

# Expressive Genetic Programming

*Tutorial*  
2012 Genetic and Evolutionary Computation Conference  
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## Instructor

**Lee Spector** is a Professor of Computer Science in the School of Cognitive Science at Hampshire College in Amherst, Massachusetts, and an adjunct professor in the Department of Computer Science at the University of Massachusetts, Amherst. He received a B.A. in Philosophy from Oberlin College in 1984 and a Ph.D. from the Department of Computer Science at the University of Maryland in 1992. His areas of teaching and research include genetic and evolutionary computation, quantum computation, and a variety of intersections between computer science, cognitive science, evolutionary biology, and the arts. He is the Editor-in-Chief of the journal *Genetic Programming and Evolvable Machines* (published by Springer) and a member of the editorial board of *Evolutionary Computation* (published by MIT Press). He is also a member of the SIGEVO executive committee and he was named a Fellow of the International Society for Genetic and Evolutionary Computation.

## Tutorial Description (1)

The language in which evolving programs are expressed can have significant impacts on the problem-solving capabilities of a genetic programming system. These impacts stem both from the absolute computational power of the languages that are used, as elucidated by formal language theory, and from the ease with which various computational structures can be produced by random code generation and by the action of genetic operators. Highly expressive languages can facilitate the evolution of programs for any computable function using, when appropriate, multiple data types, evolved subroutines, evolved control structures, evolved data structures, and evolved modular program and data architectures. In some cases expressive languages can even support the evolution of programs that express methods for their own reproduction and variation (and hence for the evolution of their offspring).

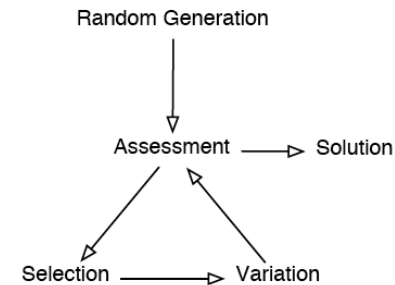
## Tutorial Description (2)

This tutorial will begin with a comparative survey of approaches to the evolution of programs in expressive programming languages ranging from machine code to graphical and grammatical representations. Within this context it will then provide a detailed introduction to the Push programming language, which was designed specifically for expressiveness and specifically for use in genetic programming systems. Push programs are syntactically unconstrained but can nonetheless make use of multiple data types and express arbitrary control structures, supporting the evolution of complex, modular programs in a particularly simple and flexible way. The Push language will be described and ten years of Push-based research, including the production of human-competitive results, will be briefly surveyed. The tutorial will conclude with a discussion of recent enhancements to Push that are intended to support the evolution of complex and robust software systems.

# Course Agenda

- Genetic Programming refresher
- Why evolve programs in expressive languages?
- Expressivity and evolvability
- Expressive trees, bits, graphs, grammars, stacks
- **Push**
- Expressing the future

# Evolutionary Computation



# Evolution, the Designer

“Darwinian evolution is itself a designer worthy of significant respect, if not religious devotion.” *Boston Globe* OpEd, Aug 29, 2005

WHAT WOULD DARWIN SAY? | LEE SPECTOR  
**And now, digital evolution**

The Boston Globe

By Lee Spector | August 29, 2005

RECENT developments in computer science provide new perspective on “intelligent design,” the view that life’s complexity could only have arisen through the hand of an intelligent designer. These developments show that complex and useful designs can indeed emerge from random Darwinian processes.

# Genetic Programming (GP)

- Evolutionary computing to produce executable computer programs
- Programs are assessed by executing them
- Automatic programming; producing software
- Potential (?): evolve software at all scales, including and surpassing the most ambitious and successful products of human software engineering

# Program Representations

- Lisp-style symbolic expressions (Koza, ...).
- Purely functional/lambda expressions (Walsh, Yu, ...).
- Linear sequences of machine/byte code (Nordin et al., ...).
- Artificial assembly-like languages (Ray, Adami, ...).
- Stack-based languages (Perkis, Spector, Stoffel, Tchernev, ...).
- Graph-structured programs (Teller, Globus, ...).
- Object hierarchies (Bruce, Abbott, Schmutter, Lucas, ...).
- Fuzzy rule systems (Tunstel, Jamshidi, ...).
- Logic programs (Osborn, Charif, Lamas, Dubossarsky, ...).
- Strings, grammar-mapped to arbitrary languages (O'Neill, Ryan, ...).

# Mutating Lisp

```
(+ (* X Y)
   (+ 4 (- Z 23)))
```

```
(+ (* X Y)
   (+ 4 (- Z 23)))
```

```
(+ (- (+ 2 2) Z)
   (+ 4 (- Z 23)))
```

# Recombining Lisp

```
Parent 1: (+ (* X Y)
              (+ 4 (- Z 23)))
```

```
Parent 2: (- (* 17 (+ 2 X))
              (* (- (* 2 Z) 1)
                 (+ 14 (/ Y X))))
```

```
Child 1: (+ (- (* 2 Z) 1)
            (+ 4 (- Z 23)))
```

```
Child 2: (- (* 17 (+ 2 X))
            (* (* X Y)
               (+ 14 (/ Y X))))
```

# Symbolic Regression

A simple example

Given a set of data points, evolve a program that produces  $y$  from  $x$ .

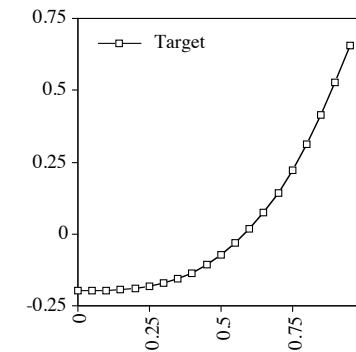
Primordial ooze: +, -, \*, %, x, 0.1

Fitness = error (smaller is better)

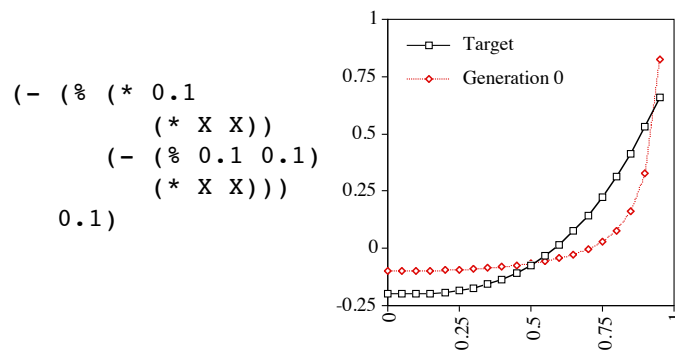
# GP Parameters

Maximum number of Generations: 51  
 Size of Population: 1000  
 Maximum depth of new individuals: 6  
 Maximum depth of new subtrees for mutants: 4  
 Maximum depth of individuals after crossover: 17  
 Fitness-proportionate reproduction fraction: 0.1  
 Crossover at any point fraction: 0.3  
 Crossover at function points fraction: 0.5  
 Selection method: FITNESS-PROPORTIONATE  
 Generation method: RAMPED-HALF-AND-HALF  
 Randomizer seed: 1.2

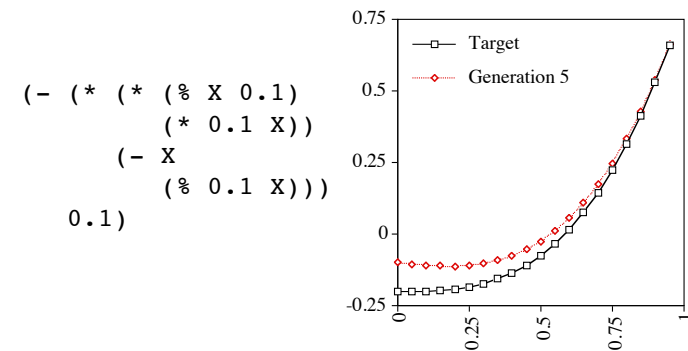
# Evolving $y = x^3 - 0.2$



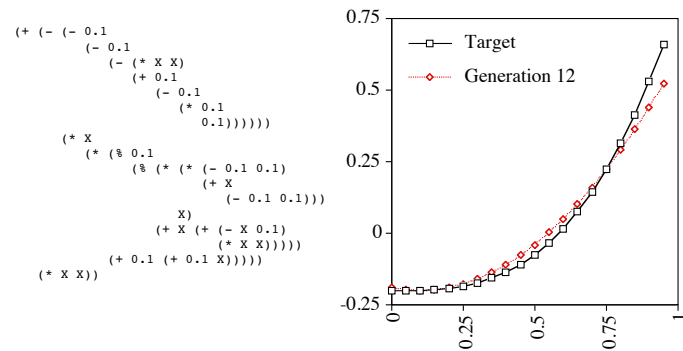
## Best Program, Gen 0



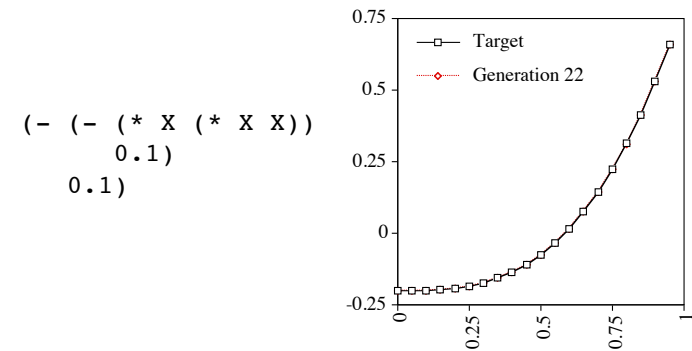
## Best Program, Gen 5



## Best Program, Gen 12



## Best Program, Gen 22



## Expressiveness

- Turing machine tables
- Lambda calculus expressions
- Register machine programs
- Partial recursive functions
- etc.

## Pragmatics

The fact that a computation *can* be expressed in a formalism does not imply that a correct expression can be produced in that formalism by a human programmer or by an evolutionary process.

