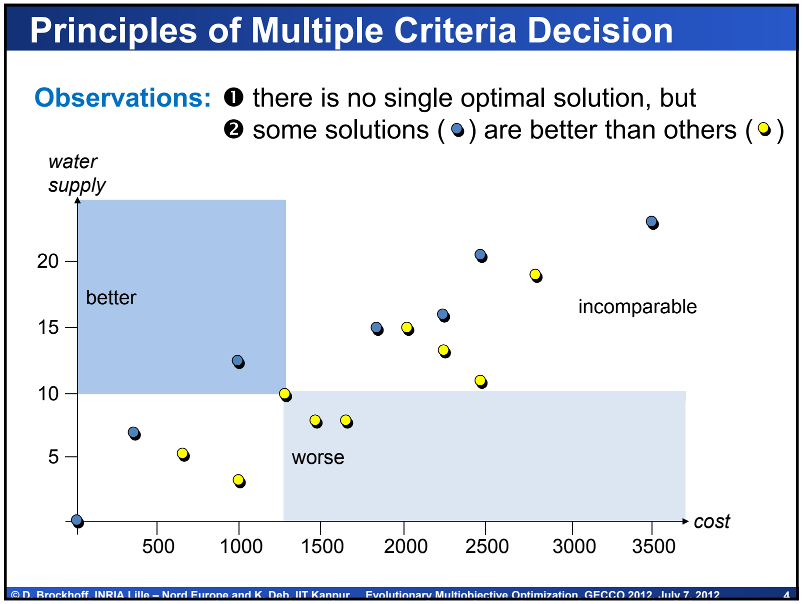
Principles of Multiple Criteria Decision

A hypothetical problem: all solutions plotted



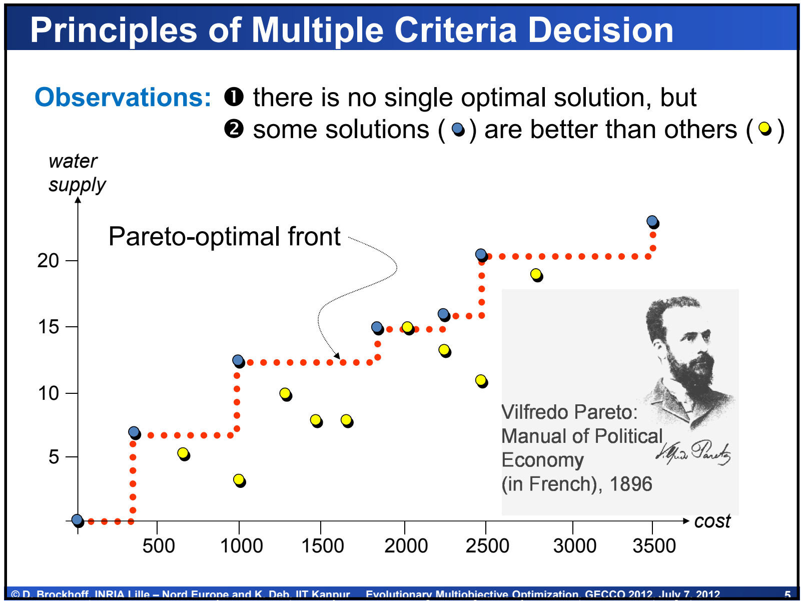
Principles of Multiple Criteria Decision

A hypothetical problem: all solutions plotted



Principles of Multiple Criteria Decision

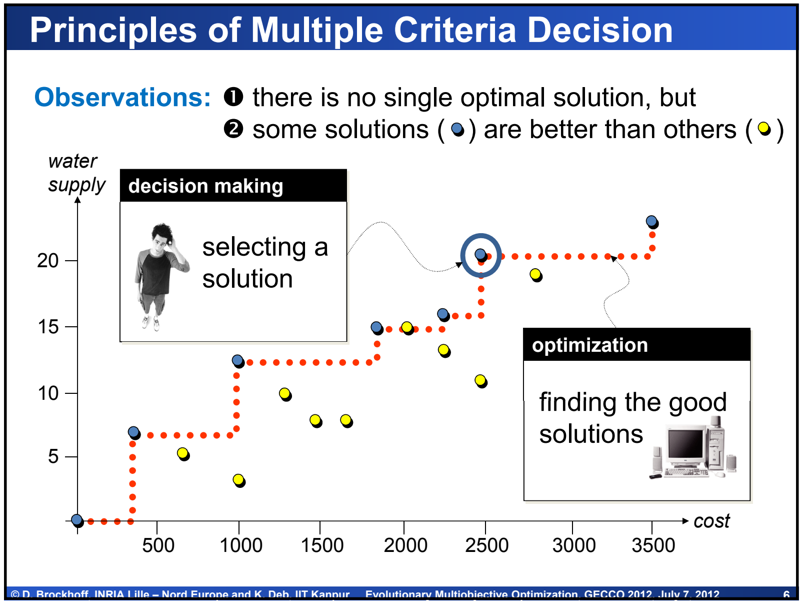
Observations: ❶ there is no single optimal solution, but
❷ some solutions (●) are better than others (●)



Principles of Multiple Criteria Decision

Observations: ❶ there is no single optimal solution, but
❷ some solutions (🔵) are better than others (🟡)

Vilfredo Pareto:
Manual of Political
Economy
(in French), 1896



Principles of Multiple Criteria Decision

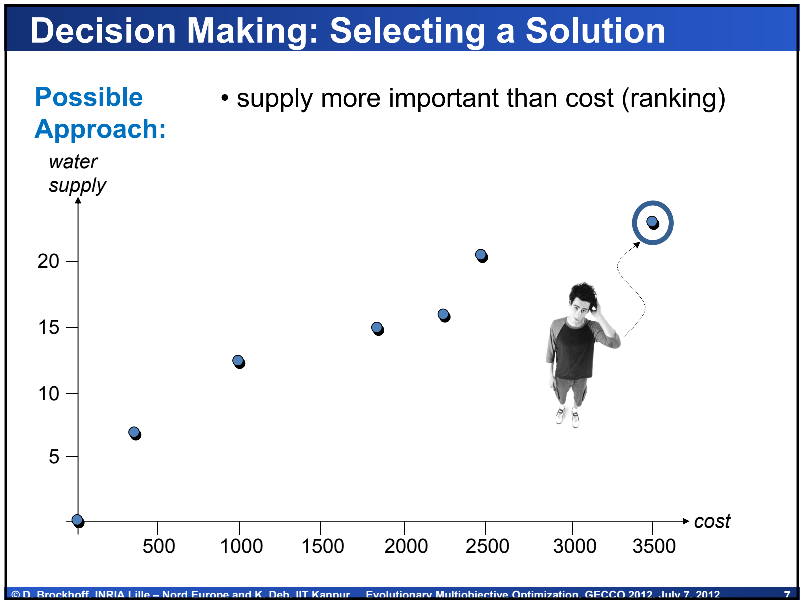
Observations: ❶ there is no single optimal solution, but
❷ some solutions (🔵) are better than others (🟡)

decision making

selecting a solution

optimization

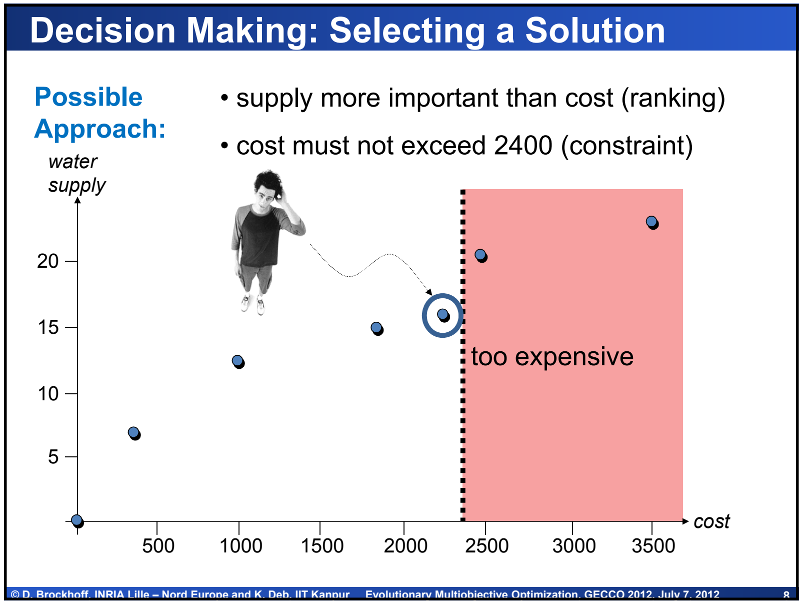
finding the good solutions



Decision Making: Selecting a Solution

Possible Approach:

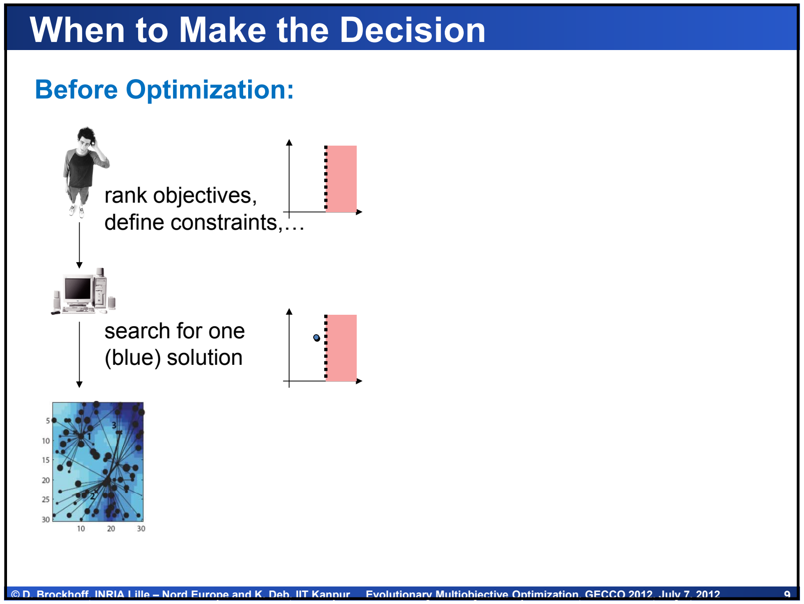
- supply more important than cost (ranking)



Decision Making: Selecting a Solution

Possible Approach:

- supply more important than cost (ranking)
- cost must not exceed 2400 (constraint)

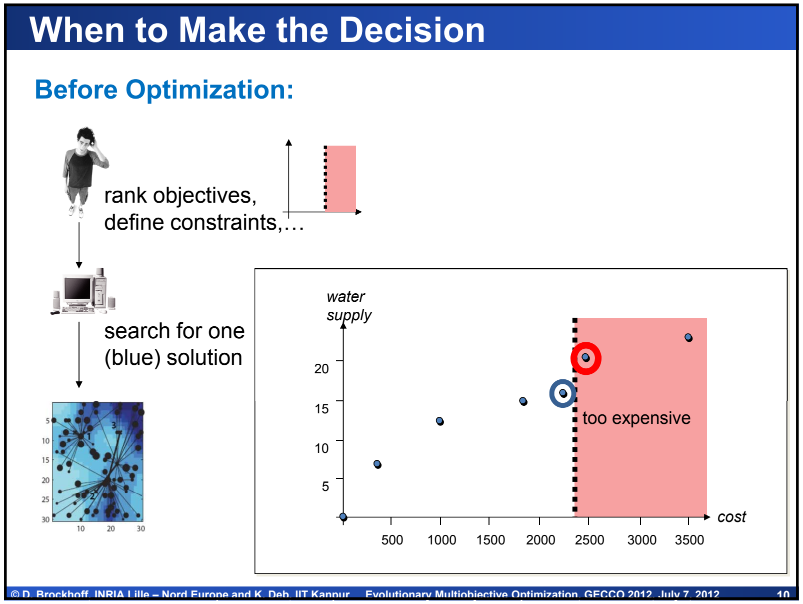


When to Make the Decision

Before Optimization:

rank objectives,
define constraints,…

search for one
(blue) solution



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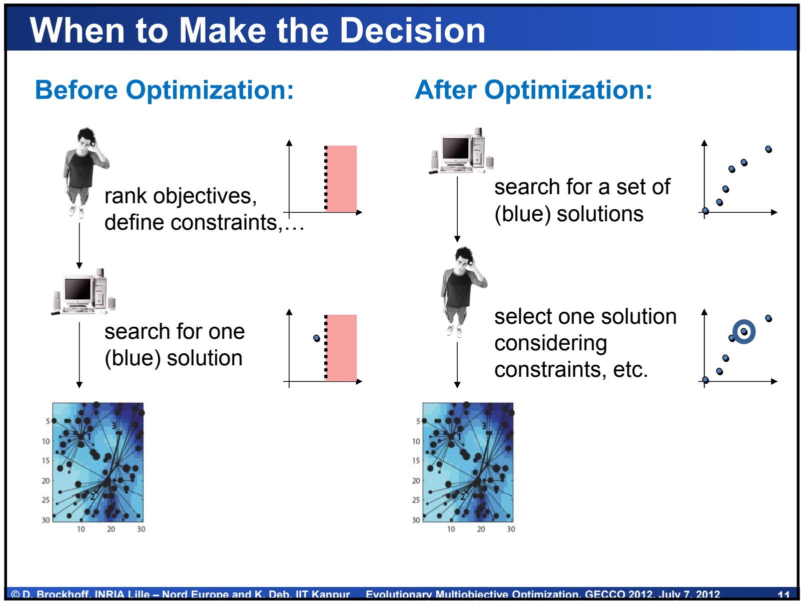


When to Make the Decision

Before Optimization:

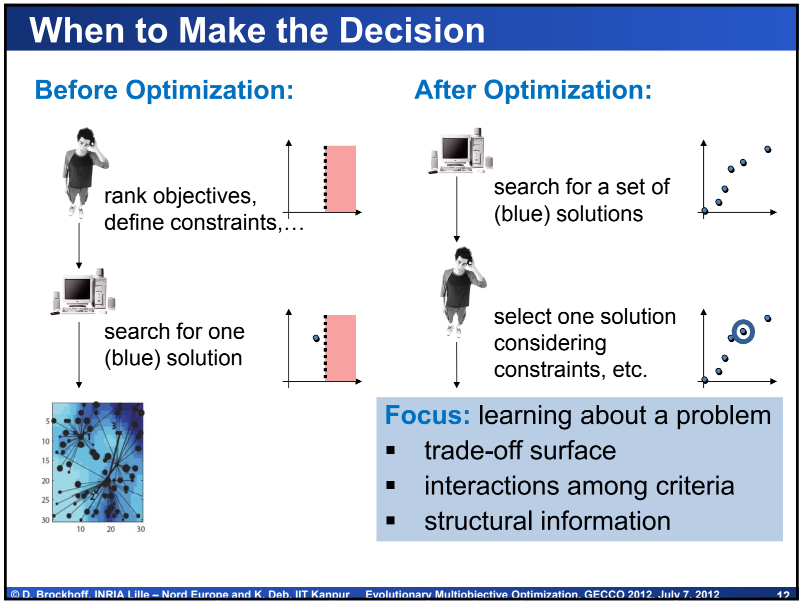
rank objectives,
define constraints,…

search for one
(blue) solution

After Optimization:

search for a set of
(blue) solutions

select one solution
considering
constraints, etc.



