A Study on Large Population MOEA Using Adaptive ε-Box Dominance and Neighborhood Recombination for Many-objective Optimization

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Abstract. Multi-objective evolutionary algorithms are increasingly being investigated to solve many-objective optimization problems. However, most algorithms recently proposed for many-objective optimization cannot find Pareto optimal solutions with good properties on convergence, spread, and distribution. Often, the algorithms favor one property at the expense of the other. In addition, in some applications it takes a very long time to evaluate solutions, which prohibits running the algorithm for a large number of generations. In this work to obtain good representations of the Pareto optimal set we investigate a large population MOEA, which employs adaptive ε-box dominance for selection and neighborhood recombination for variation, using a very short number of generations to evolve the population. We study the performance of the algorithm on some functions of the DTLZ family, showing the importance of using larger populations to search on many-objective problems and the effectiveness of employing adaptive ε-box dominance with neighborhood recombination that take into account the characteristics of many-objective landscapes.

1 Introduction

Recently, there is a growing interest on applying multi-objective evolutionary algorithms (MOEAs) to solve many-objective optimization problems, where the number of objective functions to optimize simultaneously is considered to be more than three. Historically, most applications of MOEAs have dealt with two and three objective problems leading to the development of several evolutionary approaches that work successfully in these low dimensional objective spaces. However, it is well known that conventional MOEAs [1, 2] scale up poorly with the number of objectives of the problem. The poor performance of conventional MOEAs is attributed to an increased complexity inherent to high dimensional spaces and to the use of inappropriate selection and variation operators that fail to take into account the characteristics of many-objective landscapes [3–5].

MOEAs seek to find trade-off solutions with good properties of convergence to the Pareto front, well spread, and well distributed along the front. These three properties are especially difficult to achieve in many-objective problems and most search strategies for many-objective optimization proposed recently compromise one in favor of the other [6]. In several application domains, such as multidisciplinary multi-objective design optimization, a large number of Pareto optimal solutions that give a good representation of the true Pareto front in terms of convergence, spread, and distribution of solutions are essential to extract relevant knowledge about the problem in order to provide useful guidelines to designers during the implementation of preferred solutions. Moreover, in some applications it takes a very long time to evaluate solutions, which prohibits running the evolutionary algorithm for a large number of generations. Thus, in addition to the difficulties imposed by high-dimensional spaces, we are usually constrained by time.

From this point of view, in this work to obtain good representations of the Pareto optimal set we investigate a large population MOEA, which employs adaptive ε-box dominance for selection and neighborhood recombination for variation, using a very short number of generations to evolve the population. The motivation to use large populations is twofold. One is that we need many more solutions to properly approximate the Pareto optimal set of many-objective problems. The other one is that large populations may support better the evolutionary search on high dimensional spaces. That is, large populations may be more suitable to deal with the increased complexity inherent to high dimensional spaces. We assume that all solutions in the population can be evaluated simultaneously and in parallel, i.e. the time to evaluate one generation equals the time required to evaluate one solution, independently of the number of solutions we use in the population. So, our limitations on time are directly related to the number of generations rather than to the total number of fitness function evaluations. The motivation to use adaptive ε-box dominance for selection and neighborhood recombination for variation is to enhance the design of the algorithm incorporating selection and recombination operators that interpret better the characteristics of many-objective landscapes.

We study the performance of the algorithm using some test functions of the DTLZ family [7]. Our experiments reveal the importance of using a large population to search in many-objective problems and the effectiveness of employing ε-Box dominance and neighborhood recombination.

2 Proposed Method

2.1 Concept

Two important characteristics of many-objective optimization problems are that the number of non-dominated solutions increases exponentially [3, 4] with the number of objectives of the problem, and that these solutions become spread over broader regions in variable space [5]. These characteristics of many-objective landscapes must be considered when we design the major components of the algorithm, namely ranking, density estimators, mating, and variation operators.

In this work, selection is improved by incorporating adaptive ε-box dominance during the process of ranking and selecting solutions for the next generation. The effectiveness of the recombination operator is improved by incorporating a neighborhood to mate and cross individuals that are close in objective space. In the following we describe adaptive ε-box dominance and neighborhood recombination.

2.2 Adaptive ε-Box Dominance

In the proposed method we use adaptive ε-box dominance to rank solutions and select a ε-Pareto set [8] of non-dominated solutions for the next generation. In [8] an archiving strategy that updates a ε-Pareto set with a newly generated individual was proposed to guarantee convergence and diversity properties of the solutions found. The principles of the above archiving strategy [8] is applied to modify non-domination sorting used in NSGA-II [9]. The main steps of ε-box non-domination sorting are as follow.

