Between Selfishness and Altruism: Fuzzy Nash–Berge-Zhukovskii Equilibrium

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Abstract. Nash equilibrium in many cases is not the best choice for human players. In case of trust games the Nash equilibrium is often mutual defection which is the worst possible outcome for all players. The Berge-Zhukovskii equilibrium models a more cooperative behavior, so in case of trust games, when players gain by cooperating, it is usually a better choice than Nash equilibrium. Real life results show that players rarely follow the theoretical predictions. Our aim is to find new equilibria types that offer a more realistic modeling of human players. The fuzzy Nash–Berge-Zhukovskii equilibrium is proposed which is a fuzzy combination of the Nash and Berge-Zhukovskii equilibrium. Several continuous trust games are investigated. Numerical results indicate that fuzzy Nash–Berge-Zhukovskii equilibrium is suitable to model real-life situations.

1 Introduction

The most important equilibrium concept in game theory, Nash equilibrium, is not always the most efficient solution concept. In many cases playing Nash equilibrium is not the most favorable choice since Nash equilibrium rarely assures maximal payoffs. Trust games are a class of games where players end up with greater payoffs by trusting their opponents and choosing a cooperative strategy, than by mutual defection. Also, since the payoff for defecting with a cooperative opponent is larger than the payoff for cooperation, the temptation for defection is high. In most of the cases the Nash equilibrium of trust games is mutual defection, that is the worst possible outcome for all players.

Other solution concepts, like Pareto or Berge-Zhukovskii equilibrium, are often better choices in case of trust games. Pareto equilibrium ensures optimal payoffs for all players while Berge-Zhukovskii equilibrium models a type of altruism. Berge-Zhukovskii players, when choosing their strategy, beyond their gain, also take in consideration the gain of their opponent. For trust games, both Pareto and Berge-Zhukovskii equilibria, usually ensure greater payoffs for all players than Nash equilibrium.

However, our opinion is, that standard game equilibria presume some restrictions. In real life players are rarely rational agents only acting to maximize their payoffs. Real life players can be more or less cooperative, more or less selfish and their actions are rarely uniform. One simple step towards a more realistic approach is to relax the rationality principle, and allow different rationality types in

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a single game. In [2] a fuzzy Nash-Pareto equilibrium is proposed. This concept allows players to be biased towards a certain type of rationality, which ensures a more realistic modeling of human players. According to [3] Fuzzy Nash-Pareto equilibrium is a suitable concept to model the human behavior for the discrete centipede game.

A more general approach is to explore trust games with continuous strategy set. Our intuition is that fuzzy equilibria might offer a better modeling of real-world players. Since fuzzy Nash-Pareto equilibrium does not offer promising results our goal is to find an equilibrium concept that would capture a more realistic situation.

1.1 Game Theory Prerequisites

Mathematically a finite non-cooperative one shot game is a system
$G = (N, (S_i, u_i), i = 1, ..., n)$, where:

- N represents the set of players, and n is the number of players;
- for each player $i \in N$, S_i is the set of available actions; $S = S_1 \times S_2 \times ... \times S_n$, is the set of all possible situations of the game. Each $s \in S$ is a strategy (or strategy profile) of the game;
- for each player $i \in N$, $u_i : S \to R$ represents the payoff function of i.

Denote by (s_{i_j}, s^*_{-i}) the strategy profile obtained from s^* by replacing the strategy of player i with s_{i_j} i.e. $(s_i, s^*_{-i}) = (s^*_1, s^*_2, ..., s^*_{i-1}, s_i, s^*_{i+1}, ..., s^*_n)$.

A solution of the game is called a *game equilibrium.* At equilibrium all players are contended with their outcome, and they are not willing to switch their strategies.

Nash Equilibrium. A strategy profile is a *Nash equilibrium* if none of the players have the incentive to unilaterally deviate [7] i.e. no player can improve her payoff by modifying her strategy while the others do not modify theirs.

More formally: a strategy profile $s^* \in S$ is a Nash equilibrium if the inequality holds: $u_i(s^*) \geq u_i(s_i, s^*_{-i}), \forall i = 1, ..., n, \forall s_i \in S_i$.

Berge-Zhukovskii Equilibrium. In contrast to the Nash equilibrium, where players are selfregarding, the Berge-Zhukovskii equilibrium [11] allows reaching cooperative features making it possible to determine cooperation in a non-cooperative game.