## Tricks

- Cars, airplanes, and other complex engineered artifacts...
- Evolved biological organisms...
- Large-scale software systems...  
... are each composed of millions of specialized parts, chosen, in each case, from a portfolio of domain-specialized components and processes.

## Code Tricks

- Data abstraction and organization  
Data types, variables, name spaces, data structures, ...
- Control abstraction and organization  
Conditionals, loops, modules, threads, ...

## Tricks via GP (1)

- Specialize GP techniques to directly support “code trick” syntax of human programming languages
- Strongly typed genetic programming
- Automatically defined functions
- Automatically defined macros
- Architecture altering operations

## Tricks via GP (2)

- Specialize GP techniques to **indirectly** support “code trick” syntax from human programming languages
- Repair
- Genotype/phenotype mapping
- Grammars

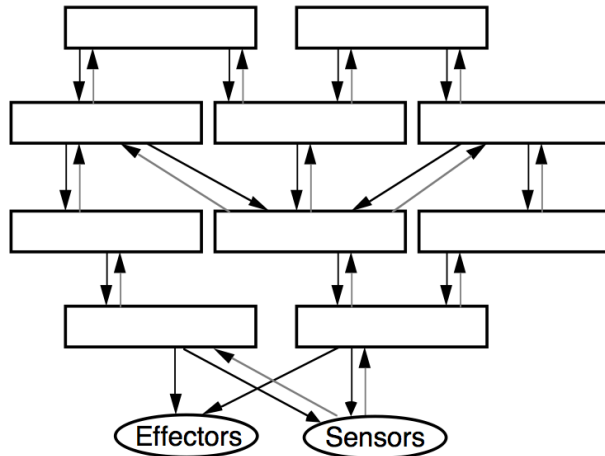
## Tricks via GP (3)

- Develop new program encodings, represented most generally as graphs
- Develop analogs of code tricks for these representations
- Specialize GP techniques to directly or indirectly support “code trick” syntax for these new program encodings

## Tricks via GP (4)

- Evolve programs in a minimal-syntax language that nonetheless supports a full range of “code tricks”
- For example: orchestrate data flows via stacks, not via syntax
- Push

## Modularity is Everywhere



## Modularity in Software

- Pervasive and widely acknowledged to be essential
- Modules may be functions, procedures, methods, classes, data structures, interfaces, etc.
- Modularity measures include coupling, cohesion, encapsulation, composability, etc.

## Modules via GP

- Automatically-defined functions
- Automatically-defined macros
- Architecture-altering operations
- Module acquisition/encapsulation systems
- Grammars for languages with modules
- Instructions that build/execute modules

## Evolving Modular Programs

With “automatically defined functions”

- All programs in the population have the same, pre-specified architecture
- Genetic operators respect that architecture
- Significant implementation costs
- Significant pre-specification
- Architecture-altering operations: more power and higher costs

## ADMs

- Macros implement control structures
- ADMs can be implemented via small tweaks to any system that supports ADFs
- Similar pros and cons to ADFs, but provide additional expressive power

## Control Structures (I)

Multiple evaluation

```
(defmacro do-twice (code)
  `(progn ,code ,code))

(do-twice (incf x))
```



## Control Structures (2)

### Conditional evaluation

```
(defmacro numeric-if (exp neg zero pos)
  `(if (< ,exp 0)
      ,neg
      (if (< 0 ,exp) ,pos ,zero)))

(numeric-if (foo) (bar) (baz) (bix))
```

## Push

- Stack-based postfix language with one stack per type
- Types include: integer, float, Boolean, name, **code**, **exec**, vector, matrix, quantum gate, [add more as needed]
- Missing argument? NOOP
- Minimal syntax:  
program → instruction | literal | ( program\* )

## Why Push?

- Highly expressive: data types, data structures, variables, conditionals, loops, recursion, modules, ...
- Elegant: minimal syntax and a simple, stack-based execution architecture
- Evolvable
- Extensible
- Supports several forms of meta-evolution

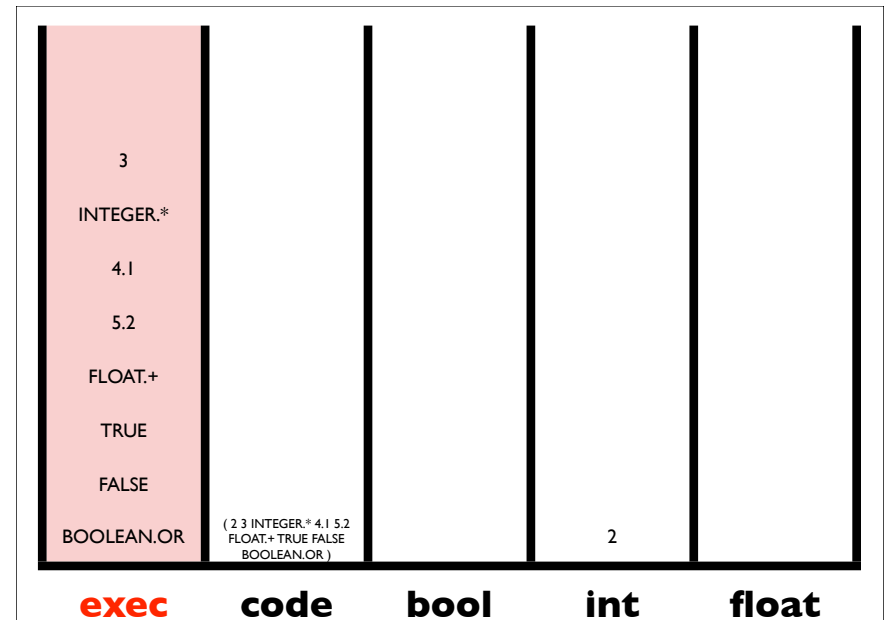
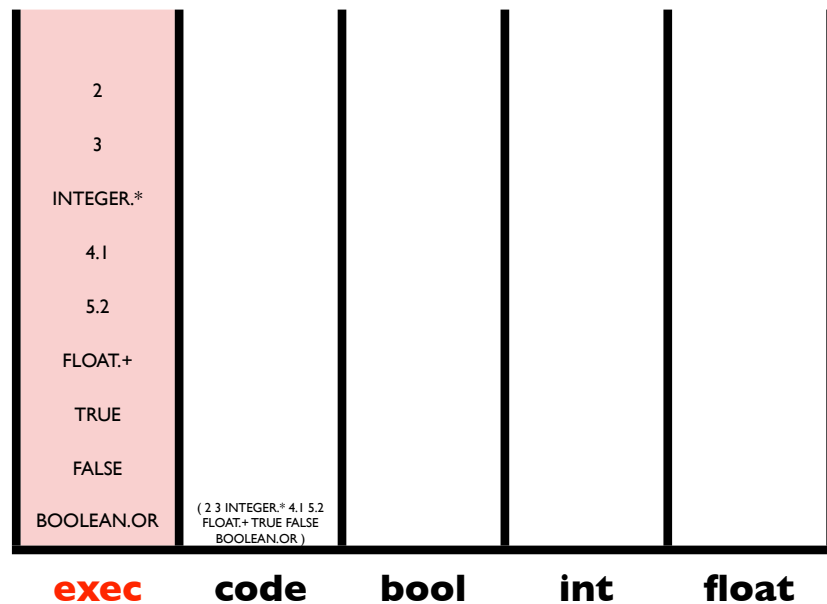
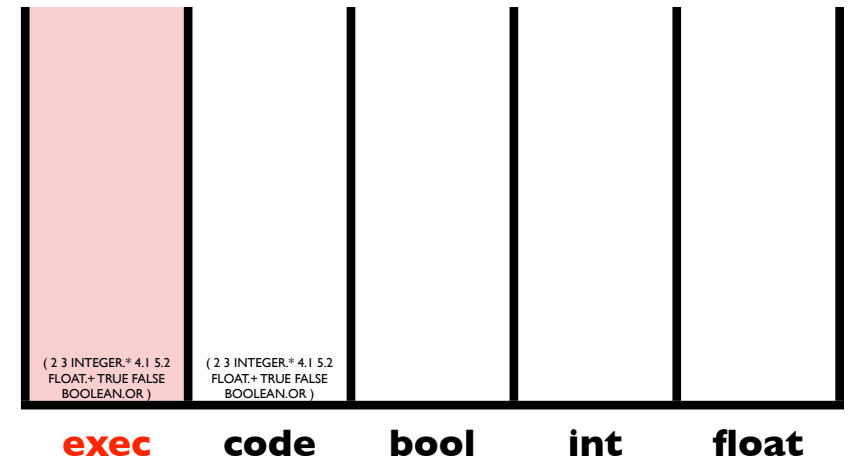
## Sample Push Instructions