Step 1 Similar to [8], ε-box non-domination sorting implicitly divides the objective space into non-overlapping boxes, where each solution is uniquely mapped to one box. That is, the box index vector $\boldsymbol{b}^{(i)} = (b_1^{(i)}, \cdots, b_m^{(i)})$ of the i-th solution in the combined population of parents P and offspring Q is calculated by

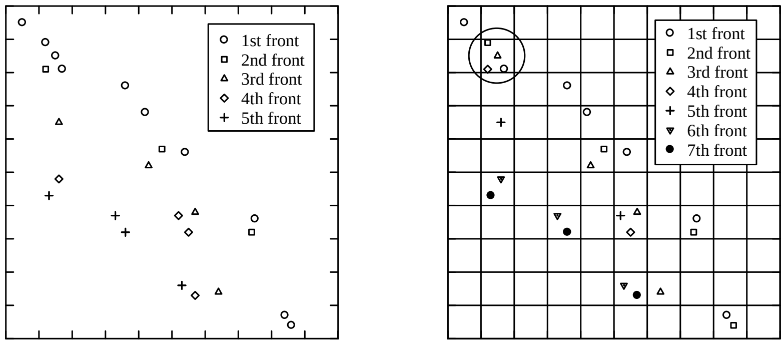
$$b_k^{(i)}(\boldsymbol{x}) = \left\lfloor \frac{\log_{10} f_k^{(i)}(\boldsymbol{x})}{\log_{10}(1+\varepsilon)} \right\rfloor (k = 1, 2, \cdots m), \tag{1}$$

where $f_k^{(i)}$ is the fitness value in the k-th objective of the i-th solution $i = 1, 2, \cdots, |P| + |Q|$, m the number of objectives, and ε a parameter that controls the size of the box.

Step 2 Pareto dominance is calculated using the box indexes $\boldsymbol{b}^{(i)}$ of solution to get a set of non-dominated ε-boxes.

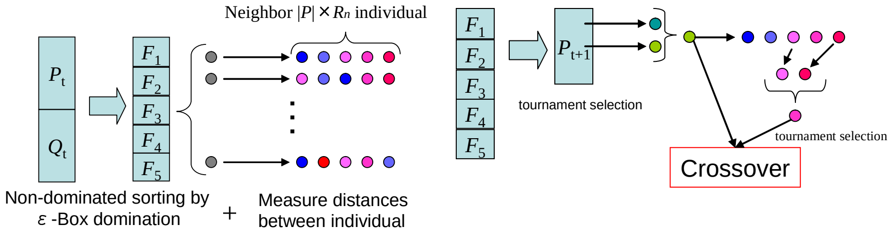
Step 3 Form a front of solutions by picking one solution from each non-dominated ε-box. If there is more than one solution in a box, we calculate Pareto dominance among solutions within the box. Thus, in each ε-box there could be either a dominating solution or several non-dominated solutions, and possibly one or more dominated solutions. To form the front we chose from each box the dominating solution, or select randomly one of the non-dominating solutions.

Step 4 Go to Step 2 to form subsequent fronts, excluding solutions already included in a previous front. Solutions located in a non-dominated ε-box but not selected as part of a previous front are considered to form the next front. Thus, compared to conventional non-domination sorting based on Pareto dominance, the proposed method reduces the ranking of some non-dominated solutions, namely those located in the same ε-box but not



(a) Front sorting by Pareto dominance (b) Front sorting by ε-Box Dominance

Fig. 1. Solution ranking by ε-Box Dominance



(a) Neighborhood creation (b) Mating for recombination

Fig. 2. Neighborhood Recombination

chosen to form the front. Note that in the archiving strategy proposed in [8], dominated solutions and not-selected non-dominated solutions within a ε-box are eliminated. Here, we keep those solutions but reduce their ranking.

Fig.1 illustrates front non-domination sorting by Pareto dominance and by ε-box dominance. In our illustration we assume a two-objective maximization problem. Note that non-dominated solutions that fall within the same ε-box are given different rank by ε-box non-domination sorting.

The ε-box non-domination sorting groups solutions in fronts of ε-box non-dominated solutions, denoted as F_j^ε, where j indicates the front number. Then, solutions are assigned a primary rank equal to the front number j it belongs to.