When to Make the Decision

Before Optimization:

rank objectives,
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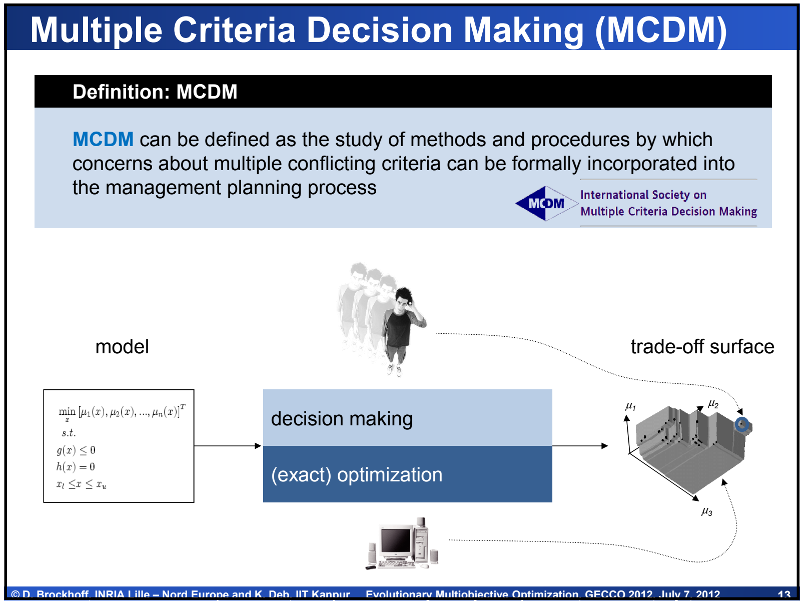
constraints, etc.

Focus: learning about a problem
- trade-off surface
- interactions among criteria
- structural information



Multiple Criteria Decision Making (MCDM)

Definition: MCDM

MCDM can be defined as the study of methods and procedures by which concerns about multiple conflicting criteria can be formally incorporated into the management planning process

International Society on Multiple Criteria Decision Making

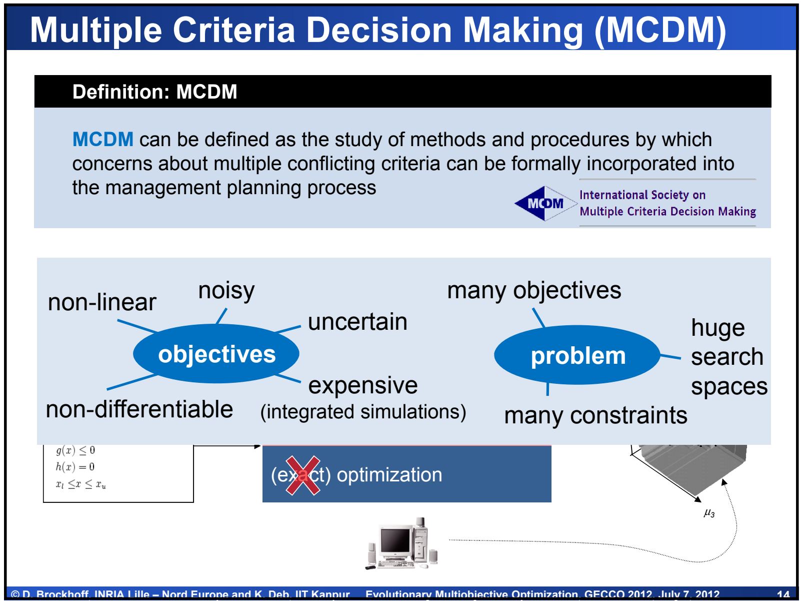
model

$$\min_{x} [\mu_1(x), \mu_2(x), ..., \mu_n(x)]^T$$
$s.t.$
$g(x) \leq 0$
$h(x) = 0$
$x_l \leq x \leq x_u$

decision making

(exact) optimization

trade-off surface



Multiple Criteria Decision Making (MCDM)

Definition: MCDM

MCDM can be defined as the study of methods and procedures by which concerns about multiple conflicting criteria can be formally incorporated into the management planning process

International Society on Multiple Criteria Decision Making

non-linear

noisy

uncertain

objectives

expensive (integrated simulations)

non-differentiable

many objectives

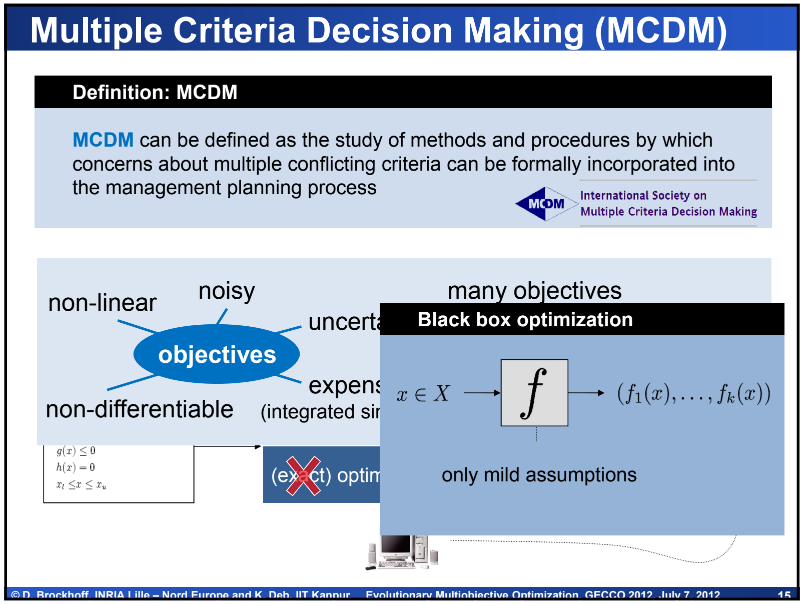
problem

huge search spaces

many constraints

$g(x) \leq 0$
$h(x) = 0$
$x_l \leq x \leq x_u$

(exact) optimization



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International Society on Multiple Criteria Decision Making

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(integrated si

non-differentiable

many objectives

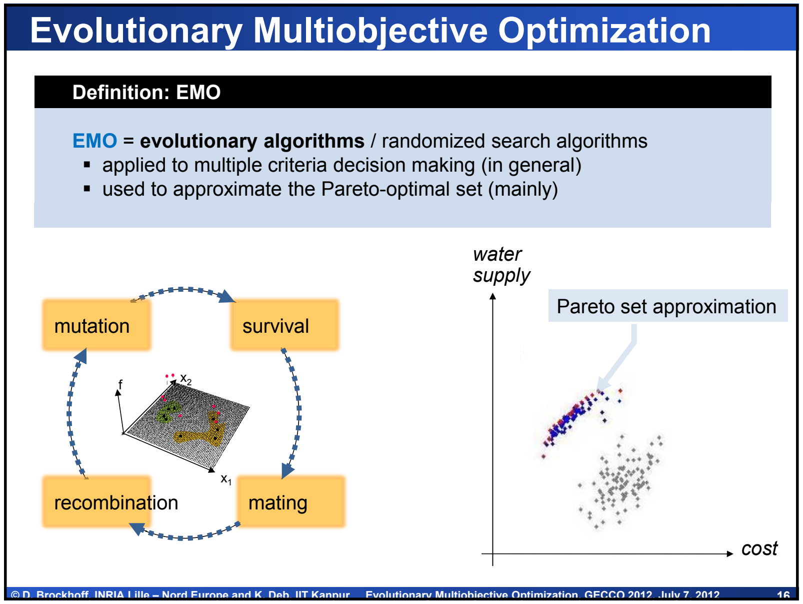
$g(x) \leq 0$
$h(x) = 0$
$x_l \leq x \leq x_u$

(exact) optim

Black box optimization

$$x \in X \longrightarrow \boxed{f} \longrightarrow (f_1(x), \ldots, f_k(x))$$

only mild assumptions



Evolutionary Multiobjective Optimization

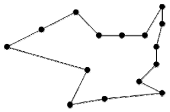
Definition: EMO

EMO = **evolutionary algorithms** / randomized search algorithms
- applied to multiple criteria decision making (in general)
- used to approximate the Pareto-optimal set (mainly)

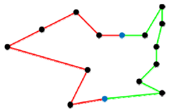
Multiobjectivization

Some problems are easier to solve in a multiobjective scenario

example: TSP
[Knowles et al. 2001]



$\pi \in S_n \rightarrow f(\pi)$



$\pi \in S_n \rightarrow (f_1(\pi, a, b), f_2(\pi, a, b))$

Multiobjectivization

by addition of new “helper objectives” [Jensen 2004]

job-shop scheduling [Jensen 2004], frame structural design [Greiner et al. 2007], theoretical (runtime) analyses [Brockhoff et al. 2009]

by decomposition of the single objective

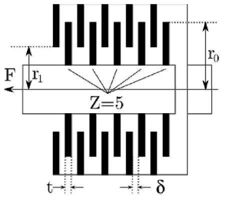
TSP [Knowles et al. 2001], minimum spanning trees [Neumann and Wegener 2006], protein structure prediction [Handl et al. 2008a], theoretical (runtime) analyses [Handl et al. 2008b]

Innovization

Often innovative design principles among solutions are found

example:
clutch brake design
[Deb and Srinivasan 2006]

min. mass +
stopping time



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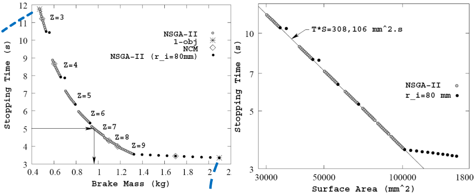

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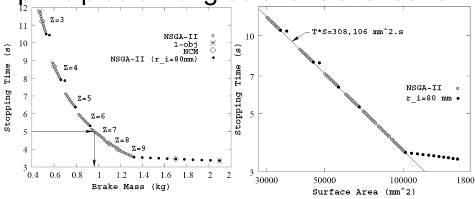
Solution	x_1	x_2	x_3	x_4	x_5	f_1	f_2
Min. f_1	70	90	1.5	1000	3	0.4704	11.7617
Min. f_2	80	110	1.5	1000	9	2.0948	3.3505

Innovization

Often innovative design principles among solutions are found

example:
clutch brake design
[Deb and Srinivasan 2006]



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Innovization [Deb and Srinivasan 2006]

= using machine learning techniques to find new and innovative design principles among solution sets

= learning about a multiobjective optimization problem

Other examples:

- SOM for supersonic wing design [Obayashi and Sasaki 2003]
- biclustering for processor design and KP [Ulrich et al. 2007]

The History of EMO At A Glance

1984 first EMO approaches

dominance-based population ranking

1990 dominance-based EMO algorithms with diversity preservation techniques

1995 attainment functions

elitist EMO algorithms preference articulation convergence proofs

2000 test problem design quantitative performance assessment multiobjectivization

uncertainty and robustness running time analyses quality measure design

MCDM + EMO quality indicator based EMO algorithms

2010 many-objective optimization statistical performance assessment

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The History of EMO At A Glance

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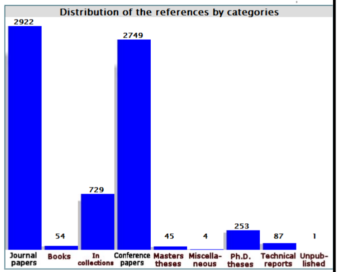
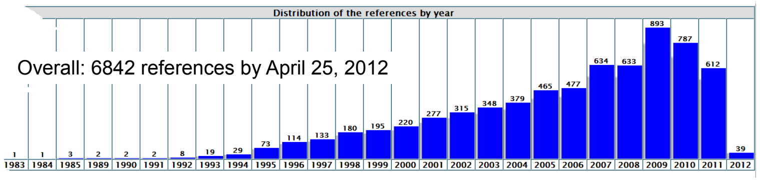
uncertainty and robustness running time analyses quality measure design

MCDM + EMO EMO algorithms based on set quality measures

2012 high-dimensional objective spaces statistical performance assessment





Overall: 6842 references by April 25, 2012

http://delta.cs.cinvestav.mx/~ccoello/EMOO/EMOOstatistics.html

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The EMO Community

The EMO conference series:

EMO2001	EMO2003	EMO2005	EMO2007	EMO2009	EMO2011	EMO2013
Zurich Switzerland	Faro Portugal	Guanajuato Mexico	Matsushima Japan	Nantes France	Ouro Peto Brazil	Sheffield UK
45 / 87	56 / 100	59 / 115	65 / 124	39 / 72	42 / 83	

Many further activities:

special sessions, special journal issues, workshops, tutorials, ...