The strategy s^* is a *Berge-Zhukovskii equilibrium* when no group of players can improve the payoff for any of the n players by changing their strategy.

More formally: Let $N - i$ denote any group of players which excludes player i (can be excluded other players, too). The strategy profile s^* is a Berge-Zhukovskii equilibrium if the inequality $u_i(s^*) \geq u_i(s^*_i, s_{N-i})$ holds for each player $i = 1, ..., n$, and $s_{N-i} \in S_{N-i}$.

1.2 Trust Games

An interesting phenomena can be observed in case of trust games. Trust games are a class of games in which players obtain much better results, higher payoffs, if they trust each other and choose a cooperative strategy than in case of mutual defection. Moreover if one player defects while the other cooperates the payoff for the defecting player is much better, so the temptation to defect is considerably high.

Very often, when players gain more by cooperating than defecting, Nash equilibrium is not the best choice for players. In these type of games the Nash equilibrium is mutual defection and players following Nash rationality end up with the worst outcome.

The Berge-Zhukovskii equilibrium models a type of altruism. Players choosing a Berge-Zhukovskii rationality are more other-regarding when choosing their strategies, as they also consider the payoffs of the other players. So usually in trust games the Berge-Zhukovskii equilibrium represents mutual cooperation which is a favorable outcome for all players.

We think that it would be interesting to investigate rationality types that are between these two extremes (mutual defection or mutual cooperation). With fuzzy Nash–Berge-Zhukovskii (N-BZ) equilibrium various intermediate states can be depicted. Players can be more or less biased towards a certain rationality, for example a player can have a membership degree of 0.7 to Nash rationality and 0.3 to Berge-Zhukovskii rationality. Thus the fuzzy N-BZ equilibrium offers a more realistic modeling of human players.

2 Generative Relations for Game Equilibria

Game equilibria may be characterized by generative relations on the set of game strategies [5]. The idea is that the non-dominated strategies with respect to the generative relation equals (or approximate) the equilibrium set.

Let us consider a relation $\mathcal{R}$ over $S \times S$. A strategy s is non dominated with respect to relation $\mathcal{R}$ if $\nexists s^* \in S : (s, s^*) \in \mathcal{R}$. Let us denote by NDR the set of non- dominated strategies with respect to relation $\mathcal{R}$. A subset $S' \subset S$ is non-dominated with respect to $\mathcal{R}$ if and only if $\forall s \in S'$, $s \in NDR$.

Relation $\mathcal{R}$ is said to be a *generative relation* for the equilibrium E if and only if the set of non-dominated strategies with respect to $\mathcal{R}$ equals the set E of strategies i.e. $NDR = E$.

2.1 Generative Relation for Nash Equilibrium

Let s and s^* be two pure strategies and $k(s^*, s)$ denotes the number of players which benefit by deviating from s^* towards s [5]:

$$k(s^*, s) = card\{i \in N, u_i(s_i, s^*_{-i}) > u_i(s^*), s_i \neq s^*_i\}.$$

Let $s^*, s \in S$. We say the strategy s^* is better than strategy s with respect to Nash equilibrium, and we write $s^* \prec_N s$, if the following inequality holds:

$k(s^*, s) < k(s, s^*)$. $k(s^*, s)$ is a relative quality measure of s and s^* - with respect to the Nash equilibrium. The relation $\prec_N$ can be considered as the *generative relation of Nash equilibrium*, i.e. that the set of non-dominated strategies with respect to $\prec_N$ induces the *Nash equilibrium* [5].

2.2 Generative Relation for Berge-Zhukovskii Equilibrium

Consider two strategy profiles s^* and s from S. Denote by $b(s^*, s)$ the number of players who lose by remaining to the initial strategy s^*, while the other players are allowed to play the corresponding strategies from s and at least one player switches from s^* to s.

We may express $b(s^*, s)$ as [4]:

$$b(s^*, s) = card[i \in N, u_i(s^*) < u_i(s_i^*, s_{N-i})].$$

Let $s, s^* \in S$. We say the strategy s^* is better than strategy s with respect to Berge-Zhukovskii equilibrium, and we write $s^* \prec_{BZ} s$, if and only if the inequality $b(s^*, s) < b(s, s^*)$ holds. We may consider relation $\prec_{BZ}$ as a *generative relation of the Berge-Zhukovskii equilibrium*. This means the set of the non-dominant strategies with respect to the relation $\prec_{BZ}$ equals the set of Berge-Zhukovskii equilibria.