In many-objective problems, the number of Pareto non-dominated solutions $|F_1|$ obtained from the combined population of parents P and offspring Q is expected to surpass the size of the parent population since early generations, i.e. $|F_1| > |P|$. Since only one solution is selected from each ε-box to form a front, the number of solutions in the first front after applying ε-box non-domination sorting is expected to be smaller than the number of Pareto optimal solutions, $|F_1^\varepsilon| < |F_1|$, and its exact number depends on the value set to $\varepsilon > 0$ and on the instantaneous distribution of solutions in objective space. In general, larger values of ε imply that the ε-boxes cover larger areas, increasing the likelihood of having more solutions within each box and therefore less solutions in the finally formed front F_1^ε. However, it is difficult to tell in advance exactly how many solutions will be in F_1^ε for a given value of ε and trying to set this parameter by hand to achieve a desired value is a difficult and problem depending task.

Instead of setting ε by hand, the proposed method adapts ε at each generation so that the actual number of solutions in F_1^ε is close to the size of the parent population P [10]. Thus, the adaptive method aims to select a sample of non-dominated individuals for the next population that are distributed according to the spacing given by Eq. (1). The appropriate value of ε that renders a number of solutions close to the desired number is expected to change as the evolution process proceeds and it is affected by the stochastic nature of the search that alters the instantaneous distributions of solutions in objective space. Thus, in addition to adapting ε, the step of adaptation Δ is also important to properly follow the dynamics of the evolutionary process on a given problem. For this reason, the proposed adaptive procedure adapts ε and its step of adaptation Δ as well.

The method to adapt ε before it is used in Eq. (1) is as follows. First, before start searching solutions, we set initial values for ε and the step of adaptation Δ, set ε's lower bound $\varepsilon_{\min} > 0.0$ and Δ's lower and upper bound, $\Delta_{\min}$ and $\Delta_{\max}$, such that $0.0 < \Delta_{\min} < \Delta_{\max}$. Next, at every generation we count the number of solutions obtained in the first front F_1^ε by ε-box non-domination sorting and compare with the size of the population P. If $|F_1^\varepsilon| < |P|$ the step of adaptation is updated to $\Delta = \Delta/2$Cand $\varepsilon = \varepsilon - \Delta$, to make the grid fine grained. Otherwise, $\Delta = \Delta \times 2$ and $\varepsilon = \varepsilon + \Delta$, to make the grid coarser. If after updating ε or Δ their values go above or below their established bounds, they are reset to their corresponding bound. In this work, the initial value set to ε is 0.01, its lower bound $\varepsilon_{\min}$ is 10^{-8}, the initial step of adaptation Δ is 0.01, its upper bound $\Delta_{\max}$ is 1, and its lower bound $\Delta_{\min}$ is 0.0001.

2.3 Neighborhood Recombination

As mentioned above, it has been shown that in many-objective problems the number of non-dominated solutions grows exponentially with the number of objectives of the problem. A side effect of this is that non-dominated solutions in problems with a large number of objectives tend to cover a larger portion of objective and variable space compared to problems with less number of objectives. Thus, in many objective problems, the difference between individuals in the instantaneous population is expected to be larger. This could affect the effectiveness of recombination because recombining two very different individuals could be too disruptive.

In this work, we encourage mating between individuals located close to each other, aiming to improve the effectiveness of recombination in high dimensional objective spaces. To accomplish that, the proposed method calculates the distance between individuals in objective space and keeps a record of the $|P| \times R_n$ closest neighbors of each individual during the ε-dominance process. However, note that when the ranked population of size $|P| + |Q|$ is truncated to form the new population of size $|P|$, some individuals would be deleted from the neighborhood of each individual. During mating for recombination, the first parent p_A is chosen from the parent population P using a binary tournament selection, while the second parent p_B is chosen from the neighborhood of p_A using another binary tournament. Then, recombination is performed between p_A and p_B. That is, between p_A and one of its neighbors in objective space. If all neighbors of individual p_A were eliminated during truncation, the second parent p_B is selected from the population P similar to p_A. **Fig.2** illustrates the neighborhood creation and mating for recombination. In this work, we set the parameter R_n that defines the size of the neighborhood of each individual to 2%C5%Cand 10% of the parent population P.

3 Test Problems and Performance Indicators

3.1 Test Problems

In this work, we study the performance of the algorithms in continuous functions DTLZ2, DTLZ3, and DTLZ4 of the DTLZ test functions family [7]. These functions are scalable in the number of objectives and variables and thus allow for a many-objective study. In our experiments, we vary the number of objectives from $m = 4$ to 6 and set the total number of variables to $n = m+9$. DTLZ2 has a non-convex Pareto-optimal surface that lies inside the first quadrant of the unit hyper-sphere. DTLZ3 and DTLZ4 are variations of DTLZ2. DTLZ3 introduces a large number of local Pareto-optimal fronts in order to test the convergence ability of the algorithm. DTLZ4 introduces biases on the density of solutions to some of the objective-space planes in order to test the ability of the algorithms to maintain a good distribution of solutions. For a detailed description of these problems the reader is referred to [7].