Overview

The Big Picture

Basic Principles of Multiobjective Optimization

- algorithm design principles and concepts
- performance assessment

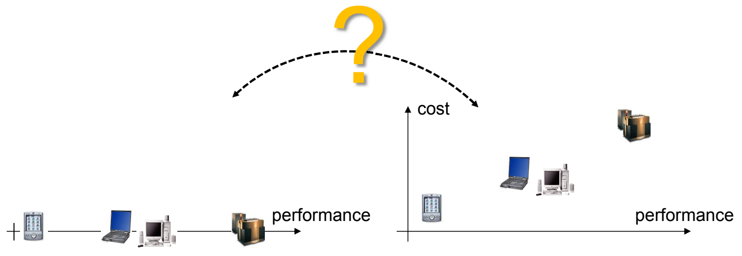
Selected Advanced Concepts

- indicator-based EMO
- preference articulation

A Few Examples From Practice

Starting Point

What makes evolutionary multiobjective optimization different from single-objective optimization?



single objective

multiple objectives

A General (Multiobjective) Optimization

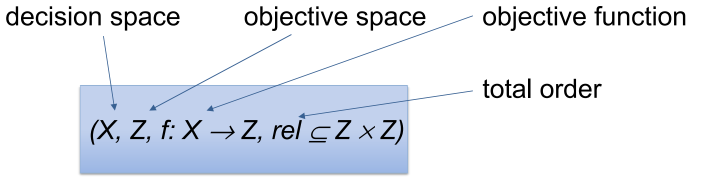
A multiobjective optimization problem: $(X, Z, \mathbf{f}, \mathbf{g}, \leq)$

X	search / parameter / decision space
$Z = \mathbb{R}^n$	objective space
$\mathbf{f} = (f_1, \ldots, f_n)$	vector-valued objective function with $f_i : X \mapsto \mathbb{R}$
$\mathbf{g} = (g_1, \ldots, g_m)$	vector-valued constraint function with $g_i : X \mapsto \mathbb{R}$
$\leq \subseteq Z \times Z$	binary relation on objective space

Goal: find decision vector(s) $\mathbf{a} \in X$ such that

- ❶ for all $1 \leq i \leq m : g_i(\mathbf{a}) \leq 0$ and
- ❷ for all $\mathbf{b} \in X$: $\mathbf{f}(\mathbf{b}) \leq \mathbf{f}(\mathbf{a}) \Rightarrow \mathbf{f}(\mathbf{a}) \leq \mathbf{f}(\mathbf{b})$

A Single-Objective Optimization Problem



decision space

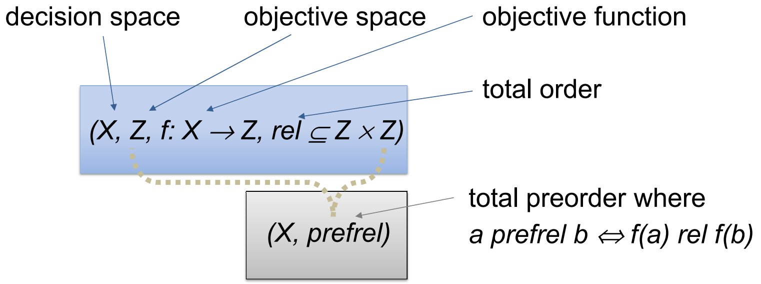
objective space

objective function

total order

(X, Z, f: X → Z, rel ⊆ Z × Z)

A Single-Objective Optimization Problem



decision space

objective space

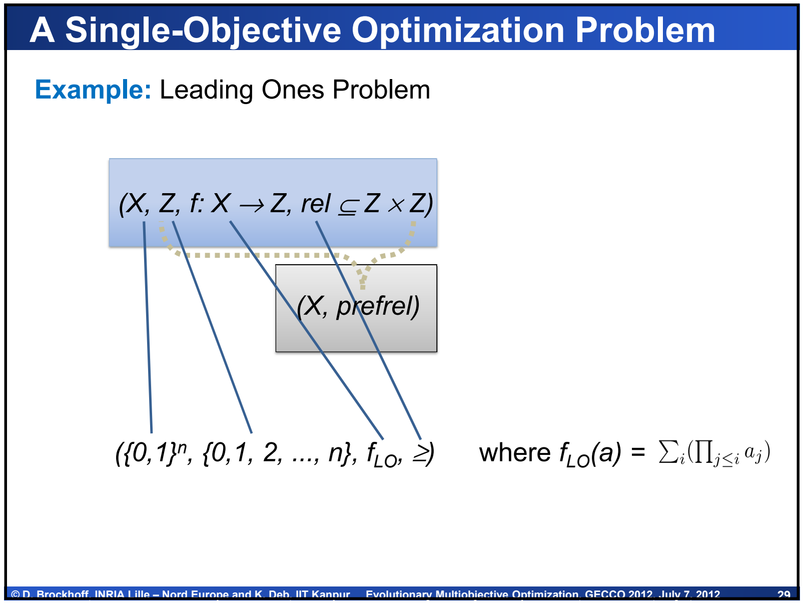
objective function

total order

(X, Z, f: X → Z, rel ⊆ Z × Z)

(X, prefrel)

total preorder where
a prefrel b ⇔ f(a) rel f(b)



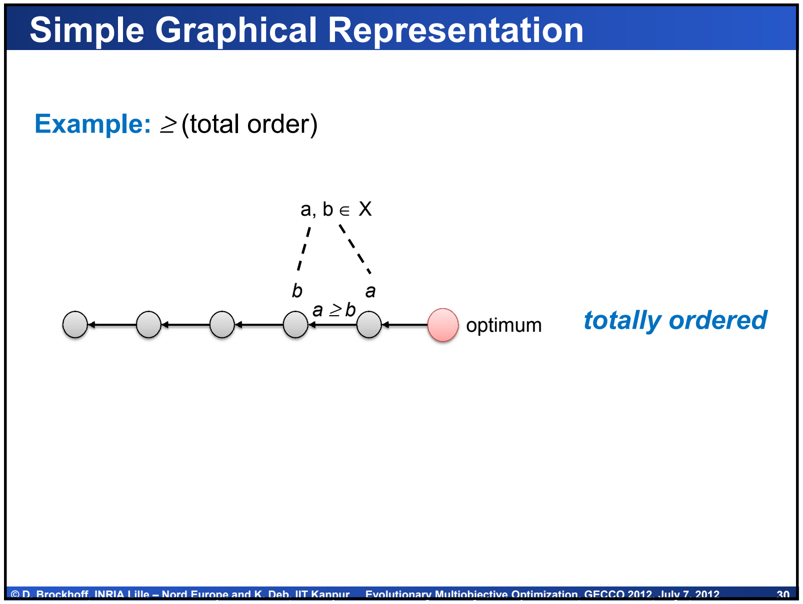
A Single-Objective Optimization Problem

Example: Leading Ones Problem

$(X, Z, f: X \rightarrow Z, rel \subseteq Z \times Z)$

$(X, prefrel)$

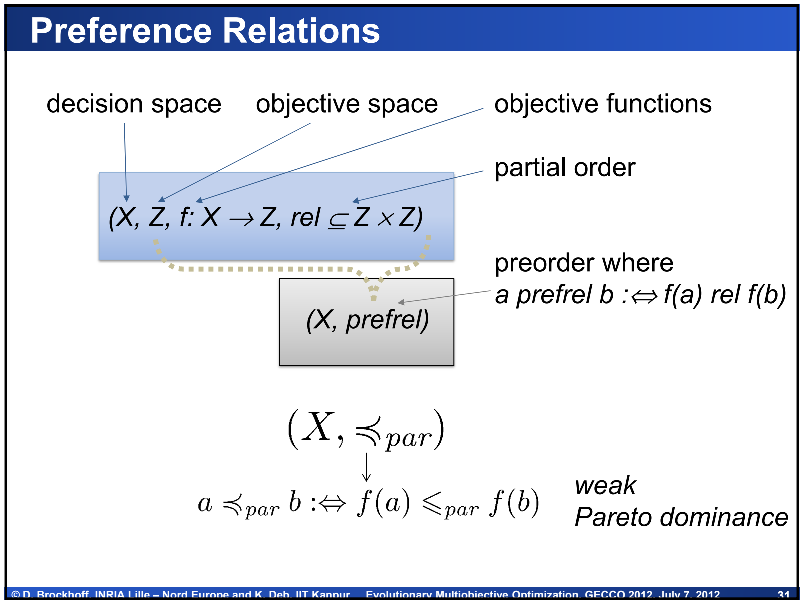
$(\{0,1\}^n, \{0,1, 2, \ldots, n\}, f_{LO}, \geq)$ where $f_{LO}(a) = \sum_i(\prod_{j\leq i} a_j)$



Simple Graphical Representation

Example: $\geq$ (total order)

totally ordered



Preference Relations

$(X, Z, f: X \rightarrow Z, rel \subseteq Z \times Z)$

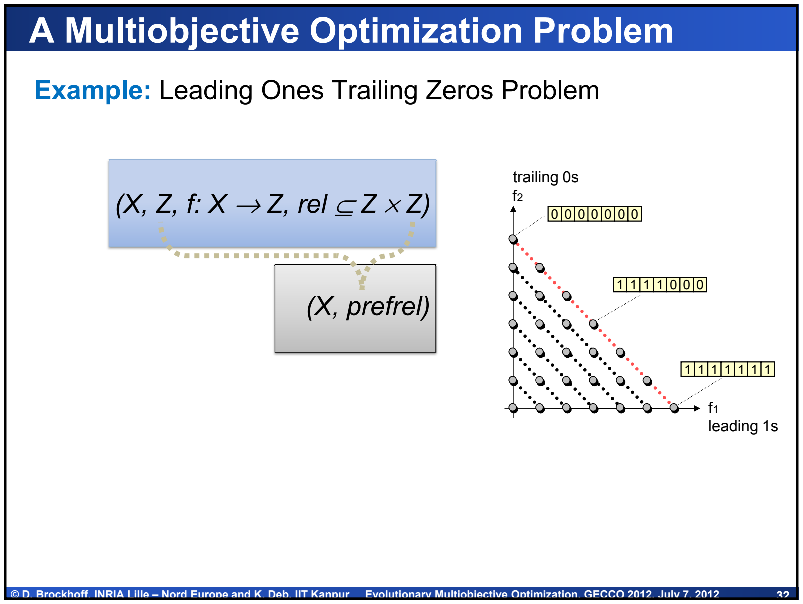
$(X, prefrel)$

preorder where
a prefrel b :⇔ f(a) rel f(b)

$$(X, \preccurlyeq_{par})$$

$$a \preccurlyeq_{par} b :\Leftrightarrow f(a) \leqslant_{par} f(b)$$

weak Pareto dominance



A Multiobjective Optimization Problem

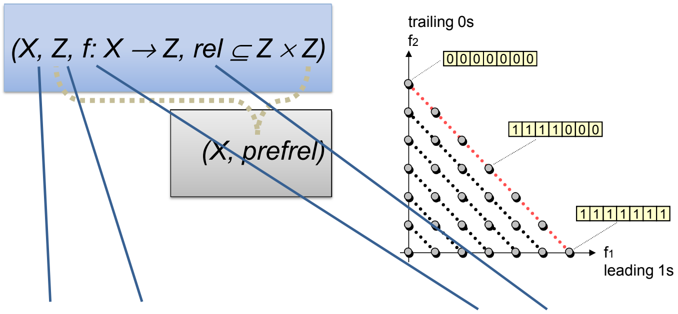
Example: Leading Ones Trailing Zeros Problem

$(X, Z, f: X \rightarrow Z, rel \subseteq Z \times Z)$

$(X, prefrel)$

A Multiobjective Optimization Problem

Example: Leading Ones Trailing Zeros Problem



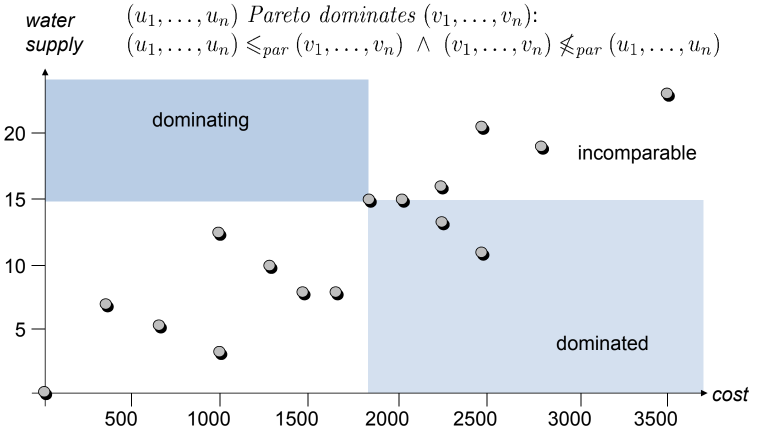
$(\{0,1\}^n, \{0,1, 2, ..., n\} \times \{0,1, 2, ..., n\}, (f_{LO}, f_{TZ}), ?)$

$f_{LO}(a) = \sum_i(\prod_{j\leq i} a_j)$ $\qquad$ $f_{TZ}(a) = \sum_i(\prod_{j\leq i}(1-a_j))$

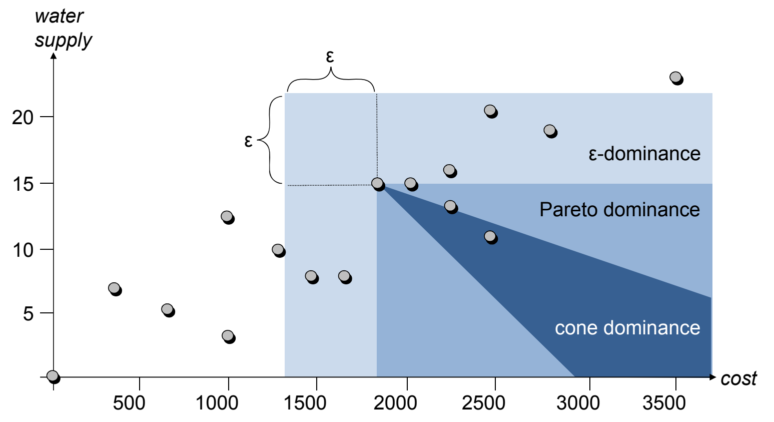
Pareto Dominance

$(u_1,\dots,u_n)$ *weakly Pareto dominates* $(v_1,\dots,v_n)$:
$(u_1,\dots,u_n) \leqslant_{par} (v_1,\dots,v_n) :\Leftrightarrow \forall 1 \leq i \leq n : u_i \leq v_i$

$(u_1,\dots,u_n)$ *Pareto dominates* $(v_1,\dots,v_n)$:
$(u_1,\dots,u_n) \leqslant_{par} (v_1,\dots,v_n) \ \wedge\ (v_1,\dots,v_n) \nleqslant_{par} (u_1,\dots,u_n)$