3 Evolutionary Equilibria Detection

Games can be viewed as multiobjective optimization problem, where the payoffs of the participating players are to be maximized. All of the objectives to be optimized are uniform and equally important. A solution of the game is called an equilibrium. At equilibrium all players are contended with their outcome, and they are not willing to switch their strategies.

An appealing technique is the use of generative relations and evolutionary algorithms for detecting equilibrium strategies. The payoff of each player is treated as an objective and the generative relation induces an appropriate dominance concept, which is used for fitness assignment purpose. Evolutionary multiobjective algorithms are thus suitable tools in searching for game equilibria.

A population of strategies is evolved. A chromosome is an n-dimensional vector representing a strategy profile $s \in S$. The initial population is randomly generated. Population model is generational. The non-dominated individuals from the population of strategy profiles at iteration t may be regarded as the current equilibrium approximation. Subsequent application of the search operators is guided by a specific selection operator induced by the generative relation. Successive populations produce new approximations of the equilibrium front, which hopefully are better than the previous ones.

For evolutionary equilibria detection any state of the art algorithm can be used. In our numerical experiments we use the NSGA2 [1] algorithm but the results were also tested with differential evolution [10]. Our goal is to focus on the detected equilibria types and not on the algorithm used.

4 Fuzzy Equilibria

In non-cooperative game theory each concept of equilibrium may be associated to a rationality type. A more realistic approach is allowing each player to be more or less biased towards a certain rationality type. This bias may be expressed by a fuzzy membership degree. This way several new types of equilibria, like fuzzy Nash-Pareto [2], can be obtained.

4.1 Fuzzy Nash–Berge-Zhukovskii Equilibrium

Let us consider a fuzzy set A_N on the player set N i.e. $A_N : N \to [0,1]$ $A_N(i)$ expresses the membership degree of the player i to the fuzzy class of Nash-biased players. Therefore A_N is the class of Nash-biased players. Similar a fuzzy set $A_{BZ} : N \to [0,1]$ may describe the fuzzy class of Berge-Zhukovskii-biased players.

A fuzzy Nash–Berge-Zhukovskii equilibrium concept is introduced in this section. Let us consider a game involving both Nash and Berge-biased players. It is natural to assume that $\{A_N, A_{BZ}\}$ represents a fuzzy partition of the player set. Therefore the condition $A_N(i) + A_{BZ}(i) = 1$ holds for each player i.

The relative quality measure of two strategies has to involve the fuzzy membership degrees. Let us consider the threshold function:

$$t(a) = \begin{cases} 1, & \text{if } a > 0, \\ 0, & \text{otherwise} \end{cases}$$

The fuzzy version of the quality measure $k(s^*, s)$ is denoted by $E_N(s^*, s)$ and may be defined as

$$E_N(s^*, s) = \sum_{i=1}^{n} A_N(i) t(u_i(s, s^*_{-i}) - u_i(s^*)).$$

$E_N(s^*, s)$ expresses the relative quality of the strategies s^* and s with respect to the fuzzy class of Nash-biased players.
The fuzzy version of $b(s^*, s)$ may be defined as

$$E_{BZ}(s^*, s) = \sum_{i=1}^{n} A_{BZ}(i) t(u_i(s, s^*_{N-i}) - u_i(s^*)).$$

The relative quality measure of the strategies s^* and s with respect to fuzzy Nash–Berge-Zhukovskii rationality may be defined as

$$E(s^*, s) = E_N(s^*, s) + E_{BZ}(s^*, s).$$

Using the relative quality measure E we can compare two strategy profiles.

Let us introduce the relation $\prec_{fNBZ}$ defined as $s^* \prec_{fNBZ} s$ if and only if the strict inequality $E(s^*, s) < E(s, s^*)$ holds.

Fuzzy Nash–Berge-Zhukovskii (N-BZ) equilibrium is the set of non-dominated strategies with respect to the relation $\prec_{fNBZ}$.

