Effect of SMS-EMOA Parameterizations on Hypervolume Decreases

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Abstract. It is possible for the $(\mu+1)$-SMS-EMOA to decrease in dominated hypervolume w.r.t. a global reference point. We study the influence of SMS-EMOA parameter settings on number and amount of the observed decreases. We show that the number of decreases drop and the number of increases rise with a higher population size. In addition, a positive correlation between mean increase and mean decrease can be observed. Our findings further indicate a substantial impact of the mutation operators on the number and amount of decreases.

Keywords: EMO, hypervolume decreases, parameter influence

1 Introduction

The dominated hypervolume was defined by Zitzler and Thiele as the the size of the space covered by a Pareto front[1] with respect to a given reference point and special properties of this indicator have been proven [6]. As recent results show, unsuspected decreases in the hypervolume progression during an optimization run are possible [4]. These results are summarised in the following section. Section 3 provides our latest results on the dependency of such decreases on EMOA parameter settings as well as some initial insight into how these decreases influence the final dominated hypervolume of an optimization run. Finally, we summarise our latest findings and give an outlook to future work in section 4.

2 Decreases in Hypervolume Progressions

The progression of the hypervolume in a 1-greedy hypervolume selection based EMOA w.r.t. a fixed global reference point was thought to never decrease in the course of an optimization run. This belief arose from the design of the algorithms, in which the individual with the least hypervolume contribution is discarded in every generation. However, Judt et al. [4] showed that this intuition is wrong for an adaptive reference point.

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[1] For details on EMO related definitions and vocabulary the reader is refered to Deb [3] or Coello Coello [2].

For the paper a total of $71\,250$ reproducible runs of the SMS-EMOA [1] on both 2-dim. and 3-dim. test cases were conducted. Different parameterizations have been considered, e.g. three different population sizes $\mu \in \{10, 20, 100\}$ were examined. Parameters of the Simulated Binary Crossover (SBX, [3]) and the Polynomial Mutation (PM, [3]) operator were chosen according to a Latin Hypercube Sample. For each combination of test function, population size and variation operator set, 50 independent runs were conducted. The hypervolume w.r.t. a fixed reference point was calculated for each generation and the number of times this hypervolume drops was stored. More details on exact parameterizations and reference points are provided in the supplementary material[2]

In both experiments, a considerable number of decreases in hypervolume were observed. In the 2-dim. case, an exceptional rule for the two boundary solutions is responsible for the drops. In the 3-dim. case, drops were explained by the hypervolume contributions being calculated w.r.t. an adaptive reference, which depends on the current population. While internally never attaining a decrease in hypervolume w.r.t. this adaptive reference point, drops may occur w.r.t. a global reference point. This 3-dim. effect can be generalized to a higher number of objectives. Nevertheless, the authors still believe that hypervolume is the most effective selection scheme for MOP known today.

3   Results

In the following analysis we focus primarily on the 3-dim. cases. More results and similar results for the 2-dim. cases are provided in the supplementary material.

3.1   Population Size Influence

It is well known that choosing a suitable population size μ is crucial for the performance of an evolutionary algorithm (EA). We therefore hypothesized that it could also have an influence on the number of decreases in hypervolume as well as the magnitude of these decreases. We find evidence for the latter hypothesis in figure 1 which looks at the absolute change of hypervolume in each iteration and depicts the trade-off between the average increase and decrease of a run.

There appears to be a barrier at 10^{-6} below which the mean decrease in hypervolume does not drop. This is a numerical artifact. The observed absolute hypervolume values are of the order of 10^9 and the double precision floating point values used for all calculations only have ≈ 16 significant digits. It is therefore not possible to observe smaller differences than $\approx 10^{-6}$ in the hypervolume directly. This problem of the hypervolume indicator seems to be largely ignored and will only worsen as the hypervolume indicator or approximations of it are used for problems with more than 3 objectives.

Another interesting thing in figure 1 is that the mean increase and decrease are positivly correlated. This implies that if we observe large increases in hypervolume during our optimization run, we can assume that we will also have large