Different Notions of Dominance

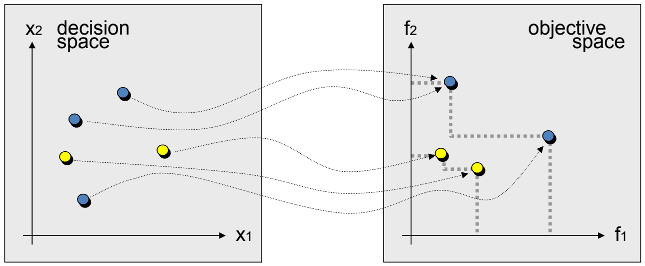


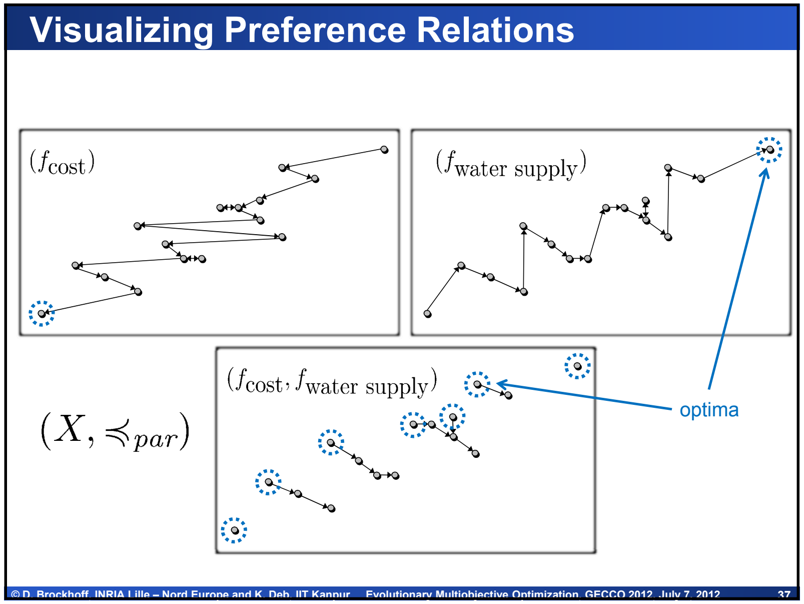
The Pareto-optimal Set

The *minimal set* of a preordered set $(Y, \leqq)$ is defined as

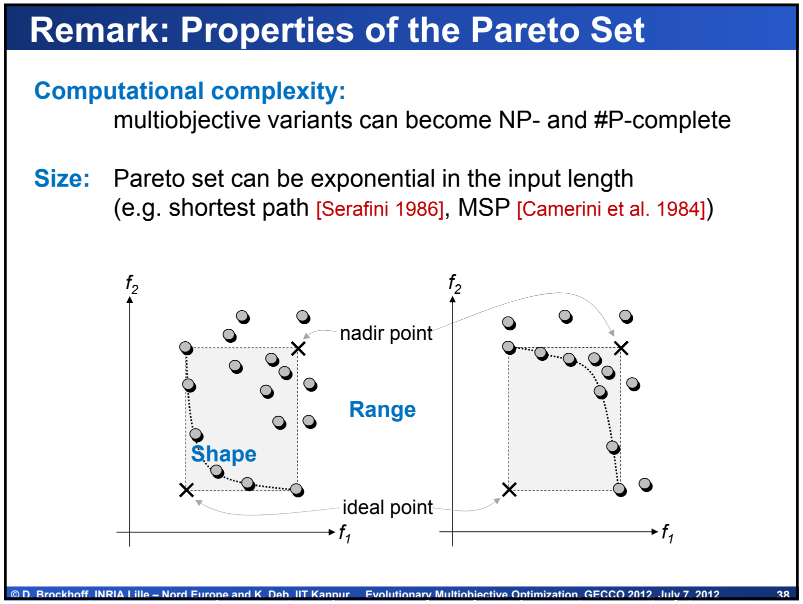
$$Min(Y, \leqq) := \{a \in Y \mid \forall b \in Y : b \leqq a \Rightarrow a \leqq b\}$$

Pareto-optimal set $Min(X, \preccurlyeq_{par})$ ● Pareto-optimal front
non-optimal decision vector ● non-optimal objective vector





Visualizing Preference Relations



Remark: Properties of the Pareto Set

Computational complexity:
multiobjective variants can become NP- and #P-complete

Size: Pareto set can be exponential in the input length
(e.g. shortest path [Serafini 1986], MSP [Camerini et al. 1984])

Approaches To Multiobjective Optimization

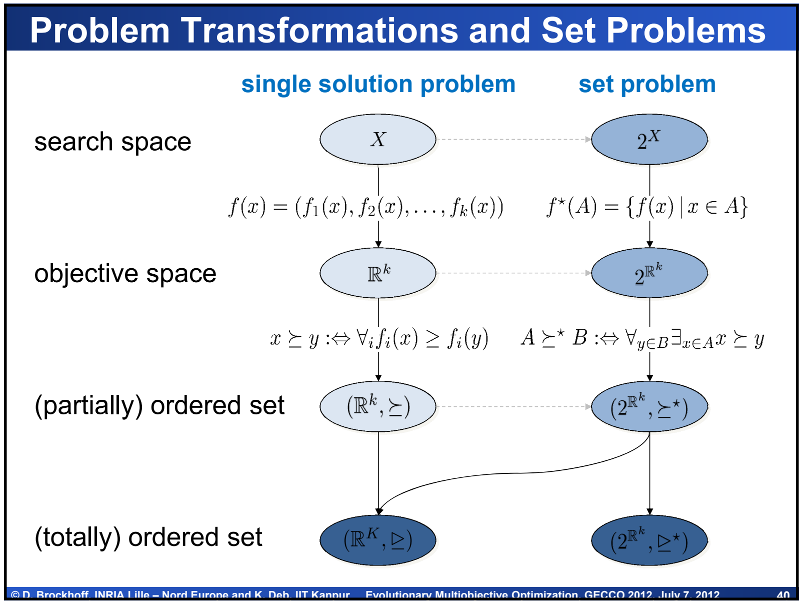
A multiobjective problem is as such underspecified
…because not any Pareto-optimum is equally suited!

Additional preferences are needed to tackle the problem:

Solution-Oriented Problem Transformation:
Induce a total order on the decision space, e.g., by aggregation.

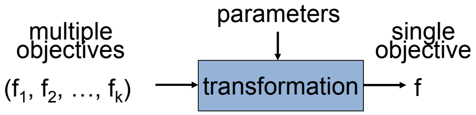
Set-Oriented Problem Transformation:
First transform problem into a set problem and then define an objective function on sets.

Preferences are needed in any case, but the latter are weaker!



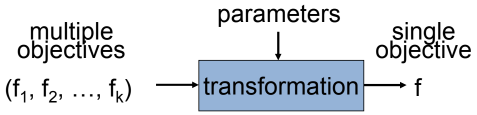
Problem Transformations and Set Problems

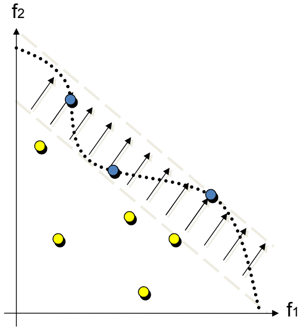
Solution-Oriented Problem Transformations



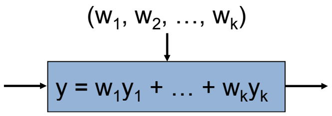
A *scalarizing function* s is a function $s : Z \mapsto \mathbb{R}$ that maps each objective vector $(u_1, \ldots, u_n) \in Z$ to a real value $s(u_1, \ldots, u_n) \in \mathbb{R}$.

Aggregation-Based Approaches





Example: weighting approach



Other example: Tchebycheff

$y = \max w_i(u_i - z_i)$

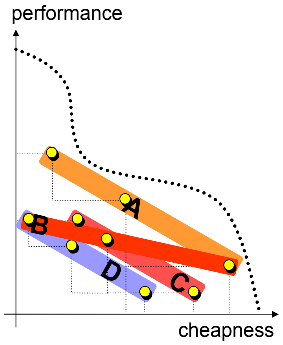
Set-Oriented Problem Transformations

For a multiobjective optimization problem $(X, Z, \mathbf{f}, \mathbf{g}, \leqslant)$, the associated *set problem* is given by $(\Psi, \Omega, F, \mathbf{G}, \preccurlyeq)$ where

- $\Psi = 2^X$ is the space of decision vector sets, i.e., the powerset of X,
- $\Omega = 2^Z$ is the space of objective vector sets, i.e., the powerset of Z,
- F is the extension of $\mathbf{f}$ to sets, i.e., $F(A) := \{\mathbf{f}(\mathbf{a}) \, : \, \mathbf{a} \in A\}$ for $A \in \Psi$,
- $\mathbf{G} = (G_1, \ldots, G_m)$ is the extension of $\mathbf{g}$ to sets, i.e., $G_i(A) := \max\{g_i(\mathbf{a}) \, : \, \mathbf{a} \in A\}$ for $1 \leq i \leq m$ and $A \in \Psi$,
- $\preccurlyeq$ extends $\leqslant$ to sets where $A \preccurlyeq B :\Leftrightarrow \forall \mathbf{b} \in B \; \exists \mathbf{a} \in A : \mathbf{a} \leqslant \mathbf{b}$.

Pareto Set Approximations

Pareto set approximation (algorithm outcome) = set of (usually incomparable) solutions

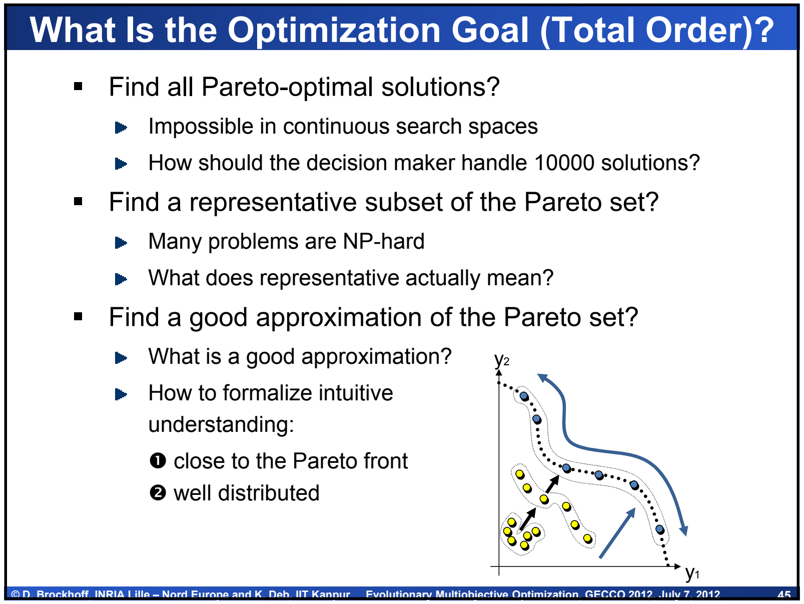


A **weakly dominates** **B**
= not worse in all objectives and sets not equal

C **dominates** **D**
= better in at least one objective

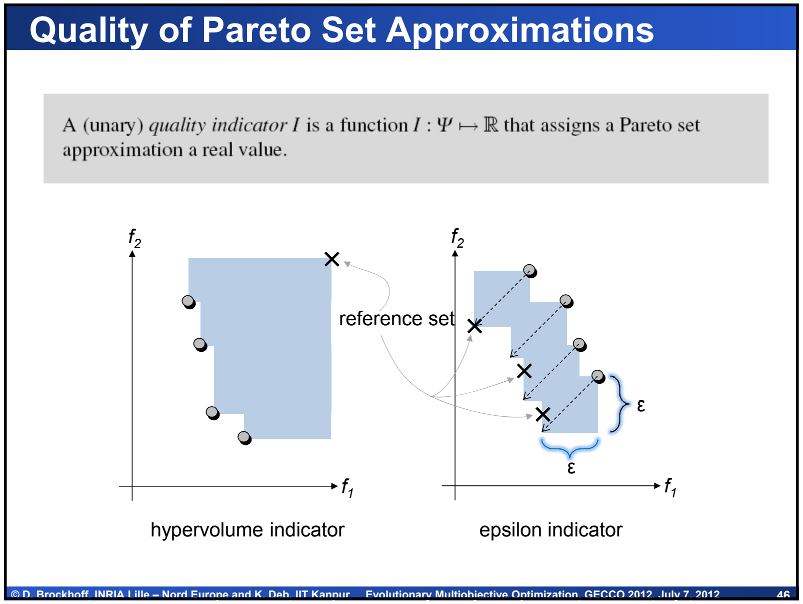
A **strictly dominates** **C**
= better in all objectives

B is **incomparable** to **C**
= neither set weakly better



What Is the Optimization Goal (Total Order)?

- Find all Pareto-optimal solutions?
  - Impossible in continuous search spaces
  - How should the decision maker handle 10000 solutions?
- Find a representative subset of the Pareto set?
  - Many problems are NP-hard
  - What does representative actually mean?
- Find a good approximation of the Pareto set?
  - What is a good approximation?
  - How to formalize intuitive understanding:
    - ❶ close to the Pareto front
    - ❷ well distributed



Quality of Pareto Set Approximations

A (unary) *quality indicator* I is a function $I : \Psi \mapsto \mathbb{R}$ that assigns a Pareto set approximation a real value.

hypervolume indicator

epsilon indicator

General Remarks on Problem

Idea:

Transform a preorder into a total preorder

Methods:

- Define single-objective function based on the multiple criteria *(shown on the previous slides)*
- Define any total preorder using a relation *(not discussed before)*

Question:

Is any total preorder ok resp. are there any requirements concerning the resulting preference relation?

⇒ Underlying dominance relation *rel* should be reflected

Refinements and Weak Refinements

❶ $\overset{\text{ref}}{\preccurlyeq}$ **refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \wedge B \overset{\text{ref}}{\not\preccurlyeq} A \qquad \textit{(better} \Rightarrow \textit{better)}$$

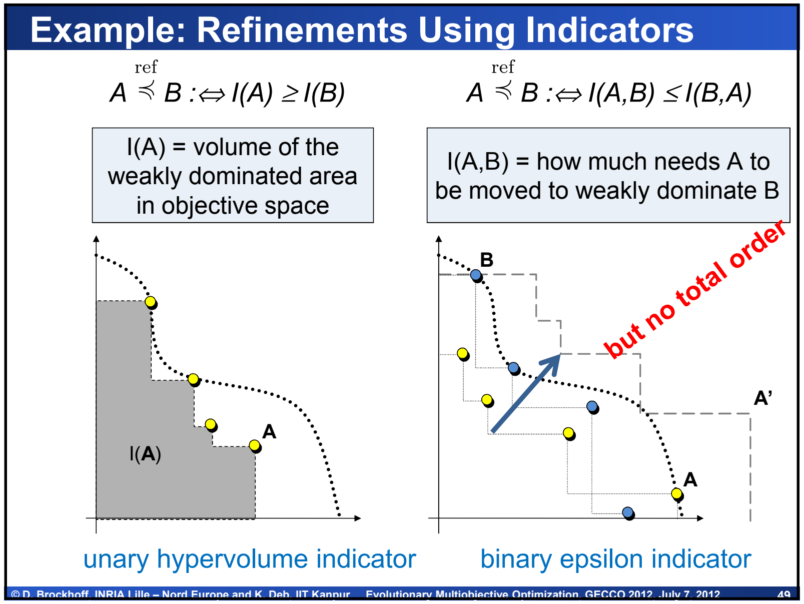
⇒ fulfills requirement

❷ $\overset{\text{ref}}{\preccurlyeq}$ **weakly refines** a preference relation $\preccurlyeq$ iff

$$A \preccurlyeq B \wedge B \not\preccurlyeq A \Rightarrow A \overset{\text{ref}}{\preccurlyeq} B \qquad \textit{(better} \Rightarrow \textit{weakly better)}$$

⇒ does not fulfill requirement, but $\overset{\text{ref}}{\preccurlyeq}$ does not contradict $\preccurlyeq$