5 Numerical Experiments

Evolutionary method described in Section 3 is used for detecting Fuzzy Nash–Berge-Zhukovskii equilibria. The multiobjective evolutionary algorithm used for equilibria detection is NSGA2 [1] with the following parameter settings: *population size=100, no. of generations=100, probability of crossover=0.9, prob. of mutation=0.5.*

5.1 Continuous Centipede Game

Consider a continuous version of the centipede game [9], the *Symmetric Real Time Trust (SRTT) Game* [6].

There is a set of n players. The strategy space of each player is continuous on the real interval $[0, T]$. Each player can make at most a single decision that "stops the clock" at time $e \in [0, T]$. The game starts at time $t = 0$ and ends either when one of the players stops the clock at some time $t < T$ or when T is reached with no player stopping the clock.

Suppose that the game ends at time $e \in [0, T]$ with player i stopping the clock. Than the payoff for the winner i is given by $r_i = \lambda(2^{(t/\theta)})$, where $\theta \geq 1$ and $\lambda > 0$.

Each of the players not stopping the clock receives only a fraction of the winners payoff. More formally, the payoff for the remaining $n - 1$ players is computed from $r_j(t) = \delta r_i(t)$, where $0 < \delta < 1$, $j = 1, 2, ..., n$, and $j \neq i$.

As time is continuous no tie is possible at times $0 < t < T$. If m players $(1 < m \leq m)$ stop the clock at exactly $t = 0$, then one of them is chosen with probability $1/m$ to receive the payoff λ, and the other $m - 1$ players receive $\delta\lambda$.

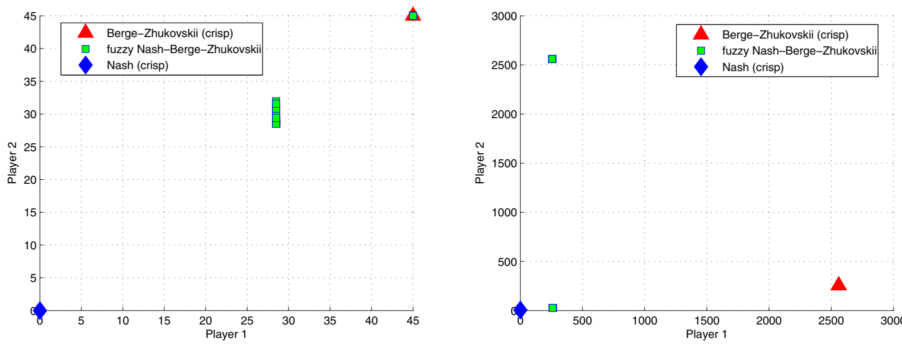
If no player stops the clock (and the game ends at time $t = T$), then the payoff for each of the n players is g, where $0 \leq g < (\lambda 2^{(T/\theta)})$.

The Nash equilibrium of the SRTT game is when all players stop the clock at zero seconds [6], so all players end up with minimal payoffs (zero for all players). However, studies on human players show that people do not play Nash equilibrium. The Berge-Zhukovskii equilibrium of the game is when all players wait until the last moment to press the button. In [6] an experiment based on the SRTT repeated game is presented. Results indicate that human players tend to stop the timer between 25 and 42 seconds (if a unique round is considered). Our aim is to find a fuzzy Nash–Berge-Zhukovskii equilibrium to model this situation.

In order to illustrate the Fuzzy Nash–Berge-Zhukovskii equilibrium we use the continuous centipede (or SRTT) game with the following parameter settings: $n = 2$, $T = 45$, $\theta = 5$, $\lambda = 5$, $\delta = 0.5$ and $g = 0$. Thus there are two players, the player who looses receives 10% of the winner's payoff and if no one stops the clock before 45 seconds the payoff for both players is zero.

Based on our experiments, the Fuzzy Nash-Pareto equilibrium fails to capture an intermediate equilibrium for the SRTT game. For any membership degree the fuzzy Nash-Pareto equilibria correspond to the Nash equilibrium.

Figures 1 and 2 depict the crisp Nash, the crisp Berge-Zhukovskii and the fuzzy N-BZ equilibrium of the SRTT game for various membership degrees. The Nash equilibrium of the game is when both players defect, meaning that they stop the clock at 0 seconds.