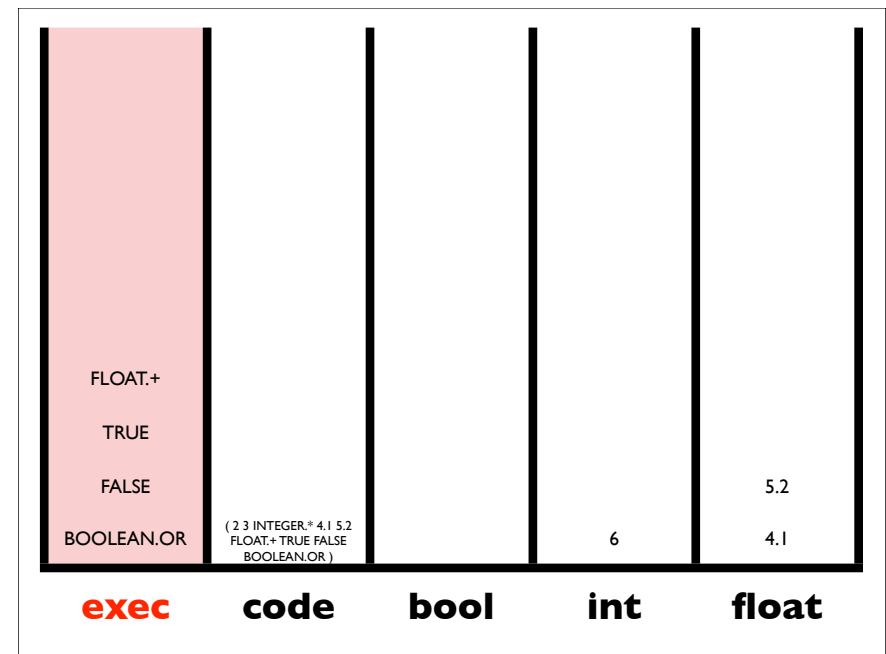
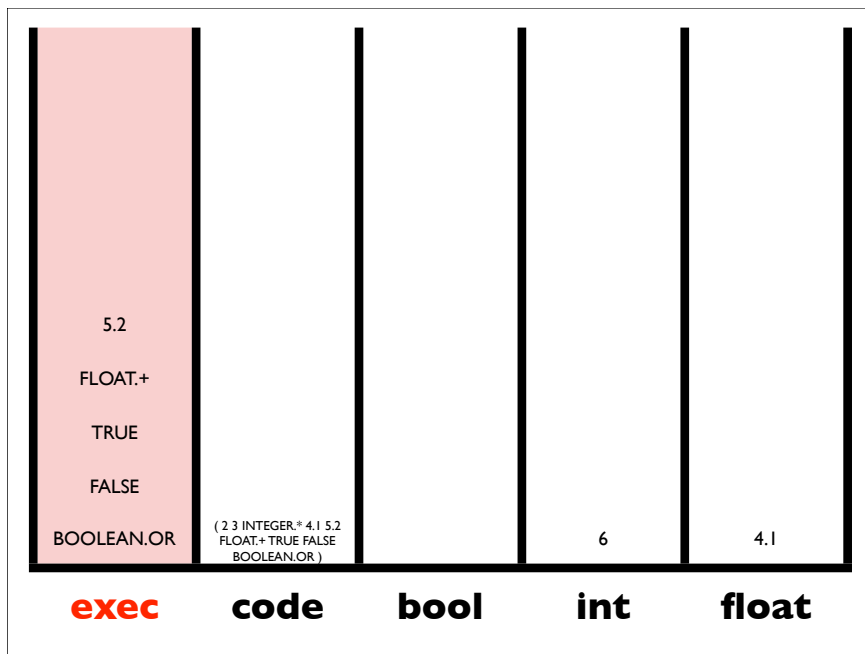
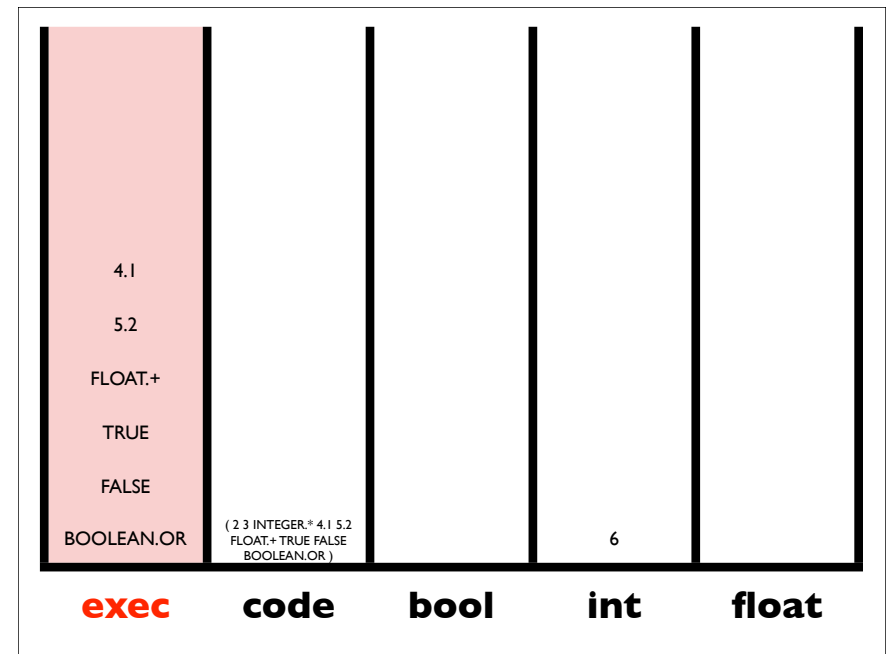
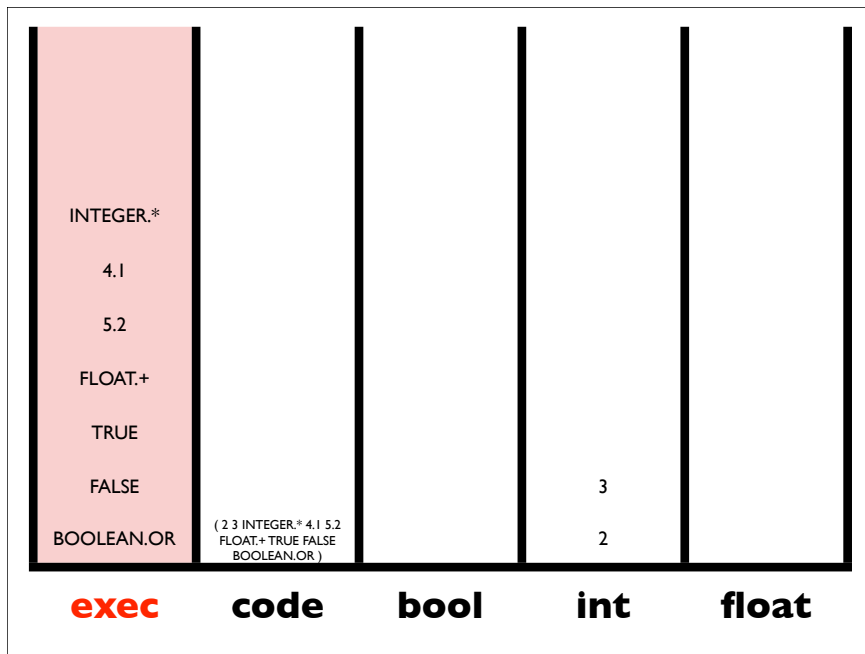
Stack manipulation instructions (all types)	POP, SWAP, YANK, DUP, STACKDEPTH, SHOVE, FLUSH, =
Math (INTEGER and FLOAT)	+, -, /, *, >, <, MIN, MAX
Logic (BOOLEAN)	AND, OR, NOT, FROMINTEGER
Code manipulation (CODE)	QUOTE, CAR, CDR, CONS, INSERT, LENGTH, LIST, MEMBER, NTH, EXTRACT
Control manipulation (CODE and EXEC)	DO*, DO*COUNT, DO*RANGE, DO*TIMES, IF

# Push(3) Semantics

- To execute program  $P$ :
  1. Push  $P$  onto the EXEC stack.
  2. While the EXEC stack is not empty, pop and process the top element of the EXEC stack,  $E$ :
    - (a) If  $E$  is an instruction: execute  $E$  (accessing whatever stacks are required).
    - (b) If  $E$  is a literal: push  $E$  onto the appropriate stack.
    - (c) If  $E$  is a list: push each element of  $E$  onto the EXEC stack, in reverse order.

( 2 3 INTEGER.\* 4.1 5.2 FLOAT.+  
TRUE FALSE BOOLEAN.OR )





TRUE FALSE BOOLEAN.OR	( 2 3 INTEGER * 4 1 5.2 FLOAT + TRUE FALSE BOOLEAN.OR )		6	9.3
<b>exec</b>	<b>code</b>	<b>bool</b>	<b>int</b>	<b>float</b>

FALSE BOOLEAN.OR	( 2 3 INTEGER * 4 1 5.2 FLOAT + TRUE FALSE BOOLEAN.OR )	TRUE	6	9.3
<b>exec</b>	<b>code</b>	<b>bool</b>	<b>int</b>	<b>float</b>

BOOLEAN.OR	( 2 3 INTEGER * 4 1 5.2 FLOAT + TRUE FALSE BOOLEAN.OR )	FALSE TRUE	6	9.3
<b>exec</b>	<b>code</b>	<b>bool</b>	<b>int</b>	<b>float</b>

	( 2 3 INTEGER * 4 1 5.2 FLOAT + TRUE FALSE BOOLEAN.OR )	TRUE	6	9.3
<b>exec</b>	<b>code</b>	<b>bool</b>	<b>int</b>	<b>float</b>