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[2] Supplementary Material is available at `http://ptr.p-value.net/lion12a`.
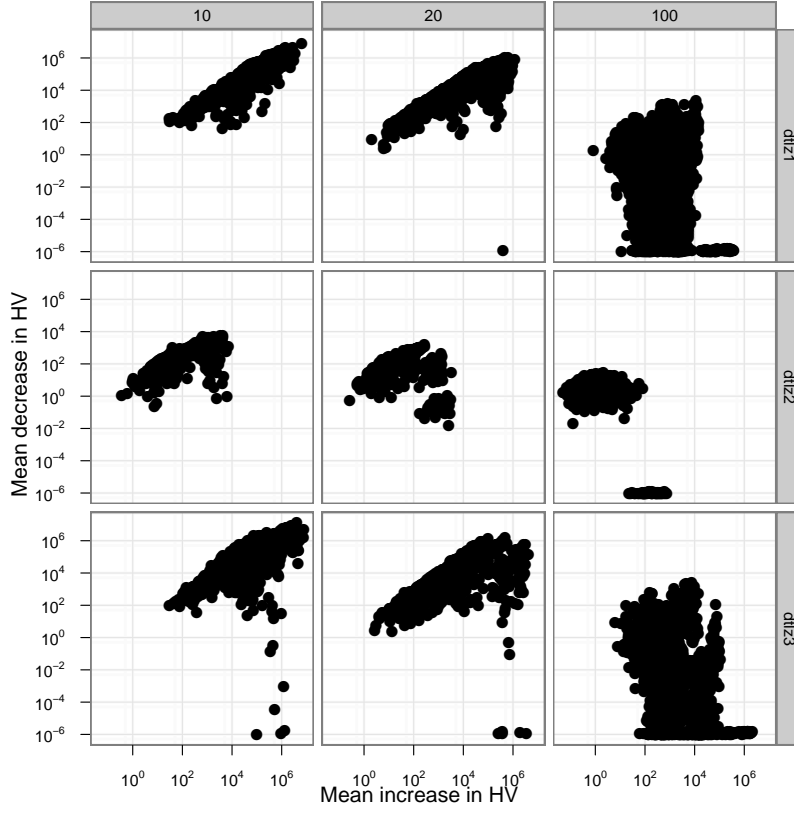
Fig. 1. Increases vs decreases in HV averaged over all iterations in each of the 22 500 3-dim. runs. The data is futher subdivided by population size μ along the top and test function along the right side.

decreases in hypervolume w.r.t. a fixed reference point. Less obvious is the effect of μ on the mean increase and decrease. Both tend to drop for larger values of μ, which is plausible given that with a larger population size the expected contribution of each individual is lower.

In addition, plots depicting the influence of the population size μ on the number of occurrences of a decrease in hypervolume can be seen in the supplementary material. It is shown there that for increasing μ the number of hypervolume decreases drops, whereas the number of increases rises.

3.2   Eliminating Parameters

The influence of variation operator parameterization on the final SMS-EMOA performance has been studied in [5]. Here, we wish to see if they also influence the number or the amount of decrease in hypervolume. The four parameters are

`sbx_n`, `sbx_p`, `pm_n`, `pm_p` from SBX and PM. The first two control the distribution and probability of crossover and the second two the distribution and probability of mutation.

Both, the mean number of decreases as well as the variance of the decreases over the different parameterizations vary widely for a fixed setting of `sbx_n` or `sbx_p`. There is no similarity between the functions and only slight similarities for a fixed function and different values of the populations size μ. The only consistent phenomenon is the decrease in variance for larger values of μ.

We therefore believe that there is little incentive to further investigate the influence of these two parameters on the number of and the amount of decrease in hypervolume. For `sbx_n` this observation is constitent with [5].

For `pm_n` and `pm_p` the situation is a bit different. While there is still strong non-linearity in the relationship between the parmeter and the number of hypervolume decreases, there are regularities in the plot. For `pm_n` there is a consistent spike between 20 and 25 for each function and setting of μ. The magnitude decreases for $\mu = 100$ and the variance seems to be approximately proportional to the mean number of hypervolume decreases. This hints at a linear relationsship between μ and both `pm_n` and `pm_p`. From [5] we know that these values lead to good overall performance of the algorithm on a wide varity of test functions.

The `pm_p` parameter also exibits some structure. There is a peak in the number of decreases in hypervolume for small values of `pm_p` and the curve drops and levels off as `pm_p` increases. The high variance is concentrated in the region of low `pm_p` values which seems peculiar.

Figure 2 visualizes all different parameter settings in a single plot. It shows the maximum attained hypervolume (rescaled to aid in visualizing the result) against the number of decreases in hypervolume. Each different shade of grey is a different parameter setting. There is obvious underlying structure. Similar parameter settings tend to 'cluster' together. That is, a parameter setting tends to produce similar number of hypervolume decreases and a similar attained hypervolume over many runs of the SMS-EMOA.

Even more interesting is the result that decreases seem to help the algorithm. Note that due to the rescaling larger values of the dominated hypervolume gap are worse than smaller values. So from figure 2 we see that by increasing the number of decreases the worst case performance (measured in dominated hypervolume) increases by 1 to 2 orders of magnitude. This would suggest that while largely ignored so far, this form of non-elitism might be beneficial to the optimization process.

However, decreases do ultimately hurt the best case performance, sometimes by several orders of magnitude. But due to the non-linear scale the dominated hypervolume is measured on, it is unclear how signigicant this result is.