(a) Strategy space (b) Payoff space

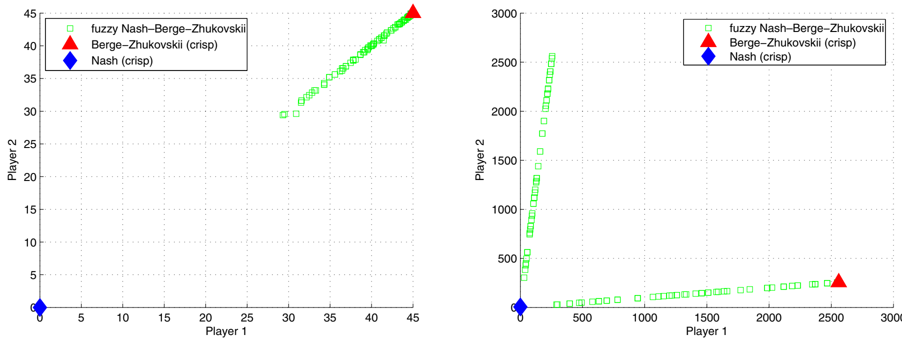
Fig. 1. The detected fuzzy N-BZ equilibrium for the SRTT game with membership degrees $A_N(1) = 0.4, A_{BZ}(1) = 0.6$ and $A_N(2) = 0.6, A_{BZ}(2) = 0.4$

Figure 1 depicts the case when Player 1 has a Nash membership degree of 0.4 (thus a Berge-Zhukovskii membership degree of 0.6) and Player 2 has a Nash membership degree of 0.6 (thus a Berge-Zhukovskii membership degree of 0.4). The fuzzy N-BZ equilibrium is when Player 1 stops the clock around 28 seconds, thus receives a higher payoff.

In cases where both players have equal memberships to both Nash and Berge-Zhukovskii equilibria ($A_N(1), A_N(2)$ and $A_{BZ}(1), A_{BZ}(2)$) the fuzzy N-BZ equilibrium converges either to crisp Nash or crisp Berge-Zhukovskii equilibria. For membership degrees $A_N(1), A_N(2) > 0.5$ (and $A_{BZ}(1), A_{BZ}(2) < 0.5$) the fuzzy N-BZ equilibrium is the same as the Nash equilibrium otherwise if $A_N(1), A_N(2) < 0.5$ (and $A_{BZ}(1), A_{BZ}(2) > 0.5$) the fuzzy N-BZ equilibrium is the same as the Berge-Zhukovskii equilibrium.

Figure 2 depicts the case when both players have equal membership degrees of 0.5. In this case the fuzzy N-BZ equilibrium consists of a set of points, meaning that both players stop the clock somewhere between 25 and 45 seconds. The payoffs for the players is between 256 and 2560 depending on the chosen strategy. Thus, both players end up with higher payoffs than in case of the Nash equilibrium. This equilibrium corresponds to the real-life results presented in [6].

Our numerical experiments show that the fuzzy Nash–Berge-Zhukovskii equilibria always lie between the crisp Nash and crisp Berge-Zhukovskii equilibria. In all the cases the payoffs for the fuzzy Nash–Berge-Zhukovskii equilibria is higher than for the Nash equilibrium. Moreover, when both players have equal biases to Nash and Berge-Zhukovskii equilibrium the detected fuzzy N-BZ equilibrium is suitable to model the human behavior presented in [6].



(a) Strategy space (b) Payoff space

Fig. 2. The detected fuzzy N-BZ equilibrium for the SRTT game with membership degrees: $A_N(1) = A_{BZ}(1) = 0.5$ and $A_N(2) = A_{BZ}(2) = 0.5$

5.2 Partnership Game

Partnership Game considers a firm with n partners. The profit of the firm depends on the partners effort expended on a certain job. The profit function is given by $p(x) = 4(\sum_{i=1}^{n} x_i + c\prod_{i=1}^{n} x_i)$, where x_i is the amount of the expended effort by partner i. The value c measures how complementary the tasks of partners are. Each partner i incur a personal cost x_i^2 of expending effort.

All partners select the level of their effort simultaneously and independently of the other partners. Each partner seeks to maximize their share of the firm's profit which is split equally. The payoff for partner i is given by $u_i(x) = \frac{p(x)}{n} - x_i^2$.

The Partnership game is used with the following parameter settings: $n = 2$, $x_1, x_2 \in [0, 4]$ and $c = 0.2$.

The fuzzy Nash-Pareto equilibrium does not offer promising results for the Partnership game. Similarly as for the SRTT game, for any membership degree the fuzzy Nash-Pareto equilibria correspond to the Nash equilibrium.