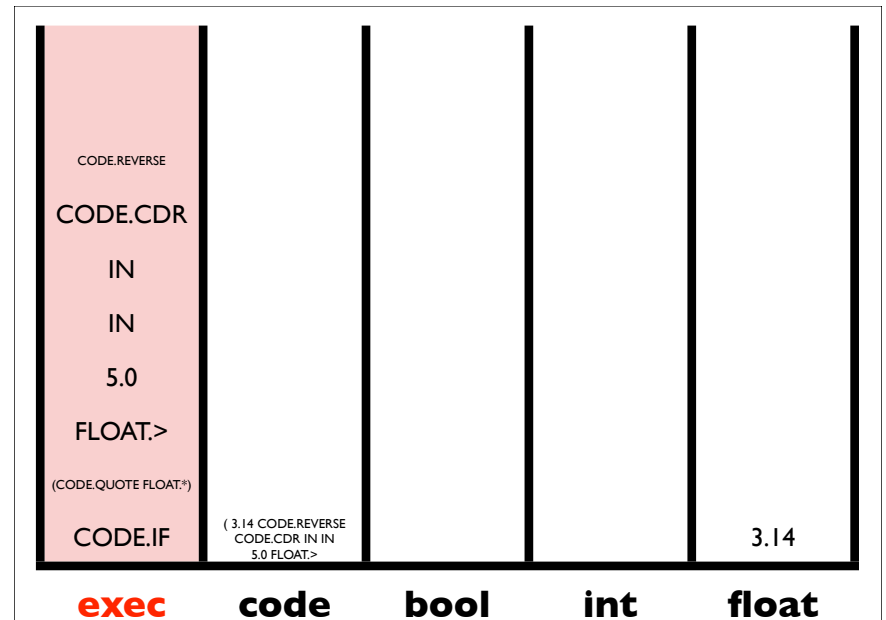
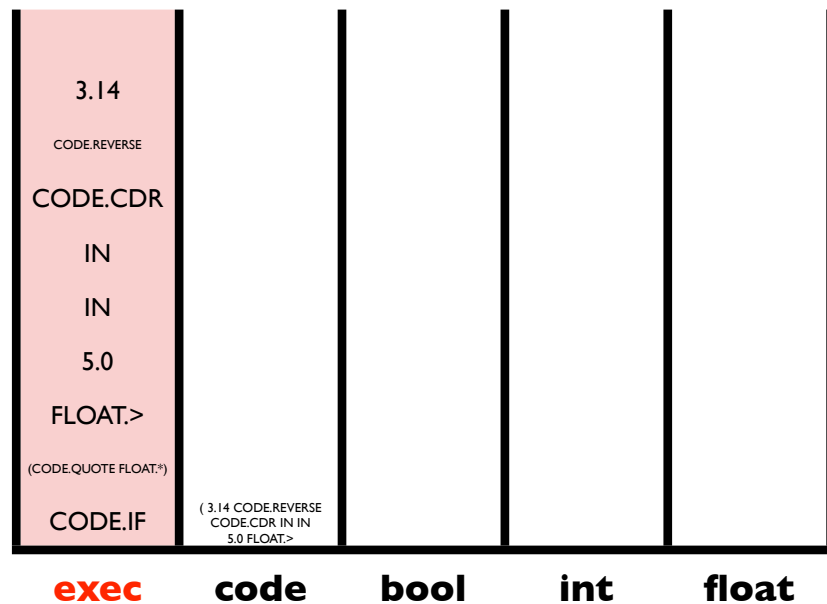
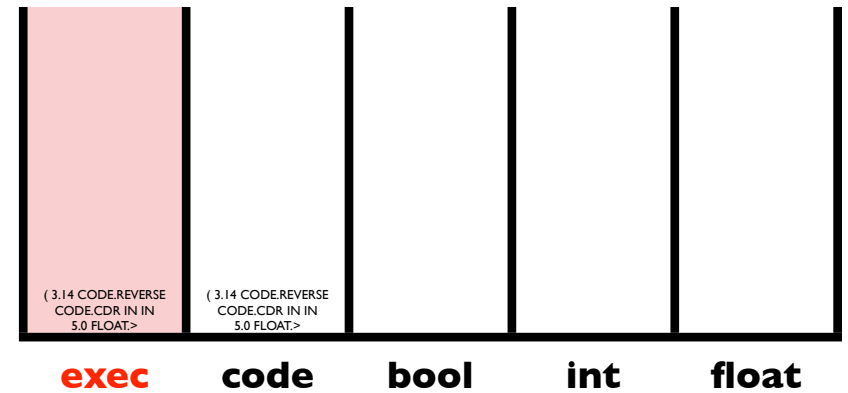
## Same Results

```
( 2 3 INTEGER.* 4.1 5.2 FLOAT.+
  TRUE FALSE BOOLEAN.OR )
```

```
( 2 BOOLEAN.AND 4.1 TRUE INTEGER./ FALSE
  3 5.2 BOOLEAN.OR INTEGER.* FLOAT.+ )
```

```
( 3.14 CODE.REVERSE CODE.CDR IN IN 5.0
  FLOAT.> (CODE.QUOTE FLOAT.*) CODE.IF )
```

IN=4.0



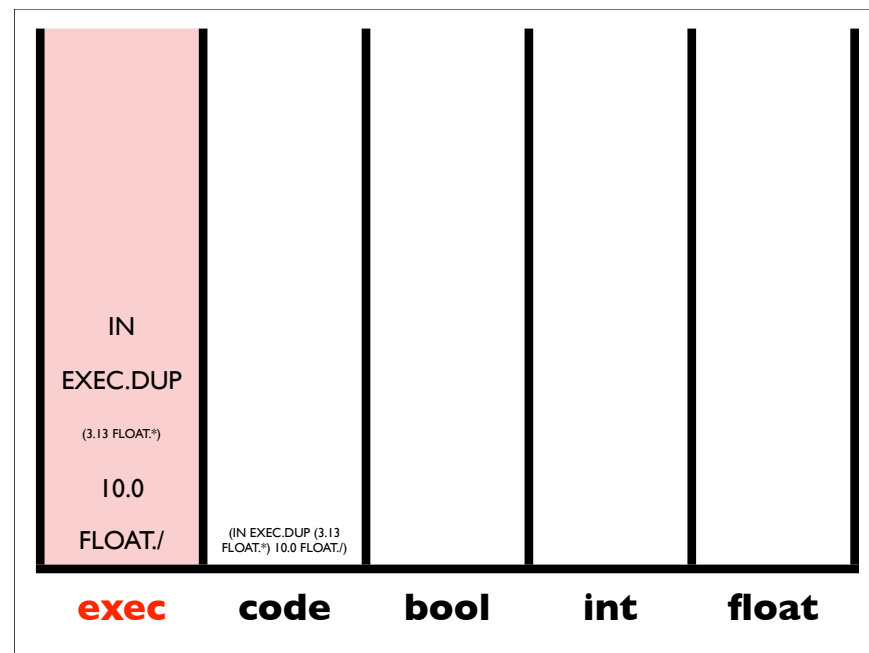
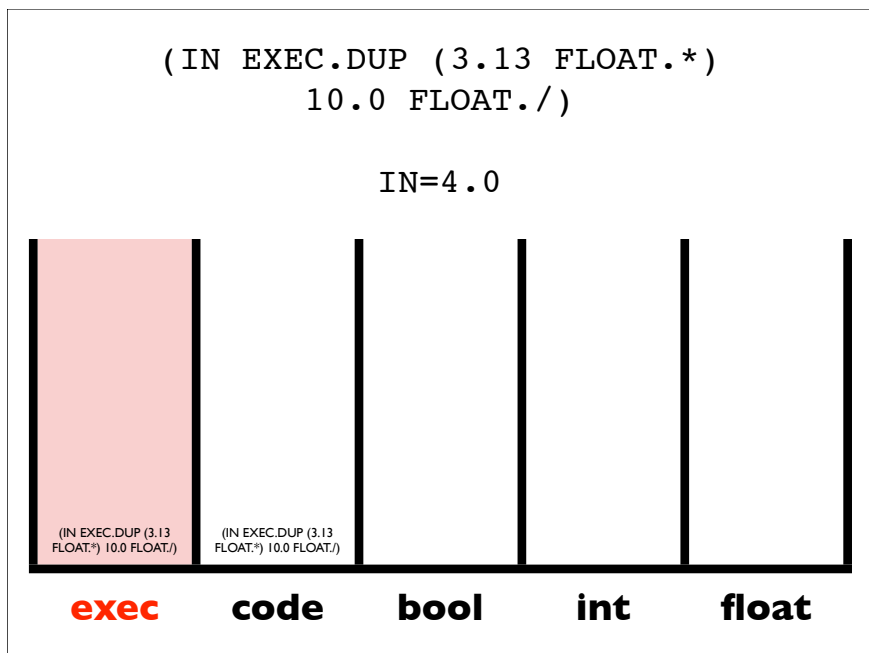
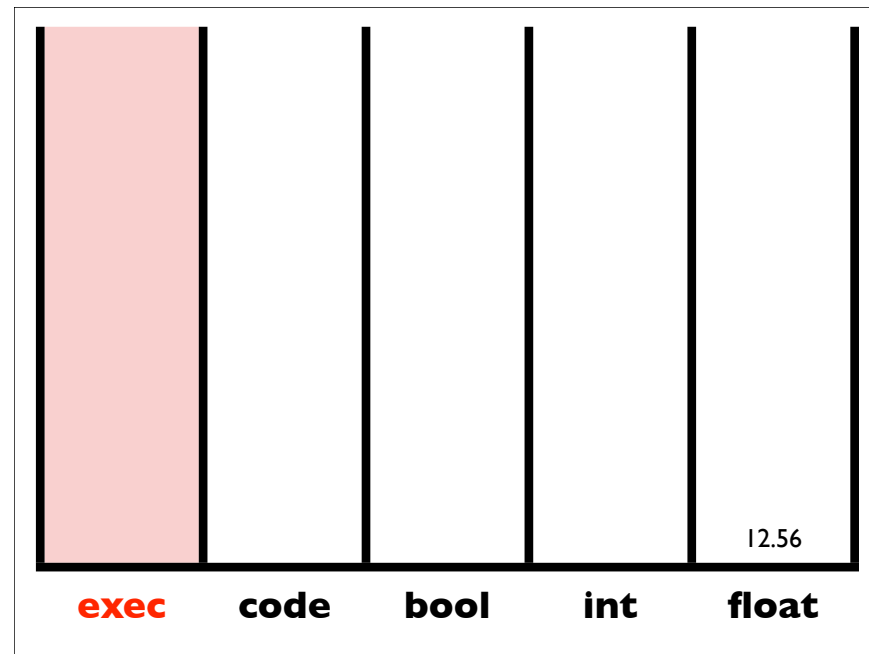
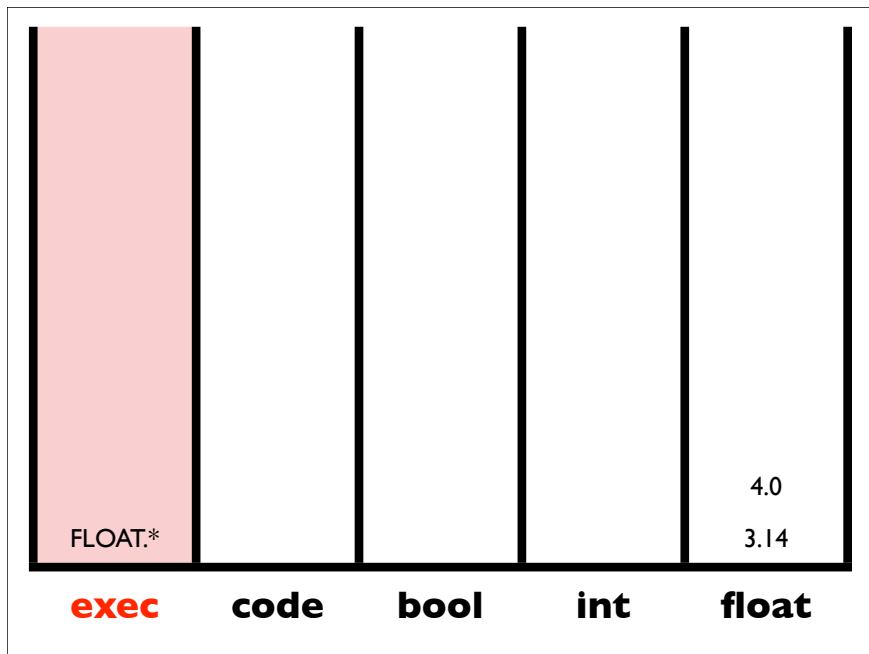
CODE.CDR				
IN				
IN				
5.0				
FLOAT.>				
(CODEQUOTE FLOAT:?)				
CODE.IF	(CODE.IF (CODEQUOTE FLOAT:?) FLOAT.> 5.0 IN IN CODE.CDR			3.14
<b>exec</b>	<b>code</b>	<b>bool</b>	<b>int</b>	<b>float</b>