3.2 Performance Indicators

In this work we evaluate the Pareto optimal solutions obtained by the algorithms using the quality indicators described below.
Proximity Indicator(I_p) [11]: Measures the convergence of solutions using equation 2, where P denotes the population and $\boldsymbol{x}$ a solution in the population. Smaller values of I_p indicate that the population P is closer to the Pareto front. That is, smaller values of I_p mean high convergence of solutions.

$$I_p = \underset{\boldsymbol{x} \in P}{median} \left\{ \left[\sum_{i=1}^{m} (f_i(\boldsymbol{x}))^2 \right]^{\frac{1}{2}} - 1 \right\} \tag{2}$$

C-metric [12]: Let us denote A and B the sets of non-dominated solutions found by two algorithms. $C(A, B)$ gives the fraction of solutions in B that are dominated at least by one solution in A. More formally,

$$C(A,B) = \frac{\mid \{\boldsymbol{b} \in B \mid \exists \boldsymbol{a} \in A : \boldsymbol{f}(\boldsymbol{a}) \succeq \boldsymbol{f}(\boldsymbol{b})\} \mid}{\mid B \mid}. \tag{3}$$

$C(A,B) = 1.0$ indicates that all solutions in B are dominated by solutions in A, whereas $C(A,B) = 0.0$ indicates that no solution in B is dominated by solutions in A. Since usually $C(A,B)+C(B,A) \neq 1.0$, both $C(A,B)$ and $C(B,A)$ are required to understand the degree to which solutions of one set dominate solutions of the other set.
Hypervolume (HV) [12]: HV calculates the volume of the m-dimensional region in objective space enclosed by a set of non-dominated solutions and a dominated reference point $\boldsymbol{r}$, giving a measure of convergence and diversity of solutions. In general, larger values of HV indicate better convergence and/or diversity of solutions. Thus, MOEAs that find Pareto optimal solutions that lead to larger values of HV are consider as algorithms with better search ability. We use Fonseca et al. [13] algorithm to calculate the hypervolume.

4 Simulation Results and Discussion

4.1 Preparation

In this work we use NSGA-II, a well known representative of the class of dominance based MOEAs. In this framework we include Adaptive ε-Box dominance and Neighborhood Recombination. In the following we call for short $A\varepsilon B$ and $A\varepsilon BNR$ the MOEAs that include Adaptive ε-Box dominance and Adaptive ε-Box dominance & Neighborhood Recombination, respectively. As genetic operators we use SBX crossover and Polynomial Mutation, setting their distribution exponents to $\eta_c = 15$ and $\eta_m = 20$, respectively. The parameter for the operators are crossover rate $p_c = 1.0$, crossover rate per variable $p_v = 0.5$, and mutation rate $p_m = 1/n$, where n is the number of variables. The number of generations is fixed to $T = 100$ and the population sizes varies from $|P| = 100$ to 5000 solutions. Result reported here are average results obtained running the algorithms 30 times.

4.2 Effects of Increasing Population Size

First, we focus our analysis on DTLZ2. **Fig.3** shows I_p obtained at the final generation varying the number of solutions in the population P from 100 to 5000. It could be seen that I_p reduces when a larger population size $|P|$ is used, regardless of the number of objectives. That is, a larger population size improves convergence of the algorithm. Especially note that for $m = 4, 5$ the reduction of I_p is remarkable and that the values of $|P|$ that lead to a pronounced decline on I_p are different, depending on the number of objectives. In the case of $m = 5$ objectives, it can be seen that a larger reduction occurs when the population size increases from $|P| = 1000$ to 2000Cthan from 2000 to 3000. In the case of $m = 6$ objectives, a sharp reduction on Ip is not observed for a population size

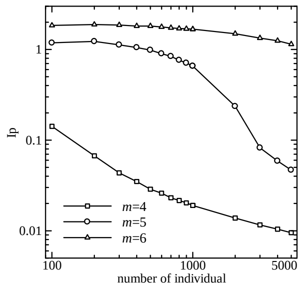
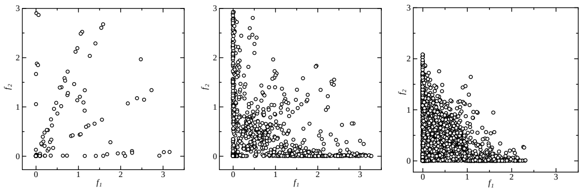


Fig. 3. Effect of increasing population size in NSGA-II (DTLZ2)