Figures 3 and 4 depict the crisp Nash, crisp Berge-Zhukovskii and the fuzzy N-BZ equilibrium for various membership degrees. The Nash equilibrium of the game ensures a smaller payoff for the players $(4, 4)$ while the Berge-Zhukovskii equilibrium offers a more favorable outcome, a payoff of 6.04 for both players.

Figure 3 depicts the detected fuzzy N-BZ equilibrium when player 1 is a pure Nash player ($A_N(2) = 0$, $A_{BZ}(2) = 1$) and player 2 is a pure Berge-Zhukovskii player ($A_N(2) = 0$, $A_{BZ}(2) = 1$).

When both players have equal membership degrees to Nash and Berge-Zhukovskii equilibria, the fuzzy N-BZ equilibrium is the same as the crisp Nash or crisp Berge-Zhukovskii equilibria, depending on the players' biases. If the Nash-bias for both players is higher than 0.5 ($A_N(1)$, $A_N(2) > 0.5$ and $A_{BZ}(1)$, $A_{BZ}(2) < 0.5$) then the corresponding fuzzy N-BZ equilibrium coincides with the Nash equilibrium. Otherwise, if both players are biased towards the Berge-Zhukovskii equilibrium ($A_N(1)$, $A_N(2) < 0.5$ and $A_{BZ}(1)$, $A_{BZ}(2) > 0.5$), the corresponding fuzzy N-BZ equilibrium coincides with the crisp Berge-Zhukovskii equilibrium.

Figure 3 depicts the detected fuzzy N-BZ equilibrium when both players have equal membership degrees of 0.5 to both Nash and Berge-Zhukovskii equilibrium ($A_N(1) = A_{BZ}(1) = A_N(2) = A_{BZ}(2) = 0.5$). The detected fuzzy N-BZ front is very close to the Pareto-optimal front.

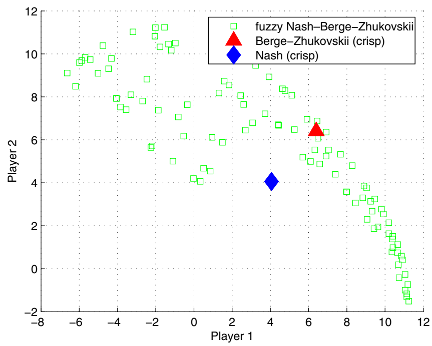


Fig. 3. The payoffs of the detected fuzzy N-BZ equilibria for the partnership game with membership degrees $A_N(1) = 1, A_{BZ}(1) = 0$ and $A_N(2) = 0, A_{BZ}(2) = 1$

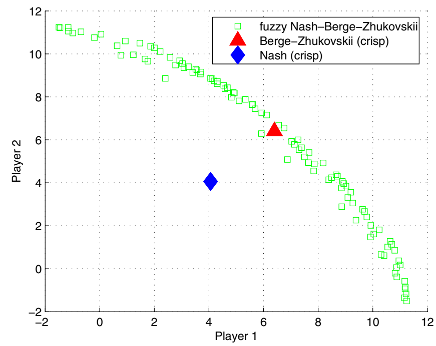


Fig. 4. The payoffs of the detected fuzzy N-BZ equilibria for the partnership game with membership degrees: $A_N(1) = A_{BZ}(1) = 0.5$ and $A_N(2) = A_{BZ}(2) = 0.5$

6 Conclusions

A new equilibrium concept, the fuzzy Nash–Berge-Zhukovskii equilibrium is proposed which is a fuzzy combination of the Nash and Berge-Zhukovskii equilibrium. Game equilibria may be described by generative relations. An evolutionary method for equilibria detection based on generative relations is considered.

Continuous trust games, the partnership game and a continuous version of the centipede game (the SRTT game), are investigated. In the case of the studied games the Nash equilibrium is mutual defection ensuring the lowest possible payoffs for all players. In contrary, the Berge-Zhukovskii equilibrium induces mutual cooperation which ensures higher payoffs.

Numerical results indicate that the proposed fuzzy Nash–Berge-Zhukovskii equilibrium offers a more realistic modeling of the real world. In case of the SRTT game fuzzy Nash–Berge-Zhukovskii equilibrium corresponds to the real life results presented in [6].

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