…sought are total refinements…



Example: Refinements Using Indicators

$A \overset{\text{ref}}{\preceq} B :\Leftrightarrow I(A) \geq I(B)$

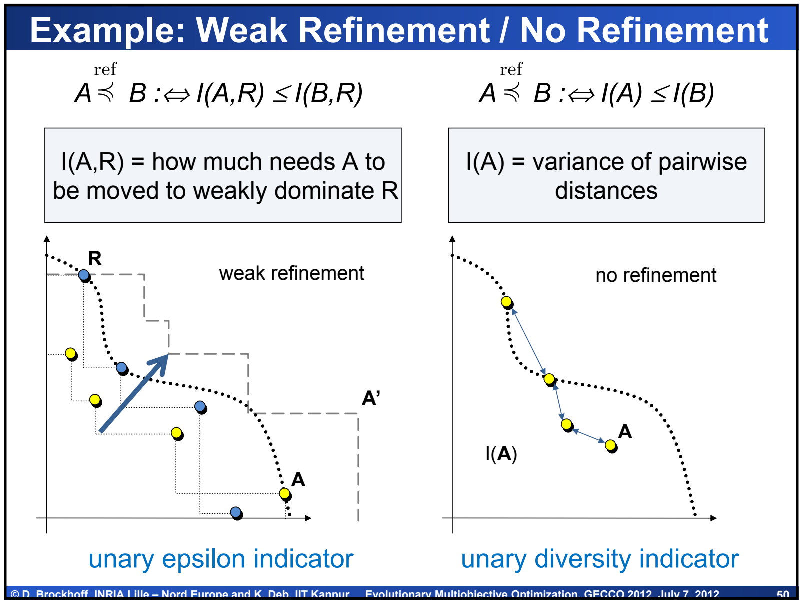
$A \overset{\text{ref}}{\preceq} B :\Leftrightarrow I(A,B) \leq I(B,A)$

I(A) = volume of the weakly dominated area in objective space

I(A,B) = how much needs A to be moved to weakly dominate B

unary hypervolume indicator

binary epsilon indicator



Example: Weak Refinement / No Refinement

$A \overset{\text{ref}}{\preceq} B :\Leftrightarrow I(A,R) \leq I(B,R)$

$A \overset{\text{ref}}{\preceq} B :\Leftrightarrow I(A) \leq I(B)$

I(A,R) = how much needs A to be moved to weakly dominate R

I(A) = variance of pairwise distances

unary epsilon indicator

unary diversity indicator

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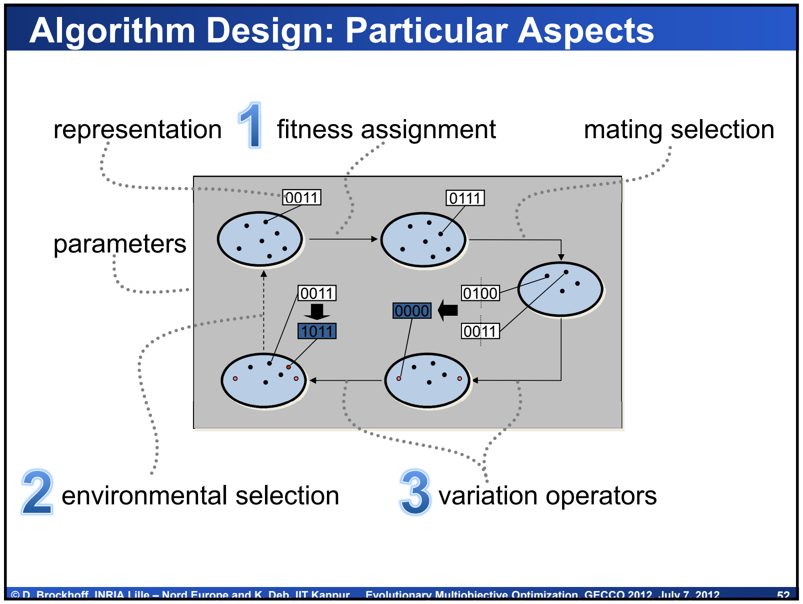


Algorithm Design: Particular Aspects

representation

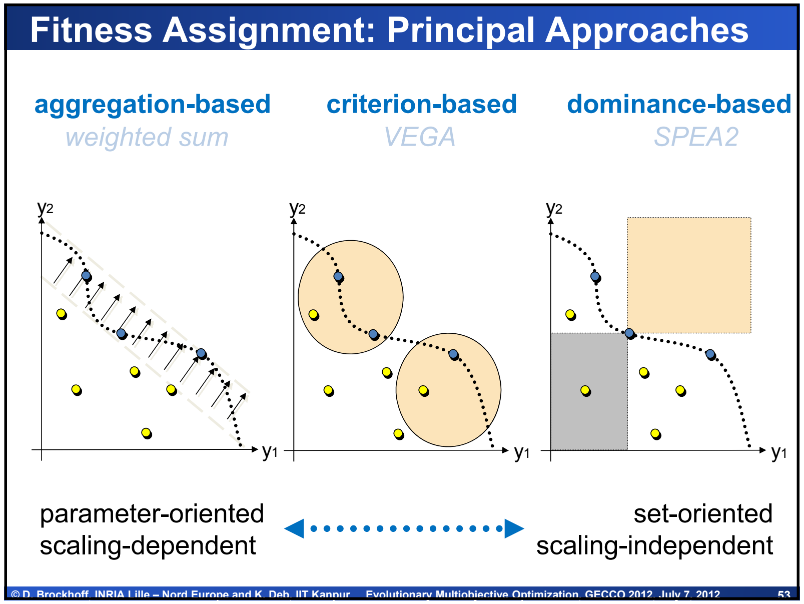
1 fitness assignment

mating selection

parameters

2 environmental selection

3 variation operators



Fitness Assignment: Principal Approaches

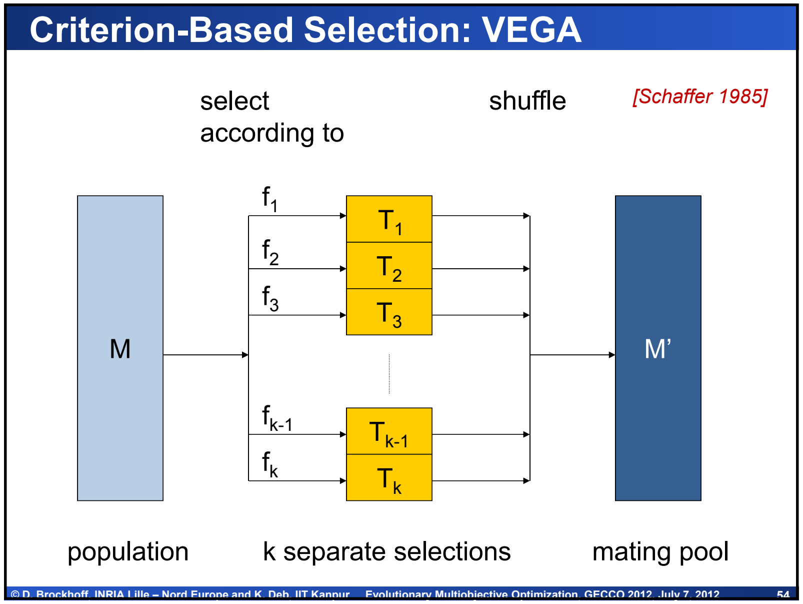
aggregation-based
weighted sum

criterion-based
VEGA

dominance-based
SPEA2

parameter-oriented
scaling-dependent

set-oriented
scaling-independent



Criterion-Based Selection: VEGA

select
according to

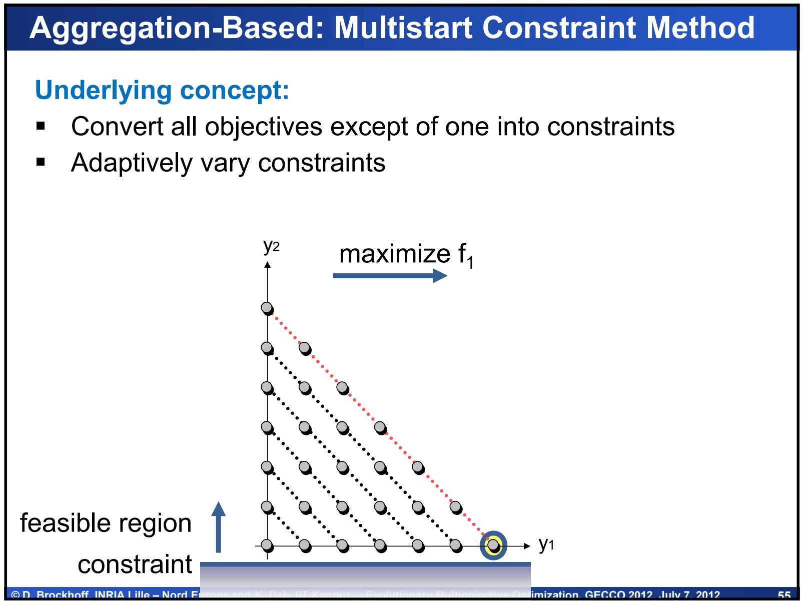
shuffle

[Schaffer 1985]

population

k separate selections

mating pool



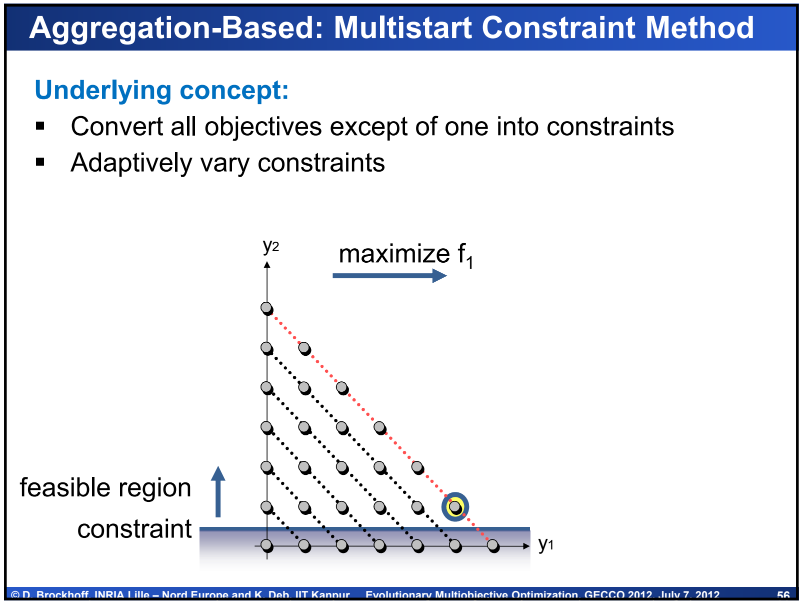
Aggregation-Based: Multistart Constraint Method

Underlying concept:

- Convert all objectives except of one into constraints
- Adaptively vary constraints

feasible region

constraint



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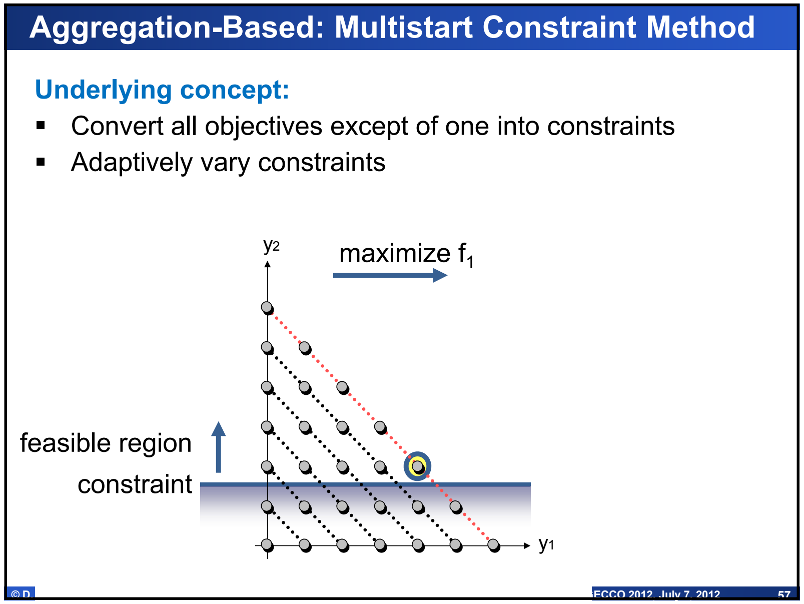
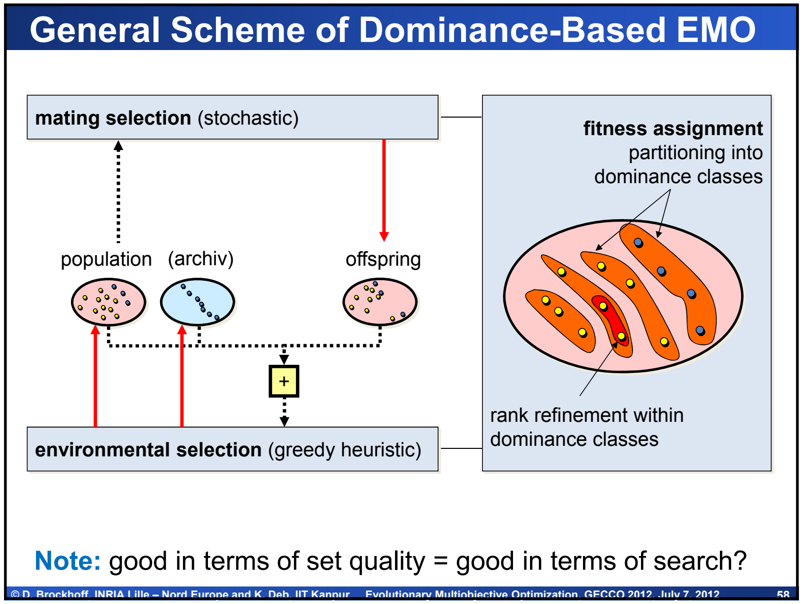


Aggregation-Based: Multistart Constraint Method

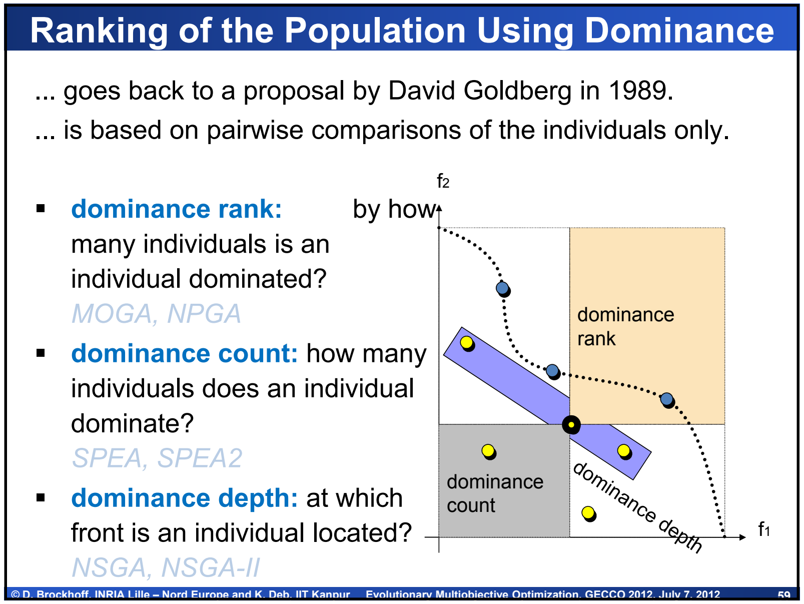
Underlying concept:

- Convert all objectives except of one into constraints
- Adaptively vary constraints



General Scheme of Dominance-Based EMO

Note: good in terms of set quality = good in terms of search?