(a) $|P| = 100$ (b) $|P| = 1000$ (c) $|P| = 5000$

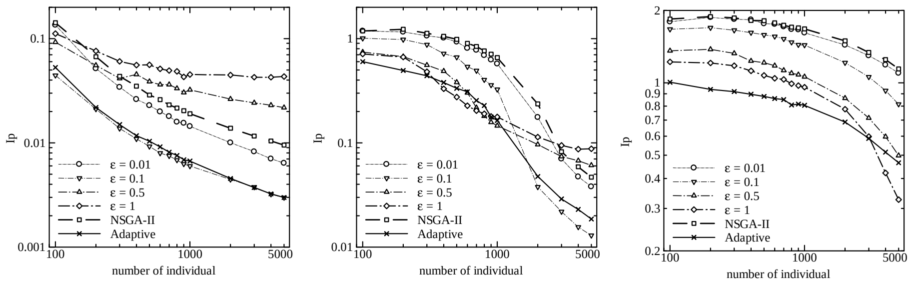
Fig. 4. Distribution of solutions by NSGA-II using various population sizes $|P|$, DTLZ2 $m = 5$ objectives

of up to 5000. It would be interesting to verify the effects of population sizes larger than 5000 on $m = 6$ objectives.

Next, **Fig.4** shows for $m = 5$ objectives the distribution of solutions in objective space projected to the $f_1 - f_2$ plane. In the DTLZ2 problem, keeping the population fixed and increasing the number of objectives, it is common to see that solutions tend to concentrate along the axis. These solutions are known as dominance-resistance solutions [14], which are favored by selection due to its inability to discriminate based on dominance while actively promoting diversity, and may cause the algorithm to diverge from the true Pareto front. From the figure, it can be seen that when the population increases the population tends to cluster towards the central regions of objective space, helping to control the presence of dominance-resistance solutions and their negative effect on convergence.

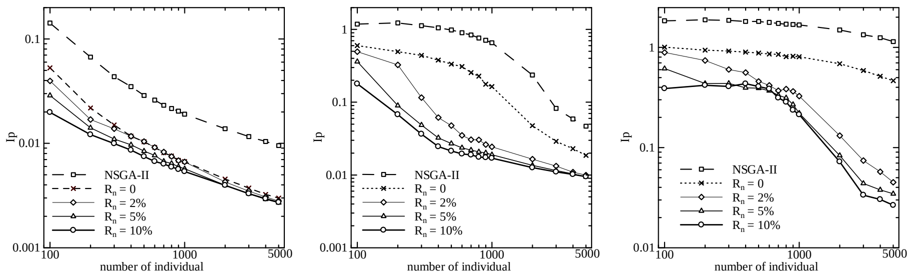
4.3 Effects of Adaptive ε-Box Dominance

In this section we compare results by conventional NSGA-II, NSGA-II enhanced with ε-box dominance setting a fixed value of $\varepsilon = \{0.01, 0.1, 0.5, 1\}$, and the Adaptive ε-Box dominance algorithm $A\varepsilon B$. The I_p by these methods is shown in **Fig.5**. Note that ε-Box dominance with fixed ε achieves better or worse I_p than conventional NSGA-II in $m = 4$ objectives, depending on the value set to ε. However, ε-Box dominance improves remarkably I_p in $m = 5, 6$, showing a bigger effect for larger population sizes $|P|$. On the other hand, using $A\varepsilon B$ stable and satisfactory performance is achieved, independently of the number of objectives. Especially, in the case of $m = 6$ objectives, $A\varepsilon B$ achieves best performance, with better effectiveness observed for larger population size.



(a) $m = 4$ objectives (b) $m = 5$ objectives (c) $m = 6$ objectives

Fig. 5. Effect of increasing population size and including ε-Box Dominance (DTLZ2)



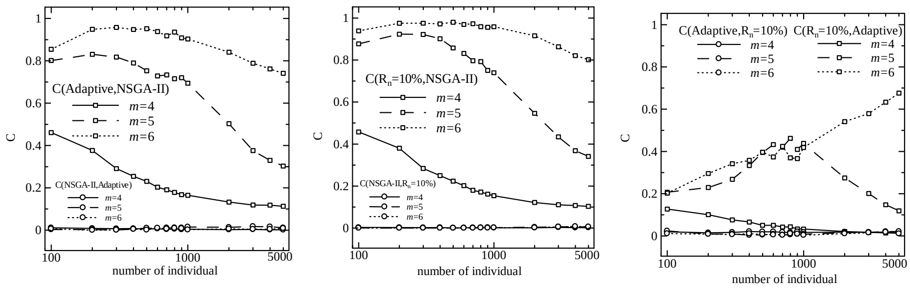
(a) $m = 4$ objectives (b) $m = 5$ objectives (c) $m = 6$ objectives