4   Summary and Outlook

The paper at hand intended to find dependencies between EMO algorithm parameter settings and the decreases in hypervolume progression [4]. The common
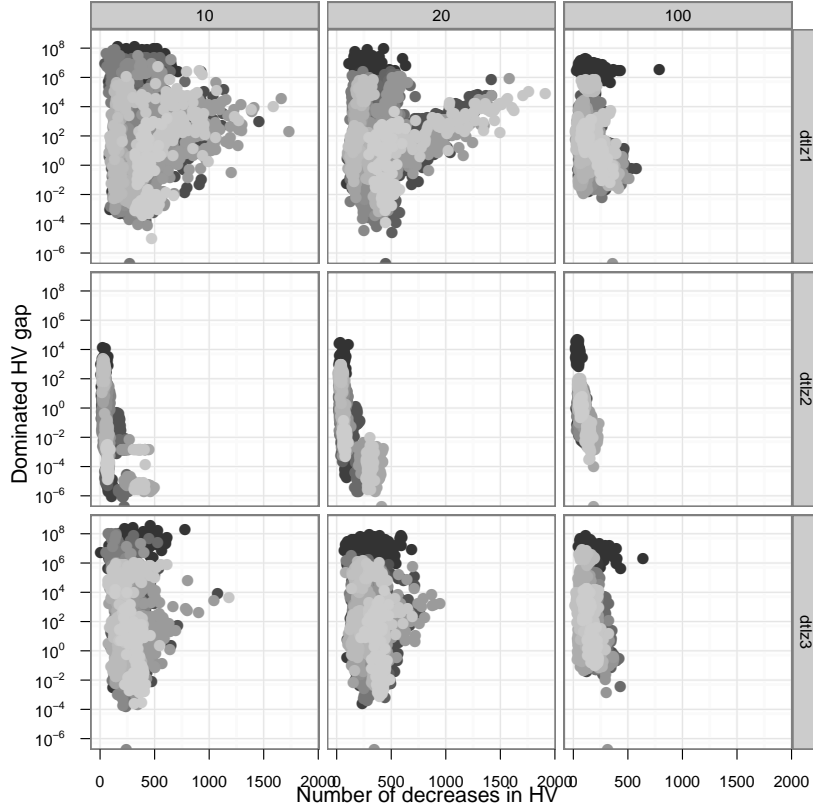
Fig. 2. Number of decreases in hypervolume in relation to the difference between the maximum hypervolume over all runs and the hypervolume of the run. The latter transformation was chosen for visualization purposes only. Note that smaller gap values correspond to a larger dominated hypervolume. The shading of the points marks different combinations of parameter settings of the algorithm.

SBX and polynomial mutation variation parameters as well as the population size of the $(\mu + 1)$ SMS-EMOA were analyzed for possible influences on the number or amount of decrease in hypervolume.

The population size has a major effect on the number and amount of hypervolume decreases. The results show that for increasing μ the number of hypervolume decreases drops. This was traced back to the larger number of boundary solutions for larger population sizes.

We found no evidence that the recombination parameters sbx_n and sbx_p influence the number of decreases in hypervolume or the amount of decreases. This observation underpins the results of [5] that the sbx_n parameter has no influence on SMS-EMOA results in general. We therefore suggest to ignore this

parameter in future experiments and thus, reduce the number of parameters of EMO algorithms and ultimately the complexity of the parameter tuning.

For `pm_n` and `pm_p` we found evidence that suggests a highly non-linear relationship between these parameters and the number and amount of hypervolume decreases. Interestingly, the standard settings from the literature for `pm_n` and `pm_p` are those that yield the largest number of decreases, suggesting that decreases might have a positive influence on the final result of the optimization process. Further proof for this was given by relating the number of decreases to the maximally attained hypervolume, w.r.t a fixed reference point, for each run. From this data, we could also deduce, that decreases help the algorithm to avoid stagnation. Runs with a large number of decreases tend to be orders of magnitude better than those with a lower number of decreases in the worst case. The price for this is a small decrease in maximally dominated hypervolume. It would be interesting to study this effect further, possibly by deliberately introducing hypervolume decreases into the optimization run and studying their influence on the final result.

Our results indicate that choosing "good" parameter settings has a measurable impact on the number and the amount of decreases in hypervolume. An effect on the overall quality of the results cannot be deduced due to missing data for comparison. There are currently no guards in the algorithm to discard points that lead to a decrease in hypervolume w.r.t. the fixed reference point. The simplest solution to this dilemma is to check each new point w.r.t a predefined fixed reference point and to forget those individuals that cause a decrease and continue with the previous population. This will form the foundation for upcoming experiments.

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