Ranking of the Population Using Dominance

... goes back to a proposal by David Goldberg in 1989.

... is based on pairwise comparisons of the individuals only.

- **dominance rank:** by how many individuals is an individual dominated?
  MOGA, NPGA
- **dominance count:** how many individuals does an individual dominate?
  SPEA, SPEA2
- **dominance depth:** at which front is an individual located?
  NSGA, NSGA-II

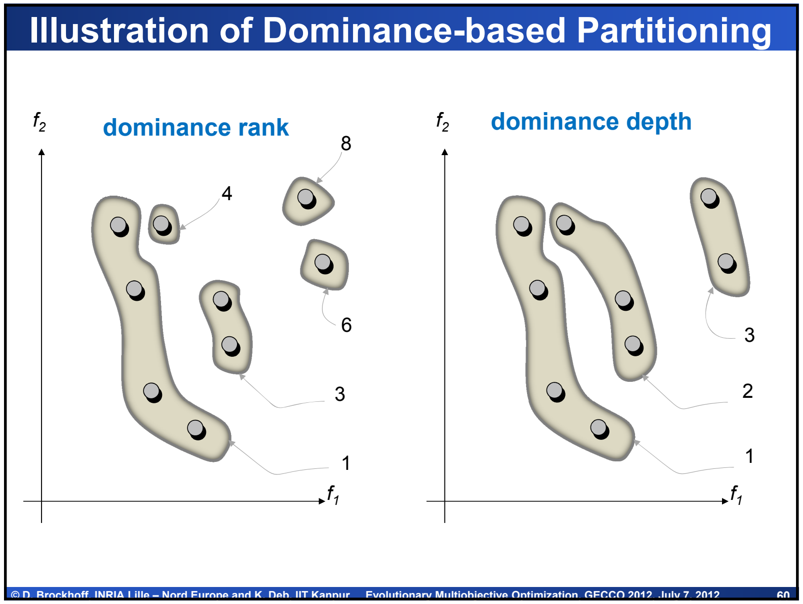
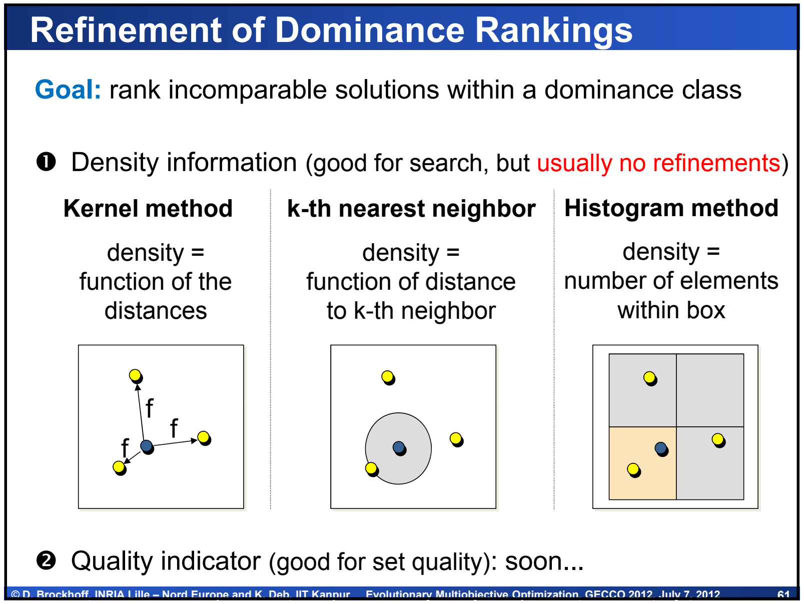


Illustration of Dominance-based Partitioning



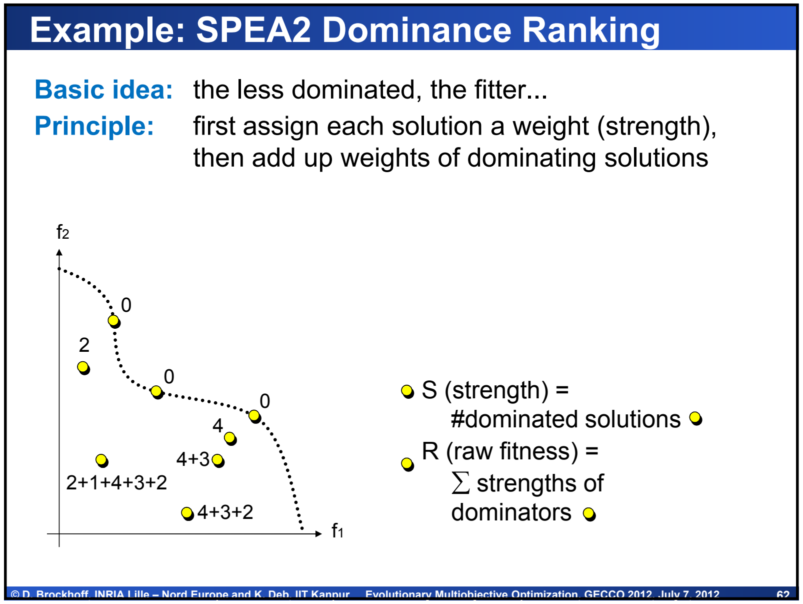
Refinement of Dominance Rankings

Goal: rank incomparable solutions within a dominance class

❶ Density information (good for search, but usually no refinements)

Kernel method	**k-th nearest neighbor**	**Histogram method**
density = function of the distances	density = function of distance to k-th neighbor	density = number of elements within box

❷ Quality indicator (good for set quality): soon...

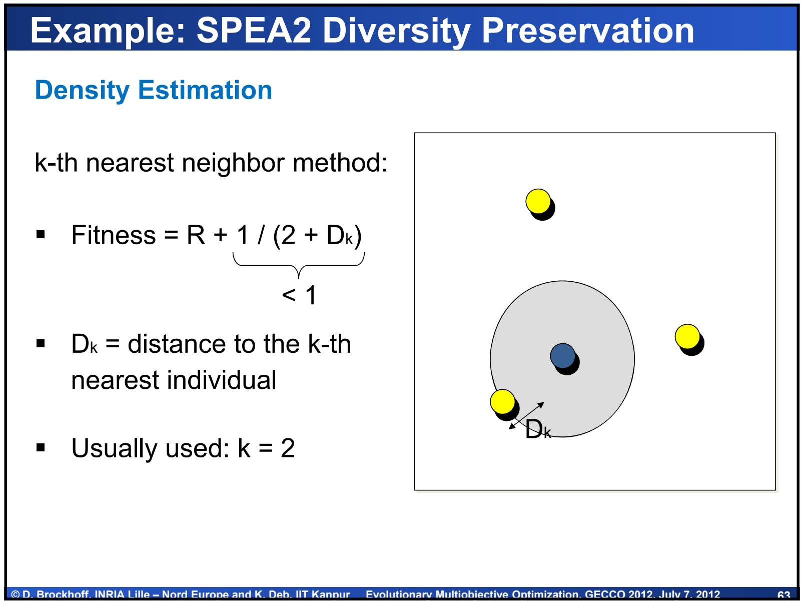


Example: SPEA2 Dominance Ranking

Basic idea: the less dominated, the fitter...

Principle: first assign each solution a weight (strength), then add up weights of dominating solutions

- S (strength) = #dominated solutions
- R (raw fitness) = ∑ strengths of dominators

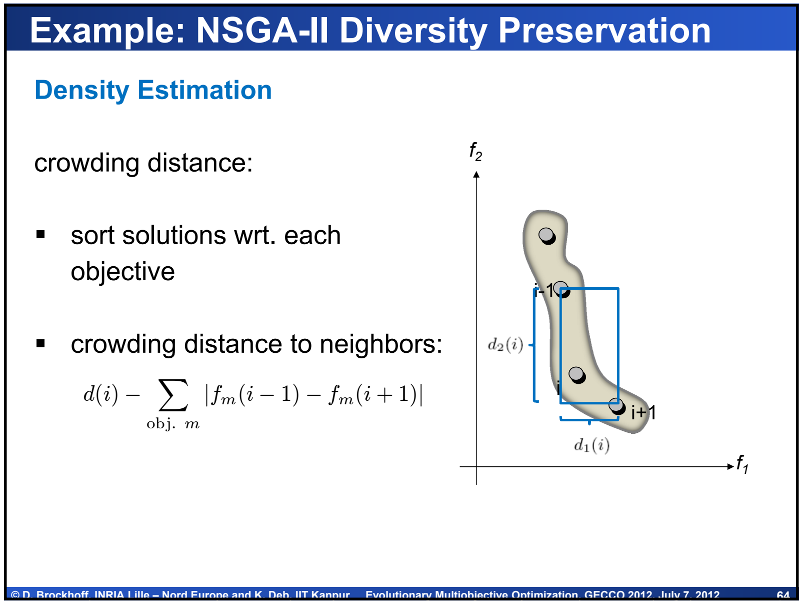


Example: SPEA2 Diversity Preservation

Density Estimation

k-th nearest neighbor method:

- Fitness = $R + \underbrace{1 / (2 + D_k)}_{< 1}$
- D_k = distance to the k-th nearest individual
- Usually used: k = 2



Example: NSGA-II Diversity Preservation

Density Estimation

crowding distance:

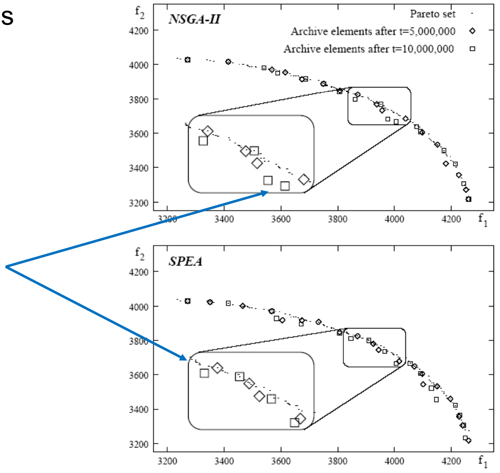
- sort solutions wrt. each objective
- crowding distance to neighbors:

$$d(i) - \sum_{\text{obj. } m} |f_m(i-1) - f_m(i+1)|$$

SPEA2 and NSGA-II: Cycles in Optimization

Selection in SPEA2 and NSGA-II can result in
deteriorative cycles

non-dominated
solutions already
found can be lost

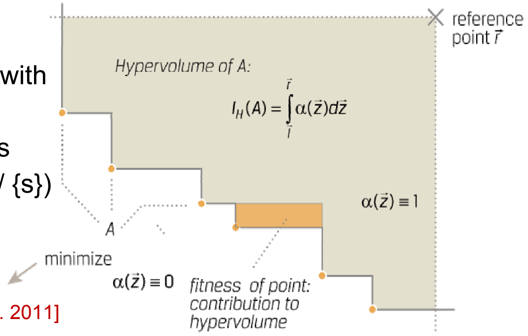


Hypervolume-Based Selection

Latest Approach (SMS-EMOA, MO-CMA-ES, HypE, …)
use hypervolume indicator to guide the search: refinement!

Main idea
Delete solutions with the smallest hypervolume loss $d(s) = I_H(P)-I_H(P / \{s\})$ iteratively

But: can also result in cycles [Judt et al. 2011] and is expensive [Bringmann and Friedrich 2009]



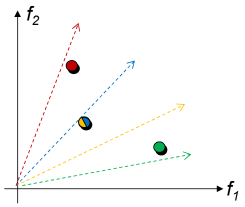
Moreover: HypE [Bader and Zitzler 2011]
Sampling + Contribution if more than 1 solution deleted

Decomposition-Based Selection: MOEA/D

MOEA/D: Multiobjective Evolutionary Algorithm Based on Decomposition [Zhang and Li 2007]

Ideas:

- Optimize N scalarizing functions in parallel
- Use only best solutions of “neighbored scalarizing function” for mating
- keep the best solutions for each scalarizing function
- use external archive for non-dominated solutions



Variation in EMO

- At first sight not different from single-objective optimization
- Most algorithm design effort on selection until now
- But: convergence to a set ≠ convergence to a point

Open Question:

- how to achieve fast convergence to a set?

Related work:

- multiobjective CMA-ES [Igel et al. 2007] [Voß et al. 2010]
- set-based variation [Bader et al. 2009]
- set-based fitness landscapes [Verel et al. 2011]

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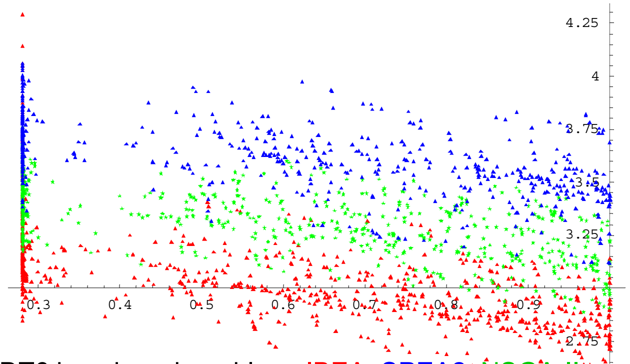
A Few Examples From Practice

Once Upon a Time...