Fig. 6. Effect of increasing population size and including Adaptive ε-Box and Neighborhood Recombination (DTLZ2)

4.4 Effects of Neighborhood Recombination

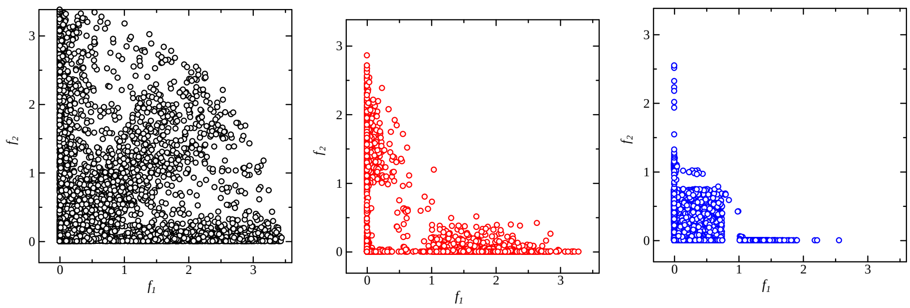
Next, we study the effects of incorporating Neighborhood Recombination in addition to Adaptive ε-Box dominance. **Fig.6** shows results by NSGA-II, $A\varepsilon B$, and $A\varepsilon BNR$ for comparison. From the figure we can see that increases in population size $|P|$ and the inclusion of Neighborhood Recombination improves further convergence. Note that the effect of Neighborhood Recombination becomes larger with the number of objectives. Especially, in $m = 6$ objectives, comparing with $A\varepsilon B$ ($R_n = 0$), note that the inclusion of Neighborhood Recombination and increasing population size $|P|$ in $A\varepsilon BNR$ ($R_n > 0$) improves convergence remarkably.

Summarizing, increasing population size $|P|$ improves convergence of MOEAs in multi-objective problems. In addition, the inclusion of Adaptive ε-Box further improves convergence, with larger improvements observed as we increase the number of objectives. Furthermore, the inclusion of Neighborhood Recombination leads to an additional remarkable improvement in convergence, which also gets larger as we increase the number of objectives. For example in the case of $m = 6$ objectives, a drastic reduction in I_p from 1.84 to 0.026 is observed



(a) NSGA-II vs. $A\varepsilon B$ (b) NSGA-II vs. $A\varepsilon BNR$ (c) $A\varepsilon B$ vs. $A\varepsilon BNR$

Fig. 7. Comparison among NSGA-II, $A\varepsilon B$, and $A\varepsilon BNR$ using the C-metric (DTLZ2)



(a) NSGA-II (b) $A\varepsilon B$ (c) $A\varepsilon BNR$

Fig. 8. Distribution of solutions found by NSGA-II, $A\varepsilon B$, and $A\varepsilon BNR$ (DTLZ2, $|P| = 5000, m = 6$)

if we compare the performance of conventional NSGA-II using population size $|P| = 100$ and $A\varepsilon BNR$ using a population size $|P| = 5000$.

4.5 Comparison using the C-metric and Distribution of Solutions

In this section, we compare the search ability of conventional NSGA-II, $A\varepsilon B$ ($R_n = 0\%$) and $A\varepsilon BNR$ ($R_n = 10\%$) using the C-metric performance indicator and analyze the distribution of solutions rendered by these algorithms.

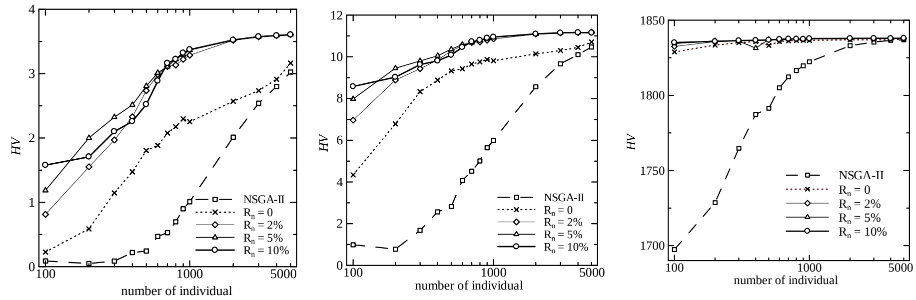
Fig.7 shows results of a pairwise comparison between algorithms using the C-metric. First, from **Fig.7** (a) it can be seen that a significant fraction of Pareto optimal solutions (POS) by the enhanced algorithm with $A\varepsilon BNR$ dominate POS by conventional NSGA-II; whereas no solution by conventional NSGA-II dominates solutions by $A\varepsilon BNR$. Also note that the fraction of dominated solutions gets larger as we increase the number of objectives. Second, from **Fig.7** (b), comparing conventional NSGA-II and the enhanced $A\varepsilon BNR$, a similar tendency as the one describe above can be observed, but with better fractions of dominated solutions in favor of the enhanced algorithm. Third, from **Fig.7** (c), comparing the enhanced algorithms, note that $A\varepsilon BNR$ that includes both Adaptive ε-Box dominance and Neighborhood Recombination dominates a fraction of solutions found by the algorithm $A\varepsilon B$ that only includes Adaptive ε-Box dominance; whereas the opposite is not true. Note that this effect becomes remarkable when the number of objectives increases. From these results on the C-metric we conclude that the inclusion of Adaptive ε-Box dominance and Neighborhood Recombination leads to remarkable increase on the number of solutions with better convergence in the POS found by the algorithm.