IN				
IN				
5.0				
FLOAT.>				
(CODEQUOTE FLOAT:?)				
CODE.IF	((CODEQUOTE FLOAT:?) FLOAT.> 5.0 IN IN CODE.CDR			3.14
<b>exec</b>	<b>code</b>	<b>bool</b>	<b>int</b>	<b>float</b>

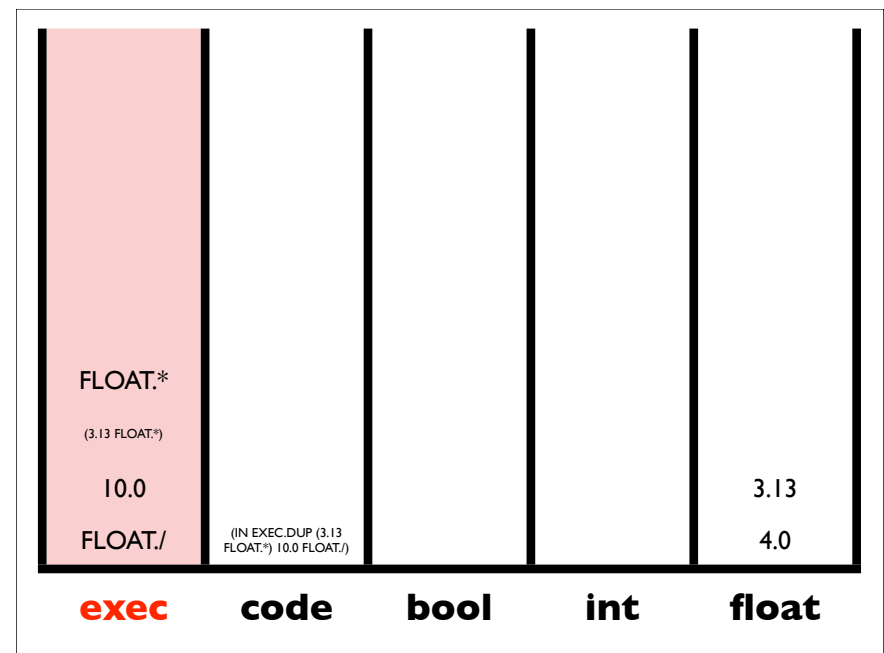
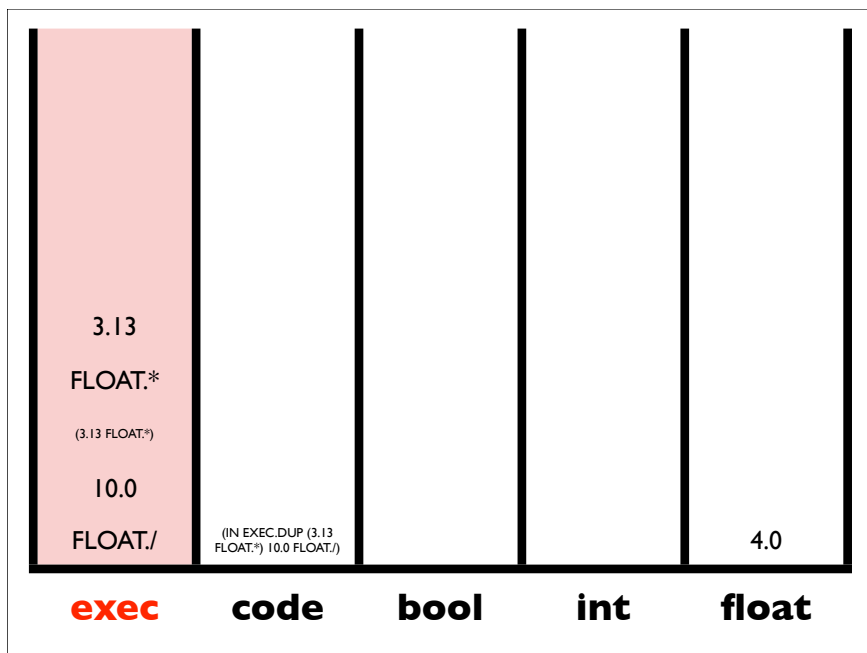
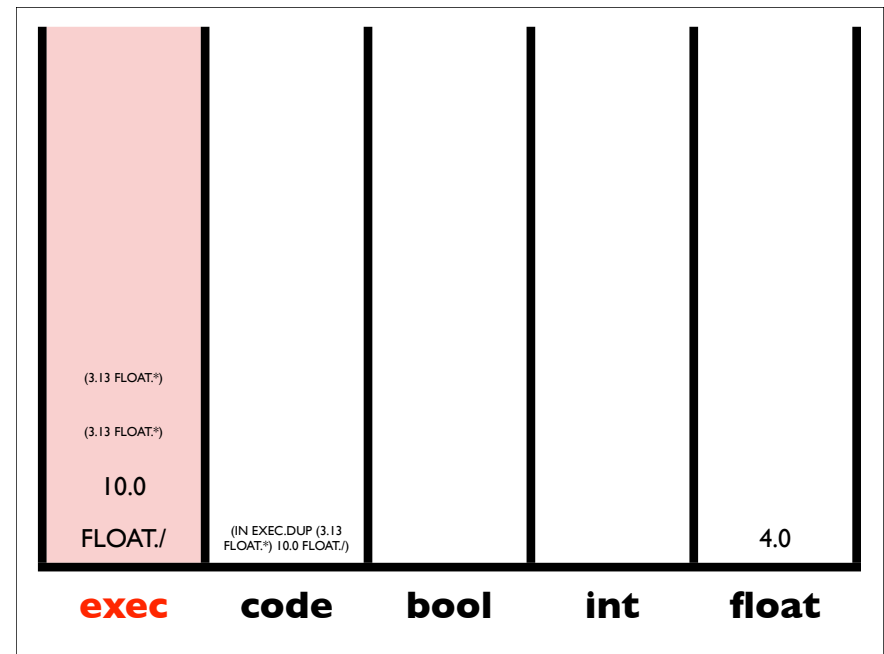
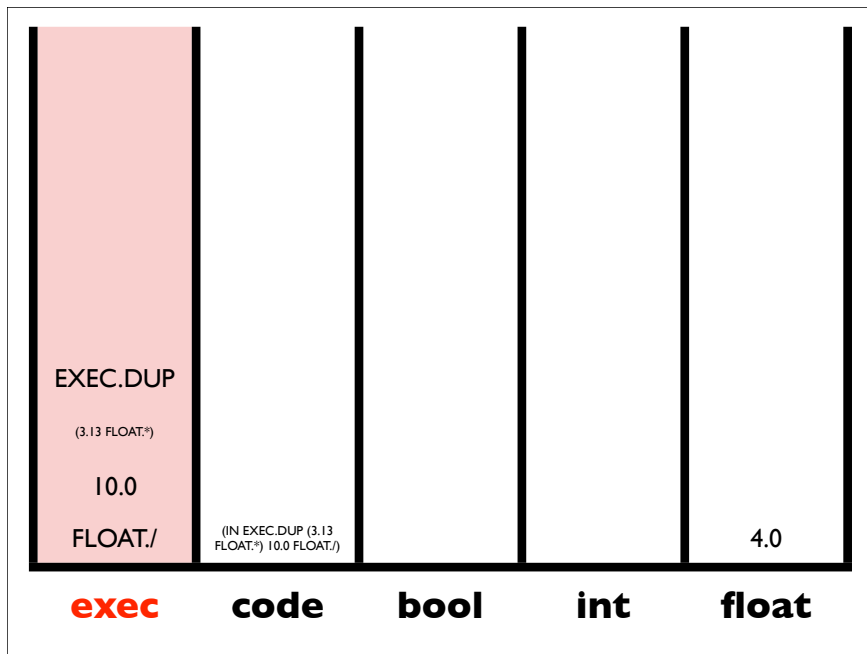
IN				
5.0				
FLOAT.>				
(CODEQUOTE FLOAT:?)				
CODE.IF	((CODEQUOTE FLOAT:?) FLOAT.> 5.0 IN IN CODE.CDR			4.0
				3.14
<b>exec</b>	<b>code</b>	<b>bool</b>	<b>int</b>	<b>float</b>

