Tailoring ε-MOEA to Concept-Based Problems

Amiram Moshaiov and Yafit Snir

Faculty of Engineering, Tel-Aviv University, Israel
moshaiov@eng.tau.ac.il, yafit_co@eng.tau.ac.il

Abstract. Concept-based MOEAs are tailored MOEAs that aim at solving problems with a-priori defined subsets of solutions that represent conceptual solutions. In general, the concepts' subsets may be associated with different search spaces and the related mapping into a mutual objective space could have different characteristics from one concept to the other. Of a particular interest are characteristics that may cause premature convergence due to local Pareto-optimal sets within at least one of the concept subsets. First, the known ε-MOEA is tailored to cope with the aforementioned problem. Next, the performance of the new algorithm is compared with C_1-NSGA-II. Concept-based test cases are devised and studied. In addition to demonstrating the significance of premature convergence in concept-based problems, the presented comparison suggests that the proposed tailored MOEA should be preferred over C_1-NSGA-II. Suggestions for future work are also included.

1 Introduction

In the concept-based approach a design concept (in short – concept) is represented by a set of potential solution alternatives [1]. Such a representation has been termed Set-Based Concept (SBC). In contrast to the traditional way of evaluating concepts, the SBC approach allows concept selection to be based not only on optimality considerations, but also on performance variability, which is inherent to the SBC representation [2].

The SBC approach unfolds various ways to compare concepts by their associated sets of performances in objective space [3]. The most studied approach is known as the s-Pareto approach [4]. It involves finding which particular solutions, of which concepts, are associated with the Pareto-front that is obtained by domination comparisons among all individual solutions from all concepts. The interested reader is referred to [5] for some concrete engineering examples of the s-Pareto approach. The current study focuses on such an approach, yet it is restricted to algorithmic aspects rather than to engineering examples.

Concept-based Multi-Objective Evolutionary Algorithms (C-MOEAs) have been originated as a part of the development of a concept-based approach to support conceptual design [1]. C-MOEAs can be obtained by modifying existing MOEAs. This, however, should be done with care. Classical MOEAs are tested for problems where the decision space is common to all solutions. C-MOEAs have to deal with situations where some or all concepts may have, each their own search space. A concept-related premature convergence problem is highly expected when SBCs are

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evolved. This is due to situations where at least one SBC may exhibit a local Pareto whereas the other has none. In such a case the algorithm might, at the extreme case, abandon a good concept. Current C-MOEAs have neither been designed to specially cope with this problem, nor have they been tested to examine their performance under such conditions (e.g., [3], [6]). The current work attempts to fill this gap, by tailoring ε-MOEA, [7], to finding the global s-Pareto front for SBCs. It also includes a comparison of the proposed algorithm with a previously reported C-MOEA (of [6]). The comparison is executed with a special focus on the aforementioned computational problem. To simulate situations each concept may have a different decision space, we adapt the common testing approach of MOEAs by running each concept with a different test function. As presented here, the proposed algorithm is proven to be promising for dealing with the local Pareto problem in the context of concept-based problems.

The rest of this paper is organized as follows. Section 2 provides the background for this paper. Section 3 describes the fundamental issues concerning our methodology. Section 4 presents the suggested algorithm, and section 5 provides the details of the executed tests. Finally, section 6 concludes this paper.

2 Background

2.1 MOEAs' Coping with Local Pareto

In complex problems, and in particular those with a large number of local optima, many existing algorithms are likely to return a sub-optimal solution. This phenomenon is termed premature convergence. In multi-objective problems, MOEAs might get stuck at a local Pareto, and hence, could fail to find the global one. There have been several MOEAs developed in recent years, which show promising results concerning the problem of premature convergence. Nevertheless, none promises convergence to the global Pareto-front. One way for tackling this issue is to use epsilon dominance (e.g. [7]). According to [7], the ε-dominance does not allow two solutions within any of predefined hyper-cubes (using ε_i in the i-th objective) to be non-dominated to each other, thereby allowing a good diversity to be maintained in a population. Furthermore, as pointed out in [7], the method is quite pragmatic because it allows the user to choose a suitable ε_i depending on the desired resolution in the i-th objective.

As explained in the introduction concept-related premature convergence problem is highly expected when SBCs are evolved. Due to their promising characteristics, epsilon-based MOEAs are potential candidates to be transformed into C-MOEAs. As demonstrated here, such tailored algorithms can cope with the peculiarities of the concept-based premature convergence problem.

2.2 Overview of Relevant Algorithms

C_1-NSGA-II and C_2-NSGA-II, which are presented in [6], are C-MOEAs that involve tailoring of the original NSGA-II, of [8], to deal with SBCs. Both are based on a simultaneous approach to the search of optimal concepts. Instead of sequentially evolving a single concept in each run using a classical MOEA such as NSGA-II, in C_1-NSGA-II, and also in C_2-NSGA-II, the population contains solutions from several concepts, and they evolve simultaneously. In C_1-NSGA-II solutions of a more fitted concepts spread on the expense of a less fitted concepts, whereas C_2-NSGA-II involves a reduction of the population size on the expense of the less fitted concepts. These algorithms have been investigated for several interesting computational aspects [6]; however, the issue of concept-based local Pareto has neither been examined in testing C_1-NSGA-II, nor in testing C_2-NSGA-II. In the current study, we use C_1-NSGA-II to compare the proposed algorithm with. No comparisons are made with C_2-NSGA-II since that its search mechanism is in principle the same as that of C_1-NSGA-II.