... multiobjective EAs were mainly compared visually:



ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

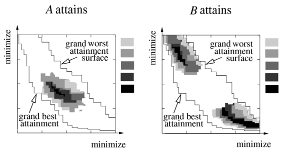
Two Approaches for Empirical Studies

Attainment function approach:

- Applies statistical tests directly to the samples of approximation sets
- Gives detailed information about how and where performance differences occur

Quality indicator approach:

- First, reduces each approximation set to a single value of quality
- Applies statistical tests to the samples of quality values

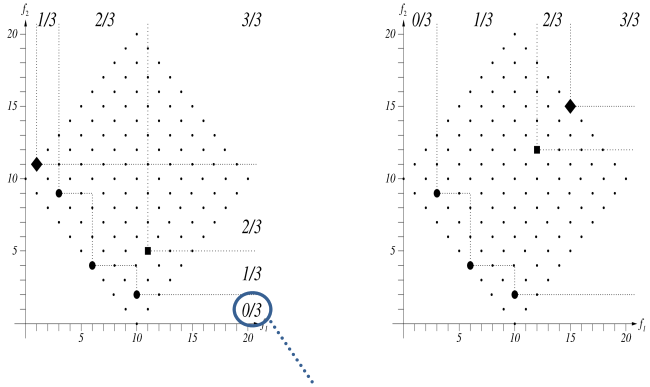


Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

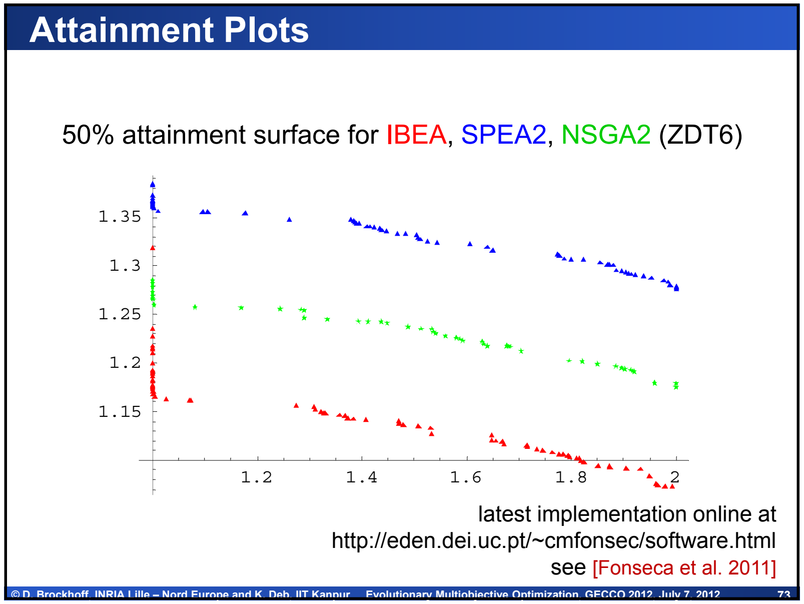
see e.g. [Zitzler et al. 2003]

Empirical Attainment Functions

three runs of two multiobjective optimizers



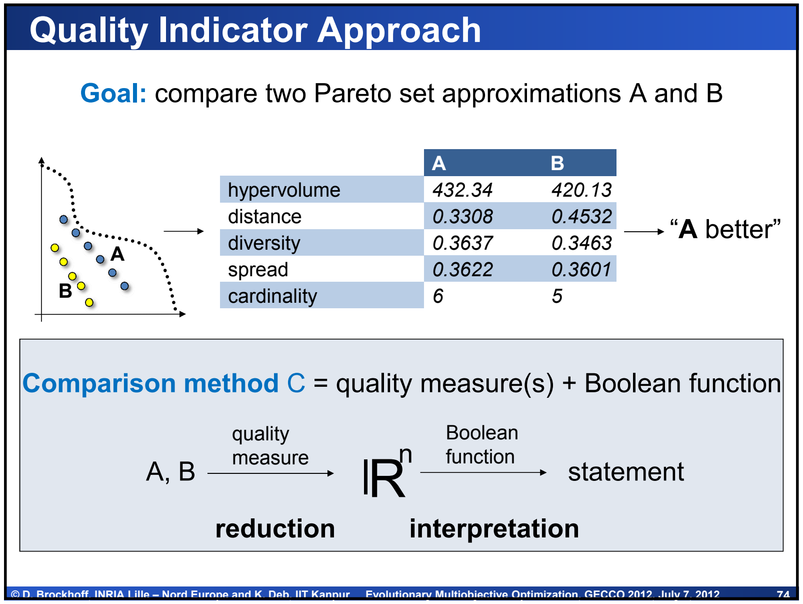
frequency of attaining regions



Attainment Plots

50% attainment surface for IBEA, SPEA2, NSGA2 (ZDT6)

latest implementation online at
http://eden.dei.uc.pt/~cmfonsec/software.html
see [Fonseca et al. 2011]



Quality Indicator Approach

Goal: compare two Pareto set approximations A and B

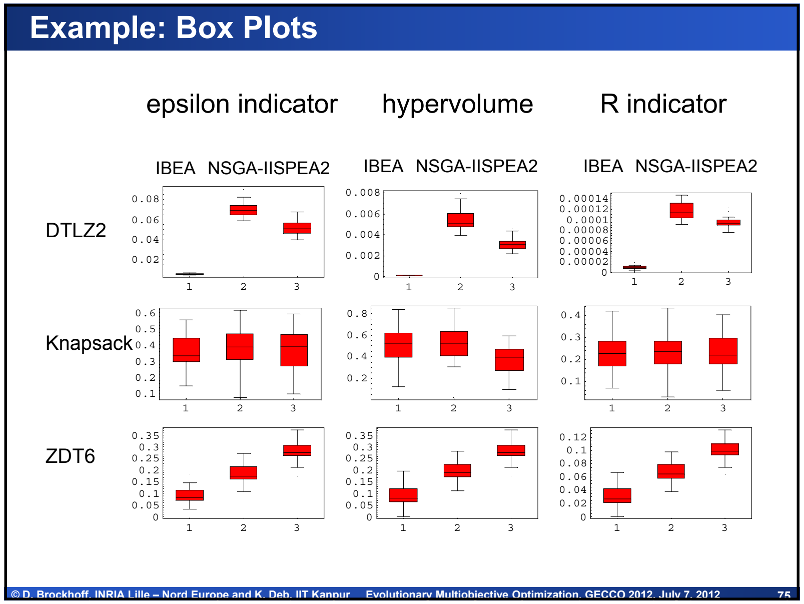
	A	B
hypervolume	*432.34*	*420.13*
distance	*0.3308*	*0.4532*
diversity	*0.3637*	*0.3463*
spread	*0.3622*	*0.3601*
cardinality	*6*	*5*

→ "**A** better"

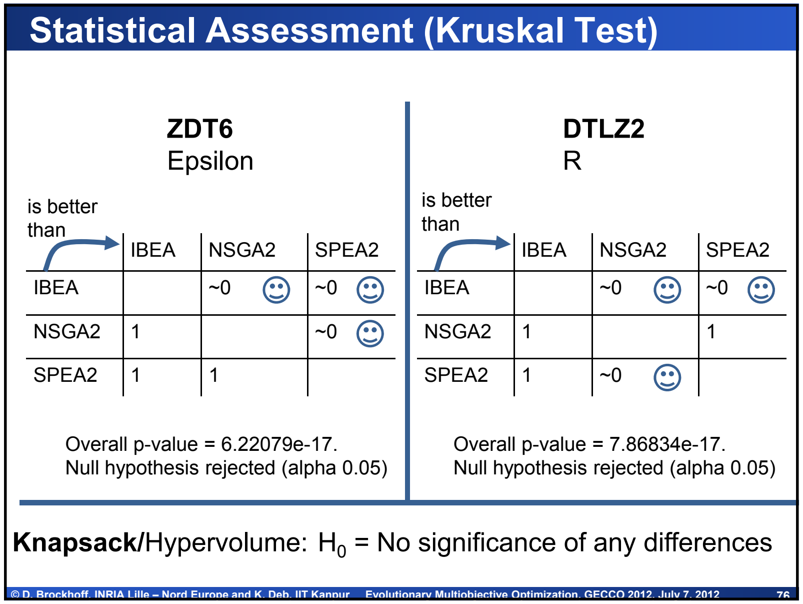
Comparison method C = quality measure(s) + Boolean function

$$A, B \xrightarrow{\text{quality measure}} \mathbb{R}^n \xrightarrow{\text{Boolean function}} \text{statement}$$

reduction **interpretation**



Example: Box Plots



Statistical Assessment (Kruskal Test)

ZDT6
Epsilon

is better than	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		~0 ☺
SPEA2	1	1	

Overall p-value = 6.22079e-17.
Null hypothesis rejected (alpha 0.05)

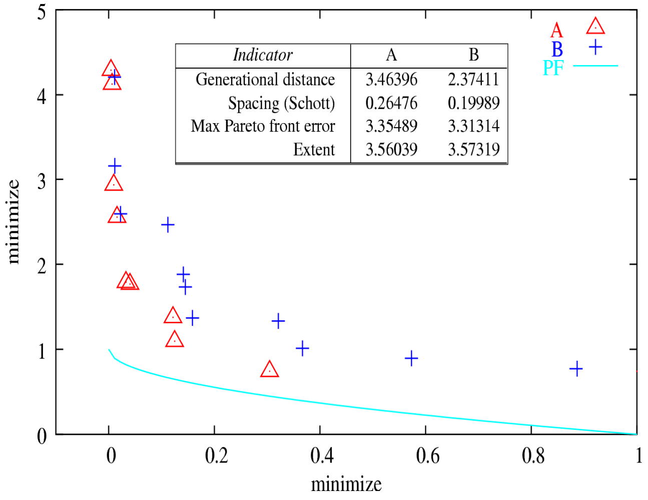
DTLZ2
R

is better than	IBEA	NSGA2	SPEA2
IBEA		~0 ☺	~0 ☺
NSGA2	1		1
SPEA2	1	~0 ☺	

Overall p-value = 7.86834e-17.
Null hypothesis rejected (alpha 0.05)

Knapsack/Hypervolume: H_0 = No significance of any differences

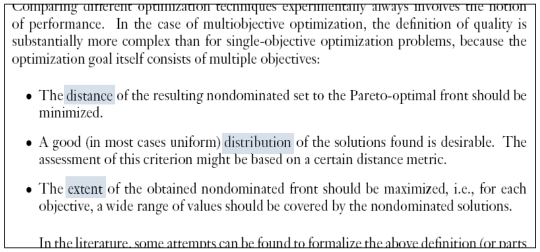
Problems With Non-Compliant Indicators



Indicator	A	B
Generational distance	3.46396	2.37411
Spacing (Schott)	0.26476	0.19989
Max Pareto front error	3.35489	3.31314
Extent	3.56039	3.57319

What Are Good Set Quality Measures?

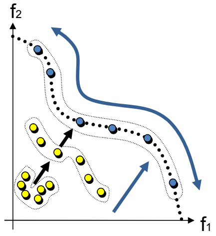
There are **three aspects** [Zitzler et al. 2000]



Comparing different optimization techniques experimentally always involves the notion of performance. In the case of multiobjective optimization, the definition of quality is substantially more complex than for single-objective optimization problems, because the optimization goal itself consists of multiple objectives:

- The distance of the resulting nondominated set to the Pareto-optimal front should be minimized.
- A good (in most cases uniform) distribution of the solutions found is desirable. The assessment of this criterion might be based on a certain distance metric.
- The extent of the obtained nondominated front should be maximized, i.e., for each objective, a wide range of values should be covered by the nondominated solutions.

In the literature, some attempts can be found to formalize the above definition (or parts



Wrong! [Zitzler et al. 2003]

An infinite number of unary set measures is needed to detect in general whether A is better than B

Set Quality Indicators

Open Questions:

- how to design a good benchmark suite?
- are there other unary indicators that are (weak) refinements?
- how to achieve good indicator values?

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Indicator-Based EMO: Optimization Goal

When the goal is to maximize a unary indicator…

- we have a single-objective set problem to solve
- but what is the optimum?
- important: population size μ plays a role!

Optimal μ-Distribution:

A set of μ solutions that maximizes a certain unary indicator I among all sets of μ solutions is called optimal μ-distribution for I. [Auger et al. 2009a]

Optimal μ-Distributions for the Hypervolume

Hypervolume indicator refines dominance relation

$\Longrightarrow$ most results on optimal μ-distributions for hypervolume

Optimal μ-Distributions (example results)

[Auger et al. 2009a]:

- contain equally spaced points iff front is linear
- density of points $\propto \sqrt{-f'(x)}$ with f' the slope of the front

[Friedrich et al. 2011]:

optimal μ-distributions for the hypervolume correspond to ε-approximations of the front

OPT	$1 + \frac{\log(\min\{A/a, B/b\})}{n}$
HYP	$1 + \frac{\sqrt{A/a} + \sqrt{B/b}}{n-4}$
logHYP	$1 + \frac{\sqrt{\log(A/a)\,\log(B/b)}}{n-2}$

! (probably) does not hold for > 2 objectives

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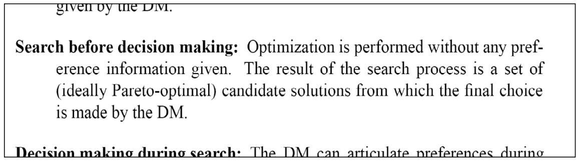
Articulating User Preferences During Search

What we thought: EMO is preference-less



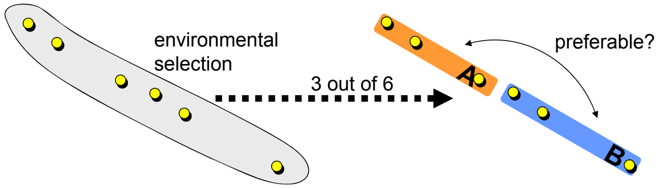
given by the DM.

Search before decision making: Optimization is performed without any preference information given. The result of the search process is a set of (ideally Pareto-optimal) candidate solutions from which the final choice is made by the DM.

Decision making during search: The DM can articulate preferences during

[Zitzler 1999]

What we learnt: EMO just uses weaker preference information

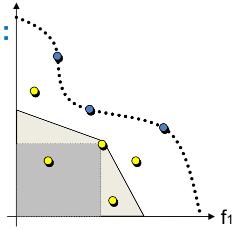


Incorporation of Preferences During Search

Nevertheless...