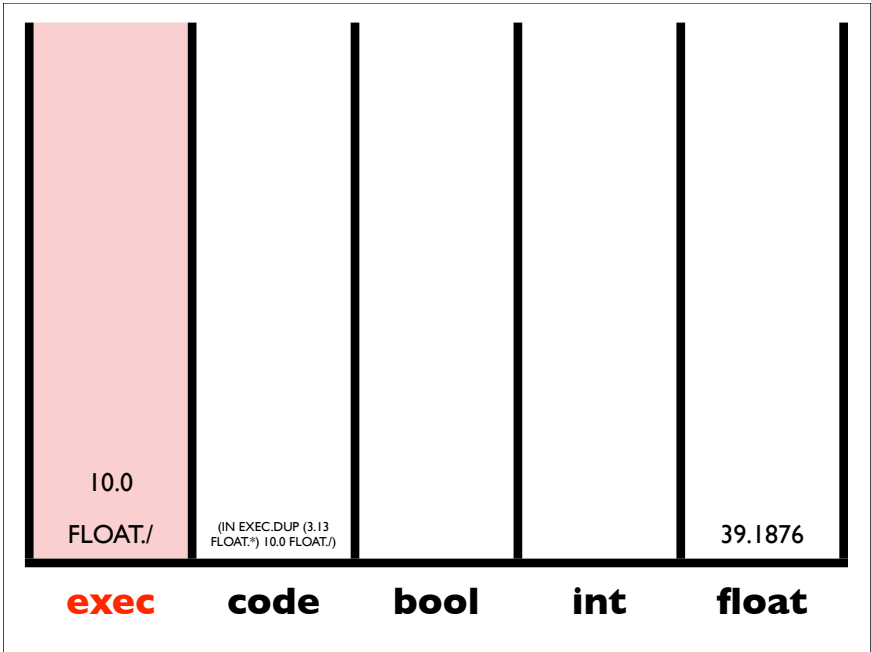
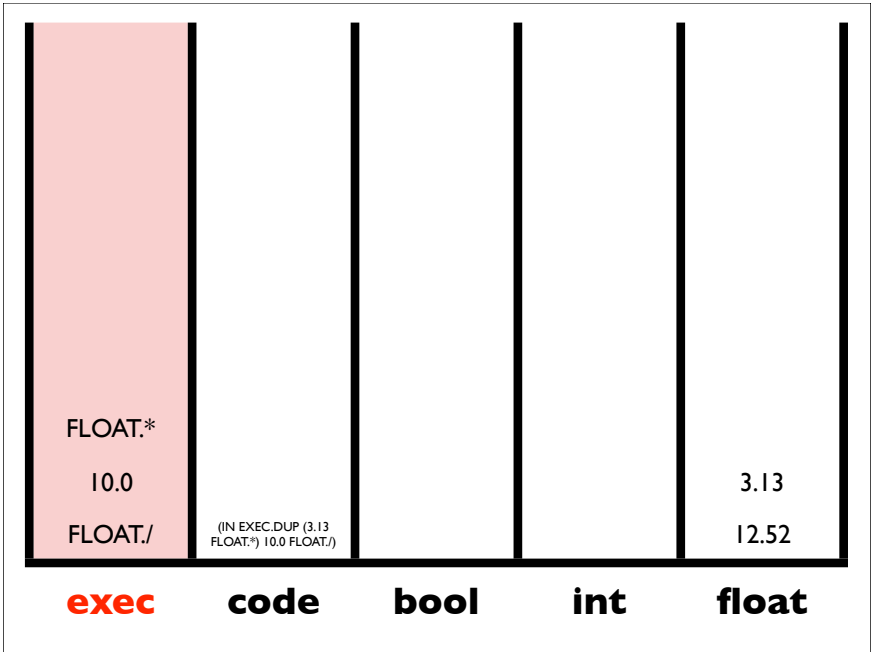
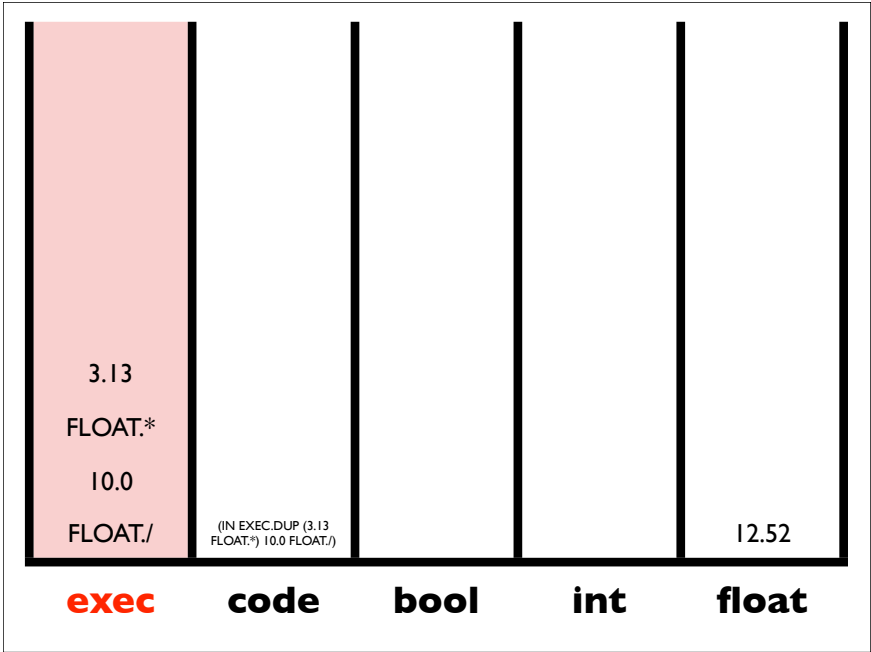
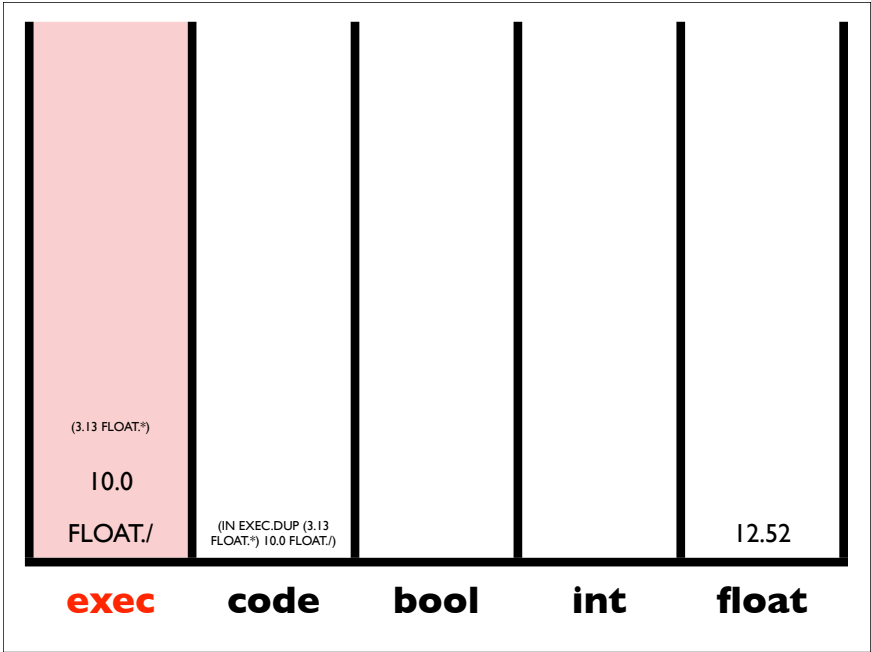
5.0				
FLOAT.>				4.0
(CODEQUOTE FLOAT:?)				4.0
CODE.IF	((CODEQUOTE FLOAT:?) FLOAT.> 5.0 IN IN CODE.CDR			3.14
<b>exec</b>	<b>code</b>	<b>bool</b>	<b>int</b>	<b>float</b>