Next, **Fig.8** shows the distribution of solutions in objective space projected to the $f_1 - f_2$ plane by these three algorithms, for $m = 6$ objectives and population size $|P| = 5000$. Note that solutions by conventional NSGA-II are broadly spread, however they lack convergence to the true Pareto front as shown in **Fig.8** (a). When Adaptive ε-Box is introduced, convergence of solutions improves but their distribution is biased towards the extreme regions of the Pareto front as seen in **Fig.8** (b). This is because Adaptive ε-Box strengthen the trend to favor solutions towards the axis of the multi-objective space. On the other hand, when Neighborhood Recombination is added convergence of the population of solutions improves further, solutions get compactly distributed around the Pareto optimal region, and the bias towards the axis of objective space becomes almost unnoticed as shown in **Fig.8** (c). This is because recombination of solutions that are neighbor in objective space allows a better exploitation of the search, especially in problems where objective and variable space are not strongly uncorrelated. However, the different density of solutions in objective space produced by the uneven granularity of the grid used by Adaptive ε-Box should be investigated with more detail in the future.

4.6 Comparison using HV

The HV measures both convergence and diversity (spread) of solutions. However, we can emphasize one over the other depending on the reference point $\boldsymbol{r}$ used to calculate HV. When the reference point is close to the Pareto front, convergence of solutions is emphasized. On the other hand, when the reference point is far away from the Pareto from, the diversity of solutions is emphasized.

Fig.9 shows the HV for DTLZ2 with $m = 6$ objectives, varying population size $|P|$ from 100 to 5000, and varying the reference point to $\boldsymbol{r}$=1.25C1.5C3.5. From this figure, note that when the reference point is set to $\boldsymbol{r} = 3.5$, which emphasizes the estimation of diversity of solutions, similar values of HV are observed by the improved algorithms for all population sizes. This is because spread of solutions by the improved algorithms is similar. The HV by NSGA-II appears very low for small populations, but approaches the HV achieved by the improved algorithm for very large population sizes. These values reflect the fact that a considerable number of solutions by NSGA-II with small populations are past the reference point, and thus do not contribute to the hypervolume calculation. In the cases of $\boldsymbol{r} = 1.5$ and $\boldsymbol{r} = 1.25$, where convergence of solutions



(a) $\boldsymbol{r} = 1.25$ (b) $\boldsymbol{r} = 1.5$ (c) $\boldsymbol{r} = 3.5$

Fig. 9. HV on DTLZ2 $m = 6$ objectives

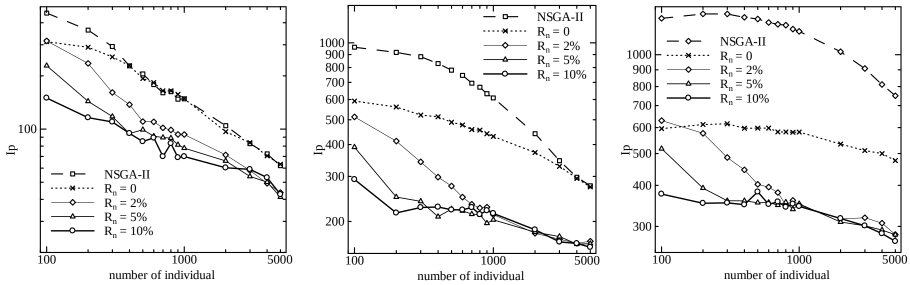
is emphasized, differences on HV between the improved algorithms and NSGA-II can be clearly seen even for very large populations. These results on the hypervolume are in accordance with our analysis discussed on previous sections.