- the more (known) preferences incorporated the better
- in particular if search space is too large

[Branke 2008], [Rachmawati and Srinivasan 2006], [Coello Coello 2000]



❶ **Refine/modify dominance relation, e.g.:**

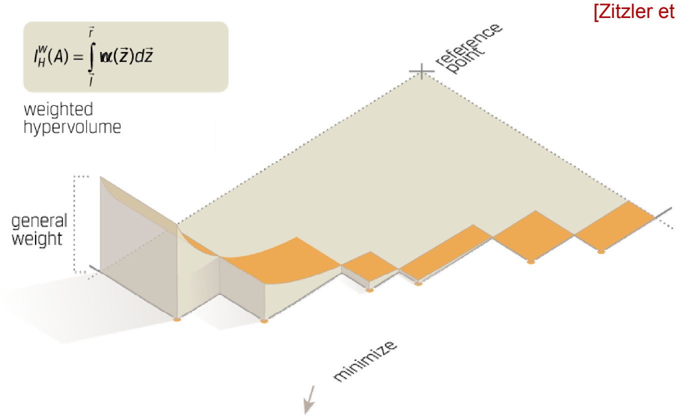
- using goals, priorities, constraints [Fonseca and Fleming 1998a,b]
- using different types of cones [Branke and Deb 2004]

❷ **Use quality indicators, e.g.:**

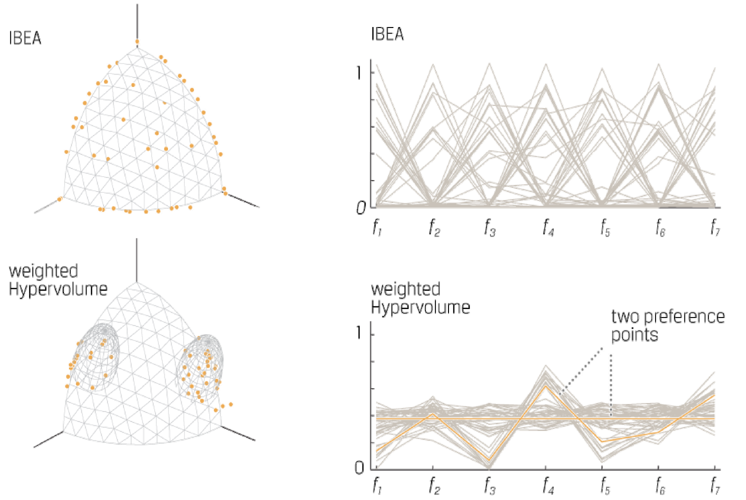
- based on reference points and directions [Deb and Sundar 2006, Deb and Kumar 2007]
- based on binary quality indicators [Zitzler and Künzli 2004]
- based on the hypervolume indicator (now) [Zitzler et al. 2007]

Example: Weighted Hypervolume Indicator

[Zitzler et al. 2007]



Weighted Hypervolume in Practice



[Auger et al. 2009b]

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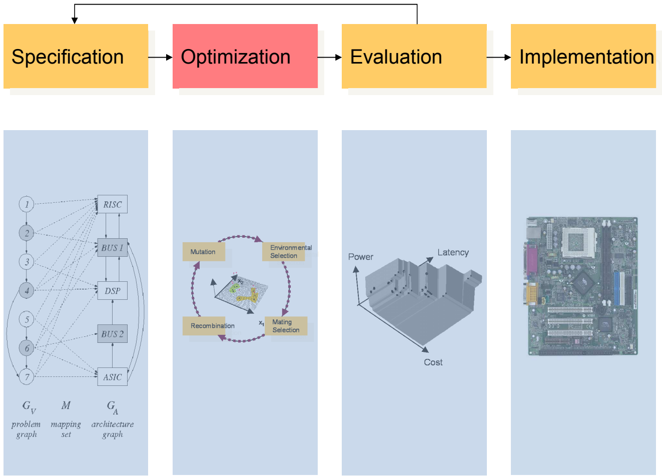
A Few Examples From Practice

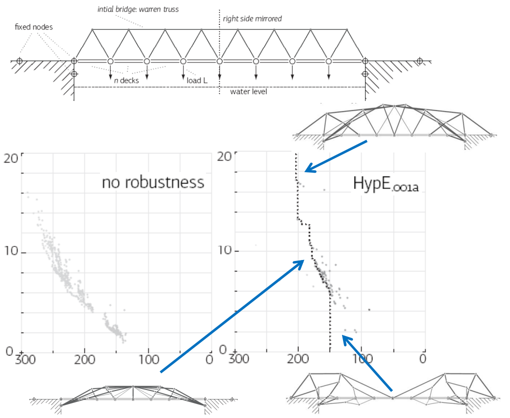
Application: Design Space Exploration

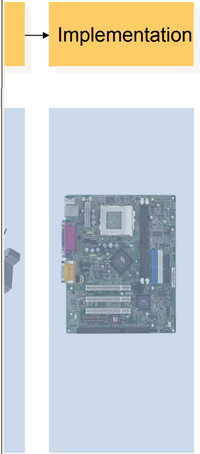


Application: Design Space Exploration

Truss Bridge Design

[Bader 2010]





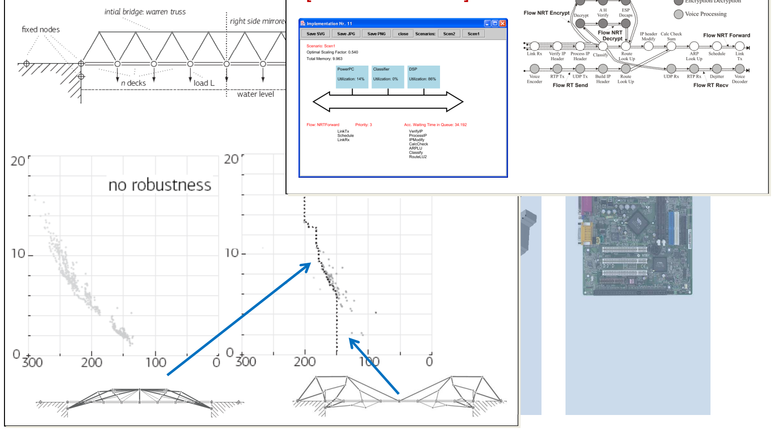
Application: Design Space Exploration

Truss Bridge Design

[Bader 2010]

Network Processor Design

[Thiele et al. 2002]



Application: Design Space Exploration

Truss Bridge Design

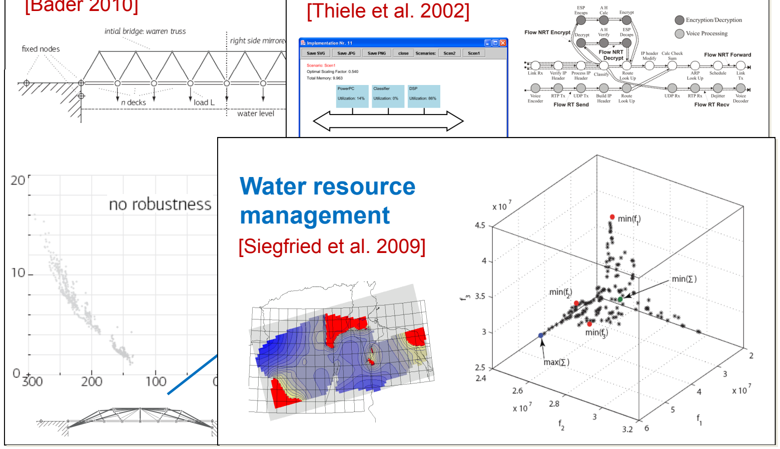
[Bader 2010]

Network Processor Design

[Thiele et al. 2002]

Water resource management

[Siegfried et al. 2009]

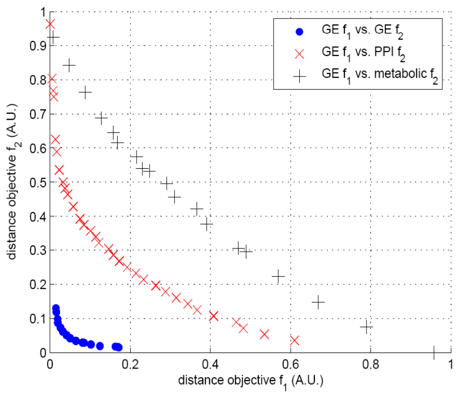


Application: Trade-Off Analysis

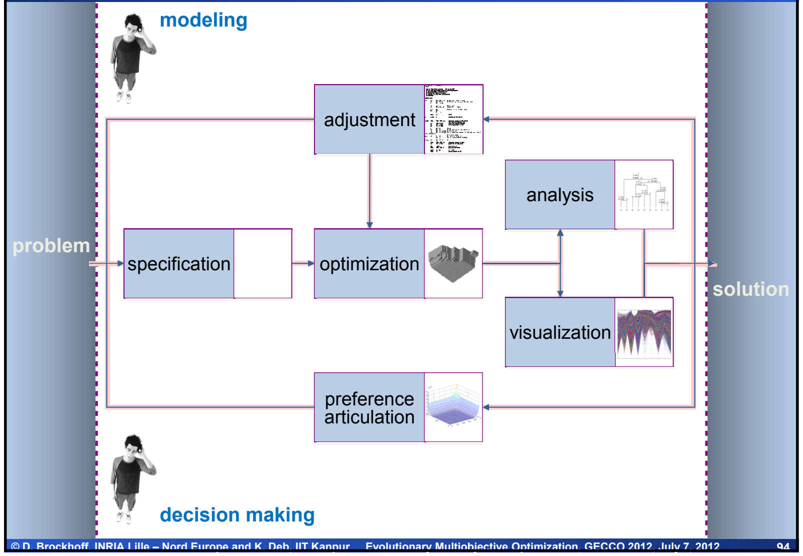
Module identification from biological data [Calonder et al. 2006]

Find group of genes wrt different data types:

- similarity of gene expression profiles
- overlap of protein interaction partners
- metabolic pathway map distances



Conclusions: EMO as Interactive Decision Support



modeling

problem

adjustment

analysis

specification

optimization

visualization

preference articulation

solution

decision making

The EMO Community

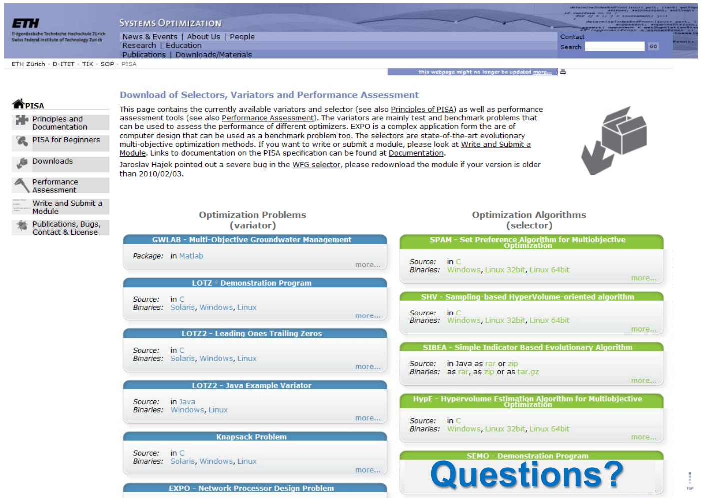
Links:

- EMO mailing list: *http://w3.ualg.pt/lists/emo-list/*
- EMO bibliography: *http://www.lania.mx/~ccoello/EMOO/*
- EMO conference series: *http://www.shef.ac.uk/emo2013/*

Books:

- ***Multi-Objective Optimization using Evolutionary Algorithms*** Kalyanmoy Deb, Wiley, 2001
- ***Evolutionary Algorithms for Solving Multi Evolutionary Algorithms for Solving Multi-Objective Problems Objective Problems***, Carlos A. Coello Coello, David A. Van Veldhuizen & Gary B. Lamont, Kluwer, 2nd Ed. 2007
- **Multiobjective Optimization—Interactive and Evolutionary Approaches**, J. Branke, K. Deb, K. Miettinen, and R. Slowinski, editors, volume 5252 of *LNCS*. Springer, 2008 [many open questions!]
- and more…

PISA: http://www.tik.ee.ethz.ch/pisa/



Download of Selectors, Variators and Performance Assessment

This page contains the currently available variators and selector (see also Principles of PISA) as well as performance assessment tools (see also Performance Assessment). The variators are mainly test and benchmark problems that can be used to assess the performance of different optimizers. EXPO is a complex application form the are of computer design that can be used as a benchmark problem too. The selectors are state-of-the-art evolutionary multi-objective optimization methods. If you want to write or submit a module, please look at Write and Submit a Module. Links to documentation on the PISA specification can be found at Documentation.

Jaroslav Hajek pointed out a severe bug in the WFG selector, please redownload the module if your version is older than 2010/02/03.

Optimization Problems (variator)

GWLAB - Multi-Objective Groundwater Management
Package: in Matlab
more...

LOTZ - Demonstration Program
Source: in C
Binaries: Solaris, Windows, Linux
more...

LOTZ2 - Leading Ones Trailing Zeros
Source: in C
Binaries: Solaris, Windows, Linux
more...

LOTZ2 - Java Example Variator
Source: in Java
Binaries: Windows, Linux
more...

Knapsack Problem
Source: in C
Binaries: Solaris, Windows, Linux
more...

EXPO - Network Processor Design Problem

Optimization Algorithms (selector)

SPAM - Set Preference Algorithm for Multiobjective Optimization
Source: in C
Binaries: Windows, Linux 32bit, Linux 64bit
more...

SHV - Sampling-based HyperVolume-oriented algorithm
Source: in C
Binaries: Windows, Linux 32bit, Linux 64bit
more...

SIBEA - Simple Indicator Based Evolutionary Algorithm
Source: in Java as rar or zip
Binaries: as rar, as zip or as tar.gz
more...

HypE - Hypervolume Estimation Algorithm for Multiobjective Optimization
Source: in C
Binaries: Windows, Linux 32bit, Linux 64bit
more...

SEMO - Demonstration Program

Questions?

Additional Slides

Instructor Biography: Dimo Brockhoff

Dimo Brockhoff

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After obtaining his diploma in computer science (Dipl.-Inform.) from University of Dortmund, Germany in 2005, Dimo Brockhoff received his PhD (Dr. sc. ETH) from ETH Zurich, Switzerland in 2009. Between June 2009 and October 2011 he held postdoctoral research positions---first at INRIA Saclay Ile-de-France in Orsay and then at Ecole Polytechnique in Palaiseau, both in France. Since November 2011 he has been a junior researcher (CR2) at INRIA Lille - Nord Europe in Villeneuve d'Ascq, France . His research interests are focused on evolutionary multiobjective optimization (EMO), in particular on many-objective optimization, benchmarking, and theoretical aspects of indicator-based search.

Instructor Biography: Kalyanmoy Deb

Kalyanmoy Deb

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He holds Deva Raj Chair Professor at Indian Institute of Technology Kanpur in India. He is the recipient of the prestigious MCDM Edgeworth-Pareto award by the Multiple Criterion Decision Making (MCDM) Society, one of the highest awards given in the field of multi-criterion optimization and decision making. He has also received prestigious Shanti Swarup Bhatnagar Prize in Engineering Sciences for the year 2005 from Govt. of India.

He has also received the `Thomson Citation Laureate Award' from Thompson Scientific for having highest number of citations in Computer Science during the past ten years in India. He is a fellow of Indian National Academy of Engineering (INAE), Indian National Academy of Sciences, and International Society of Genetic and Evolutionary Computation (ISGEC). He has received Fredrick Wilhelm Bessel Research award from Alexander von Humboldt Foundation in 2003. His main research interests are in the area of computational optimization, modeling and design, and evolutionary algorithms. He has written two text books on optimization and more than 240 international journal and conference research papers. He has pioneered and a leader in the field of evolutionary multi-objective optimization. He is associate editor of two major international journals and an editorial board members of five major journals.

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