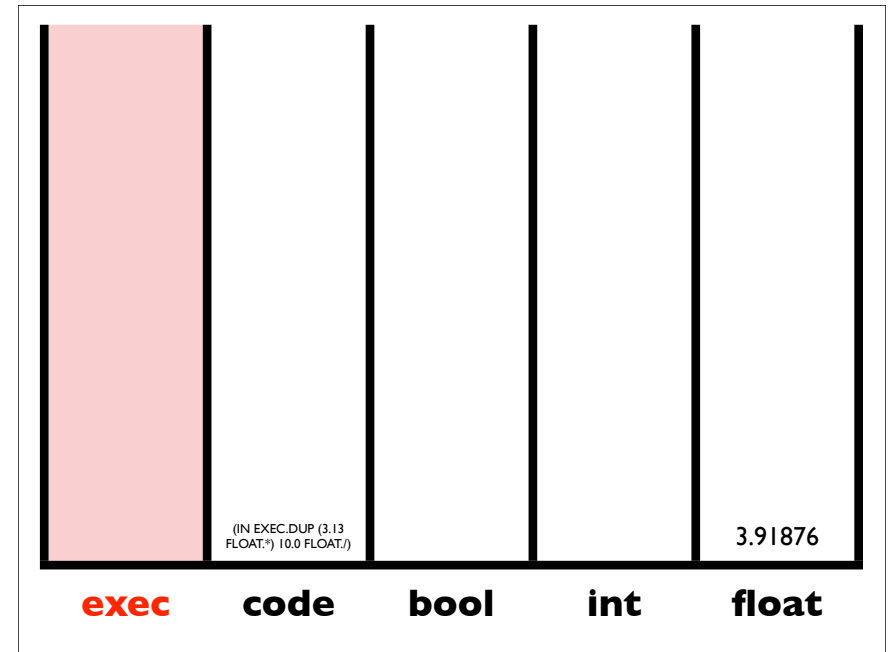
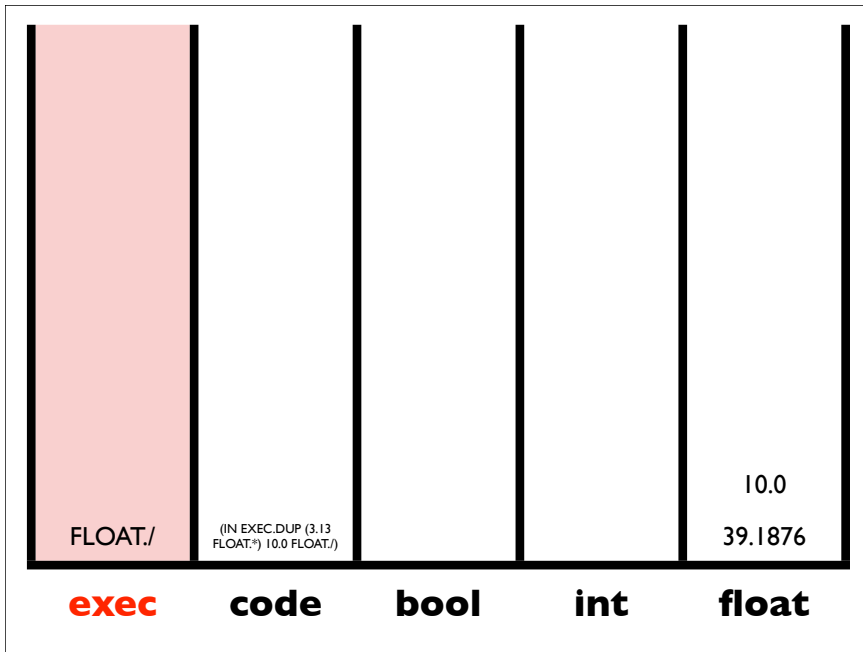












## The Odd Problem

- Integer input
- Boolean output
- Was the input odd?
- `((code.nth) code.atom)`

## Combinators

- Standard *K*, *S*, and *Y* combinators:
  - `EXEC.K` removes the second item from the `EXEC` stack.
  - `EXEC.S` pops three items (call them *A*, *B*, and *C*) and then pushes `( B C )`, *C*, and then *A*.
  - `EXEC.Y` inserts `( EXEC.Y T )` under the top item (*T*).
- A *Y*-based “while” loop:
 

```
( EXEC.Y
  ( <BODY/CONDITION> EXEC.IF
  ( ) EXEC.POP ) )
```

## Iterators

`CODE.DO*TIMES, CODE.DO*COUNT,  
CODE.DO*RANGE`

`EXEC.DO*TIMES, EXEC.DO*COUNT,  
EXEC.DO*RANGE`

Additional forms of iteration are supported  
through code manipulation (e.g. via  
`CODE.DUP CODE.APPEND CODE.DO`)

## Named Subroutines

`( TIMES2 EXEC.DEFINE ( 2 INTEGER.* ) )`

## Auto-simplification

Loop:

Make it randomly simpler

If it's as good or better: keep it

Otherwise: revert

## Problems Solved by PushGP in the GECCO-2005 Paper on Push3

- Reversing a list
- Factorial (many algorithms)
- Fibonacci (many algorithms)
- Parity (any size input)
- Exponentiation
- Sorting

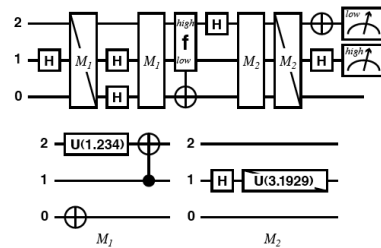
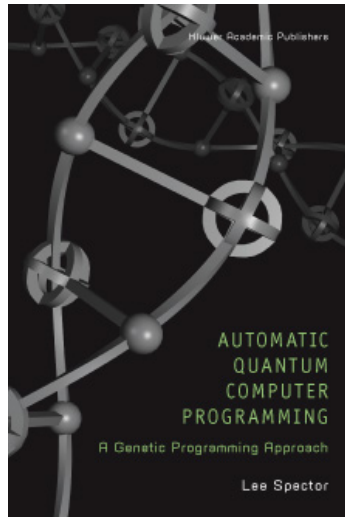


Figure 5.7. A gate array diagram for an evolved version of Grover's database search algorithm for a 4-item database. The full gate array is shown at the top, with  $M_1$  and  $M_2$  standing for the smaller gate arrays shown at the bottom. A diagonal line through a gate symbol indicates that the matrix for the gate is transposed. The "f" gate is the oracle.

**Humies 2004  
GOLD MEDAL**

## Genetic Programming for Finite Algebras

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**Humies 2008  
GOLD MEDAL**

## Autoconstructive Evolution

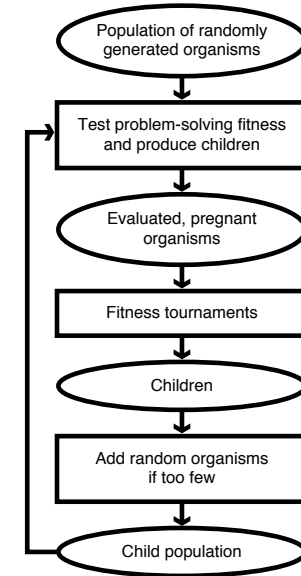
- Individuals make their own children
- Agents thereby control their own mutation rates, sexuality, and reproductive timing
- The machinery of reproduction and diversification (i.e., the machinery of evolution) evolves
- Radical self-adaptation

## Related Work

- MetaGP: but (1) programs and reproductive strategies dissociated and (2) generally restricted reproductive strategies
- ALife systems such as Tierra, Avida, SeMar: but (1) hand-crafted ancestors, (2) reliance on cosmic ray mutation, and (3) weak problem solving
- Evolved self-reproduction: but generally exact reproduction, non-improving (exception: Koza, but very limited tools for problem solving and for construction of offspring)

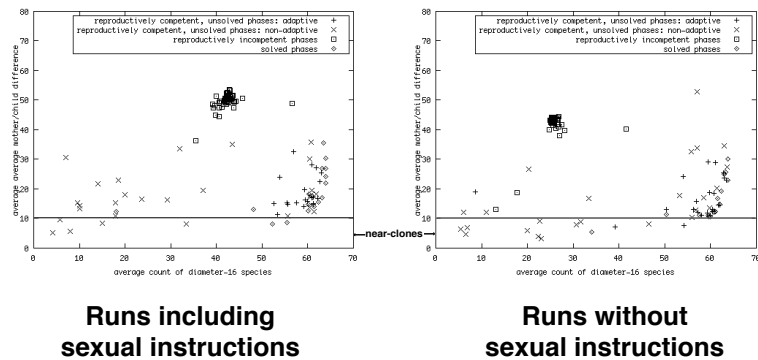
# Pushpop

- A soup of evolving Push programs
- Reproductive procedures emerge ex nihilo:
  - No hand-designed “ancestor”
  - Children constructed by any computable process
  - No externally applied mutation procedure or rate
  - Exact clones are prohibited, but near-clones are permitted.
- Selection for problem-solving performance



## # Species vs. Mother/Child Differences

Note distribution of “+” points: adaptive populations have many species and mother/daughter differences in a relatively high, narrow range (above near-clone levels).



## Pushpop Results

- In adaptive populations:
  - Species are more numerous
  - Diversification processes are more reliable
- Selection can promote diversity
- Provides a possible explanation for the evolution of diversifying reproductive systems

# SwarmEvolve 2.0

- Behavior (including reproduction) controlled by evolved Push programs
- Color, color-based agent discrimination controlled by agents
- Energy conservation
- Facilities for communication, energy sharing
- Ample user feedback (e.g. diversity metrics, agent energy determines size)

Instruction(s)	Description
DUP, POP, SWAP, REP, =, NOOP, PULL, PULLDUP, CONVERT, CAR, CDR, QUOTE, ATOM, NULL, NTH, +, *, /, >, <, NOT, AND, NAND OR, NOR, DO*, IF	Standard Push instructions (See [11])
VectorX, VectorY, VectorZ, VPlus, VMinus, VTimes, VDivide, VectorLength, Make-Vector	Vector access, construction, and manipulation
RandI, RandF, RandV, RandC	Random number, vector, and code generators
SetServoSetpoint, SetServoGain, Servo	Servo-based persistent memory
Mutate, Crossover	Stochastic list manipulation (parameters from stacks)
Spawn	Produce a child with code from code stack
ToFood	Vector to energy source
FoodIntensity	Energy of energy source
MyAge, MyEnergy, MyHue, MyVelocity, MyLocation, MyProgram	Information about self
ToFriend, FriendAge, FriendEnergy, FriendHue, FriendVelocity, FriendLocation, FriendProgram	Information about closest agent of similar hue
ToOther, OtherAge, OtherEnergy, OtherHue, OtherVelocity, OtherLocation, OtherProgram	Information about closest agent of non-similar hue
FeedFriend, FeedOther	Transfer energy to closest agent of indicated category

# SwarmEvolve 2.0



Winner, Best Paper Award, AAAA Track, GECCO-2003

# AutoPush

- Goals:
  - Superior problem-solving performance
  - Tractable analysis
- Push3
- Asexual
- Children produced on demand (not during fitness testing)
- Constraints on selection and birth
- Still work in progress

# Evolving Modular Programs

## With Code Manipulation

- Transform code as data on “code” stack
- Execute transformed code with `code.do`, etc.
- Simple uses of modules can be evolved easily
- Does not scale well to large/complex systems

# Evolving Modular Programs

## With Execution Stack Manipulation

- Code queued for execution is stored on an “execution stack”
- Allow programs to duplicate and manipulate code that on the stack
- Example: `(3 exec.dup (1 integer.+))`
- More parsimonious, but same scaling issue

# Evolving Modular Programs

## With Named Modules

- Uses Push’s “name” stack
- Example:  

```
(plus1 exec.define (1 integer.+))  
...  
plus1
```
- Coordinating definitions/references is tricky  
***and this never arises in evolution!***

# Module Identity

- How are modules recognized by other components of a system?
- Where do module identities come from?
- How can module identity co-evolve with modular architecture?