4.7 Results on DTLZ3 and DTLZ4 Functions

In previous sections we focused our analysis on the DTLZ2 function. Here, we include and analyze results on DTLZ3 and DTLZ4 functions. Results for DTLZ3 are show in **Fig.10**, whereas results for DTLZ4 are shown in **Fig.11**. Results for DTLZ2 using similar configurations are shown in **Fig.6**.

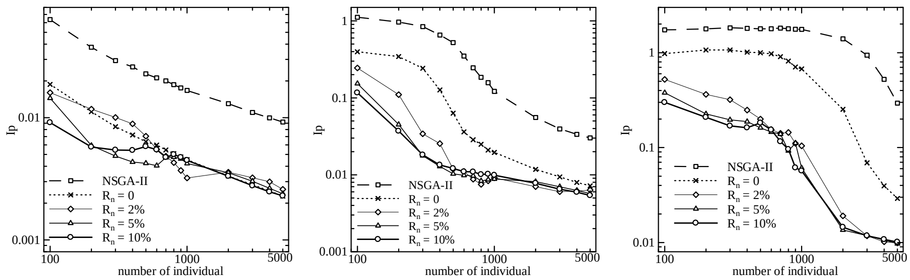
From **Fig.11** it can be seen that results on DTLZ4 are similar to those observed on DTLZ2, but note that in DTLZ4 convergence improvement due to bigger population sizes becomes more significant. This is because DTLZ4 is a problem that favors diversity of solutions, where an algorithm that selects solutions based on crowding distance[1] is expected to improve convergence of solutions specially in extreme regions of the Pareto front. On the other hand, from **Fig.10** note that in DTLZ3 although convergence improves by using $A\varepsilon BNR$, compared to the I_p values achieved on DTLZ2 and DTLZ4 it can be seen that convergence is still insufficient.

To study with more detail the insufficient convergence in DTLZ3, **Fig.12** show I_p by the three algorithms on $m = 4, 5, 6$ objectives, setting the population size $|P|$ to 100 and 1000 individuals, and extending the number of generations from 100 to $T = 500$. From the figure note that independently of the number of objectives, increasing the number of generations is effective to improve convergence of $A\varepsilon B$ and $A\varepsilon BNR$. Similarly, better results are observed using a population size of 1000 individuals than a population size of 100. However, note that the reduction speed of I_p and its achieved value differ greatly according to the number of objectives. That is, the algorithms find it more difficult to reduce I_p conform the number of objectives increases. In the future, it is necessary to investigate ways to achieve good convergence within a minimum number of generations in this kind of problem.



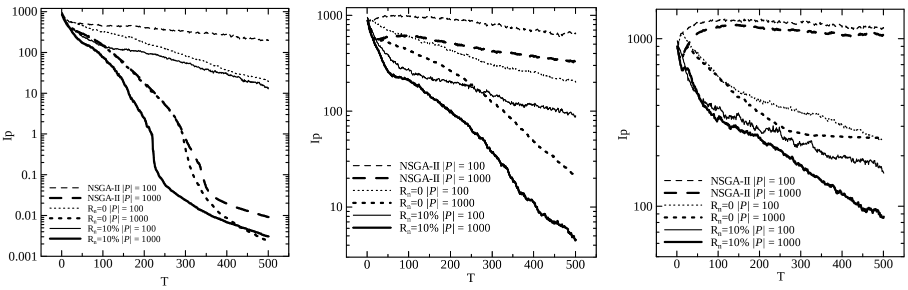
(a) $m = 4$ objectives (b) $m = 5$ objectives (c)$m = 6$ objectives

Fig. 10. Effects on DTLZ3



(a) $m = 4$ objectives (b) $m = 5$ objectives (c) $m = 6$ objectives

Fig. 11. Effects on DTLZ4



(a) $m = 4$ objectives (b) $m = 5$ objectives (c) $m = 6$ objectives

Fig. 12. I_p convergence on DTLZ3 increasing the number of generations to $T = 500$

5 Conclusions

In this work, we have studied the optimization of $m = 4, 5, 6$ many-objective problems under a fixed, restricted, small number of generations. First, we investigated the effect of population size on a conventional NSGA-II, verifying that convergence of solutions improves when we increase the population size. Next, we investigated the effects of including ε-Box dominance and Adaptive ε-Box dominance into NSGA-II, verifying that Adaptive ε-Box dominance improves convergence of solutions without the need to set the parameter ϵ by hand. Moreover, we investigated the effects of Adaptive ε-Box Dominance and Neighborhood Recombination, verifying that convergence of solutions improves substantially, especially when bigger populations are used and the number of the objectives of the problem becomes larger.

In the future, we would like to study the effects of using larger populations, explore parallelization, and develop MOEAs that can effectively evolve solutions independently of the characteristics of problem under a restricted number of generations.

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