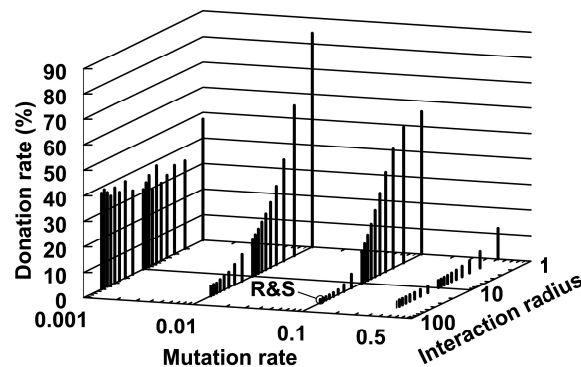


## Holland's Tags

- Initially arbitrary identifiers that come to have meaning over time
- Matches may be inexact
- Appear to be present in some form in many different kinds of complex adaptive systems
- Examples range from immune systems to armies on a battlefield
- A general tool for the support of emergent complexity

## Tag-Based Altruism

- Individuals have tags and tag-difference tolerances
- Donate when  $\Delta\text{tags} \leq \text{tolerance}$
- Riolo et al. (*Nature*, 2001) showed that tag-based altruism can evolve; Roberts & Sherratt (*Nature*, 2002) claimed it would not evolve under more realistic conditions



Spector, L., and Klein, J. Genetic stability and territorial structure facilitate the evolution of tag-mediated altruism. In *Artificial Life*.

## Evolving Modular Programs

With tags

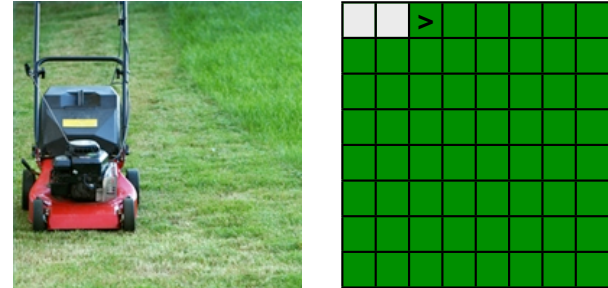
- Include instructions that tag code (modules)
- Include instructions that recall and execute modules by *closest matching tag*
- If a single module has been tagged then all tag references will recall modules
- The number of tagged modules can grow incrementally over evolutionary time
- **Expressive and evolvable**

## Tags in Push

- Tags are integers embedded in instruction names
- Instructions like `tag.exec.123` tag values
- Instructions like `tagged.456` recall values by *closest matching tag*
- If a single value has been tagged then all tag references will recall (and execute) values
- The number of tagged values can grow incrementally over evolutionary time

## Lawnmower Problem

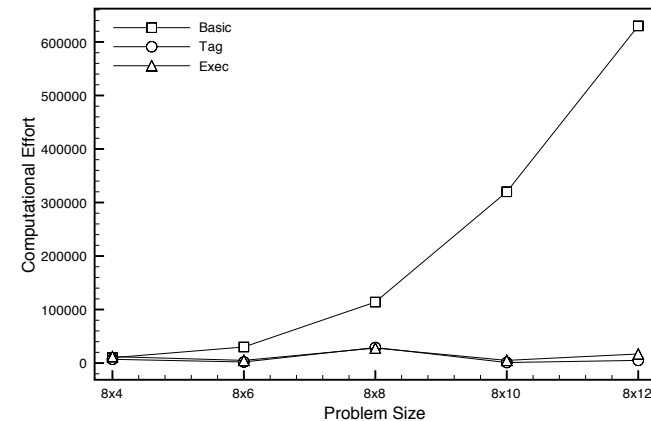
- Used by Koza to demonstrate utility of ADFs for scaling GP up to larger problems



## Lawnmower Instructions

Condition	Instructions
Basic	left, mow, v8a, frog, $\mathcal{R}_{vs}$
Tag	left, mow, v8a, frog, $\mathcal{R}_{vs}$ , tag.exec.[1000], tagged.[1000]
Exec	left, mow, v8a, frog, $\mathcal{R}_{vs}$ , exec.dup, exec.pop, exec.rot, exec.swap, exec.k, exec.s, exec.y

## Lawnmower Effort

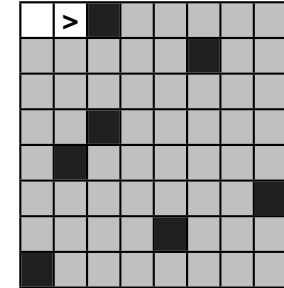


## Lawnmower Effort

	problem size				
instr set	8x4	8x6	8x8	8x10	8x12
basic	10000	30000	114000	320000	630000
tag	7000	2000	29000	<1000	5000
exec	12000	5000	28000	5000	17000

## Dirt-Sensing, Obstacle-Avoiding Robot Problem

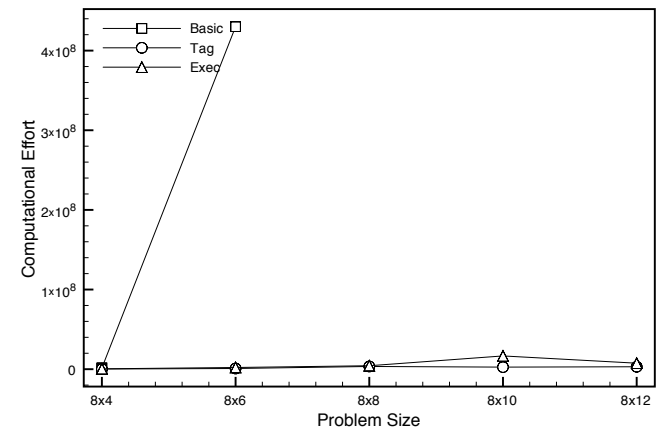
Like the lawnmower problem but harder and less uniform



## DSOAR Instructions

Condition	Instructions
Basic	if-dirty, if-obstacle, left, mop, v8a, frog, $\mathcal{R}_{v8}$
Tag	if-dirty, if-obstacle, left, mop, v8a, frog, $\mathcal{R}_{v8}$ , tag.exec.[1000], tagged.[1000]
Exec	if-dirty, if-obstacle, left, mop, v8a, frog, $\mathcal{R}_{v8}$ , exec.dup, exec.pop, exec.rot, exec.swap, exec.k, exec.s, exec.y

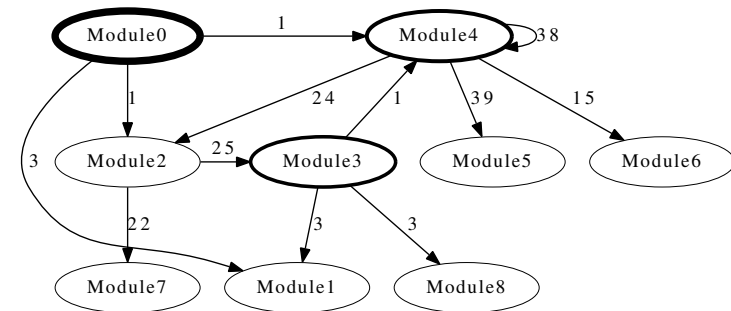
## DSOAR Effort



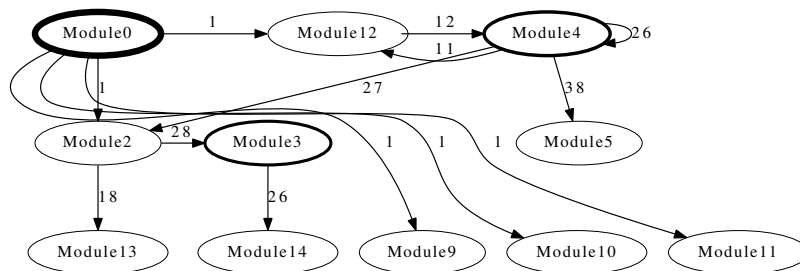
## DSOAR Effort

	problem size				
	8x4	8x6	8x8	8x10	8x12
instr set					
basic	1584000	430083000	inf	inf	inf
tag	216000	864000	3420000	2599000	3051000
exec	450000	2125000	4332000	16644000	7524000

## Evolved DSOAR Architecture (in one environment)



## Evolved DSOAR Architecture (in another environment)



## Tags in Trees

- Example:  

```
(progn (tag.123 (+ a b))
      (+ tagged.034 tagged.108))
```
- Must do something about endless recursion
- Must do something about return values of tagging operations and references prior to tagging
- Non-trivial to support arguments in a general way
- Utility not clear from experiments conducted to date

## Expressiveness and Assessment

- Expressive languages ease representation of programs that over-fit training sets
- Expressive languages ease representation of programs that work only on subsets of training sets
- Lexicase selection may help: Select parents by starting with a pool of candidates and then filtering by performance on individual fitness cases, considered one at a time

## Future Work

- Expression of variable scope and local environments
- Expression of concurrency, parallelism, and time-based structures
- Applications for which expressiveness is likely to be essential, e.g. complete software applications and programs for agents in complex, dynamic, heterogeneous environments

## Conclusions

- GP in expressive languages may allow for the evolution of complex software
- Minimal-syntax languages can be expressive, and GP systems that evolve programs in such languages can be simple
- Push is expressive, evolvable, successful, and extensible
- Tags appear to allow for the evolvable expression of program modularity

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