The ε-MOEA, presented in [7], is a classical MOEA, which is computationally fast and capable of finding a well-converged and well-distributed set of solutions. It uses two co-evolving populations: an EA population $P(t)$ and an archive population $A(t)$ (where t is the iteration counter). The run begins with an initial population $P(0)$. The initial archive population $E(0)$ is assigned with the ε-non-dominated solutions of $P(0)$. Thereafter, two solutions, one from $P(t)$ and one from $A(t)$, are chosen for mating and an offspring solution c is created. Thereafter, the offspring solution c can enter either one of the two populations with different strategies. In section 4 ε-MOEA is modified into the proposed C-ε-MOEA.

3 Fundamentals

Section 3.1, which is provided here for the sake of clarity and completeness, briefly describes the concept-based problem that is dealt with in this paper (based on [4], and [6]). Next, section 3.2 provides a discussion on the need to tailor existing MOEAs into C-MOEAs. This discussion is required since that, in general, existing MOEAs can also be used, as-is, to find the s-Pareto.

3.1 Problem Description

In the following, we consider a finite set C of SBCs, namely of candidate-sets of particular solutions, where $|C| = cs$ is the number of the examined concepts (SBCs). Each $s_m \in C$, m= 1,…., cs, represents the solutions belonging to the m-th SBC. Also considered, for each $s_m \in C$, is the feasible-set $X_m \subseteq S_m$ resulting from possible constraints on using members of S_m. Next, let any i-th member of any X_m be denoted as $x_i^m \in X_m$, and let the set X be the union of the feasible members from all candidate-sets. In general, $X_m \cap X_n$ is an empty set for any $m \neq n$. It is noted that for each S_m there is an associated decision-variable space. In general, for any $m \neq n$, it should be assumed that S_m and S_n do not have a mutual decision-variable-space. For a given mapping $F:\ X \to Y$, the members of the union X are mapped into a mutual multi-objective space $Y \subseteq R^k$, such that for any $x_i^m \in X$ there is one and only one associated vector $y_i^m \in Y$, where $y_i^m = (y_1^{m,i},....y_j^{m,i},....,y_k^{m,i})$.

A concept-based problem involves determining, for each $s_m \in C$, which represents the m-th concept, an evaluation-set E_m that consists of "passed-members" among the members of X_m. For the s-Pareto approach, [4], [6], the "passed-members" are members of the Pareto-optimal set of *X* based on domination comparisons in *Y* among all members of *X*.

In other words, without loss of generality, the problem amounts to *min* $y_i^m \in Y$, over all m and i, to find the s-Pareto front and the associated optimal set. Implicit to the above is that the evaluation sets are meant to be used for concept selection. Another implicit aspect is that the mapping, *F*, may involve numerical characteristics, which may vary from one concept to the other.

It can be argued that the s-Pareto optimality is essentially no different from the Pareto-optimality [6]. Hence it is valid to ask why C-MOEAs are needed or why traditional MOEAs cannot be used as are.

3.2 Why Tailoring Is Needed?

The intention in using the s-Pareto approach is to find all the Pareto-optimal concepts, where each such concept has at least one member of its set being a non-dominated solution with respect to the entire feasible set of solutions. A tailored MOEA for finding the s-Pareto should ensure adequate representation of the concepts along the s-Pareto-front [6]. This means that the resulting set should contain individuals from all the Pareto-optimal concepts. Furthermore, an adequate representation means that the resulting subsets are well distributed on the front.

As seen in the above section, a concept-based problem is almost equivalent to a classical MOP. It is therefore legitimate to ask why we cannot use traditional MOEAs, as are, to solve a concept-based problem. A sequential search approach is certainly possible, where the front of each SBC is separately found. Yet, as discussed in [6], the use of such an approach could mean the waste of resources on finding the fronts of inferior concepts. In contrast, while carrying efficiency promise, the simultaneous SBC search approach, involves the numerical risk of a concept-related premature convergence problem (see introduction).

A simultaneous search technique could be conceived, in which the entire set of solutions from all concepts is treated by a traditional MOEA without any special tailoring to the problem. This assumes that, posterior to the evolutionary run, the obtained Pareto-optimal set and front can be analyzed to identify the parts associated with each concept. Under the assumption that no crossover can take place among individuals from different concepts, such a search approach is restrictive. Namely, a large part of existing MOEAs use a genetic algorithm approach rather than an evolutionary strategy one and therefore cannot be used as-is. Furthermore, as discussed in [6], the use of any traditional MOEA, without some tailoring, may fail to provide adequate representation of the concepts along the s-Pareto front even under the case of a mutual decision space. This is further explained below.

Even in the case of a mutual search space and assuming that individuals from different concepts can mate, the search is inherently divided into different regions to explore the behavior of concepts rather than just specific solutions. The end result, which is the s-Pareto optimal set and its associated front, should provide an understanding of the distribution of the concepts' representatives on the front, rather than just the distribution of particular solutions without their associated concepts' labels. A simple tailoring of a MOEA, for a simultaneous search of the s-Pareto, would amount to making sure that each concept has sufficient representatives in the population, such that even if its proportional part in the s-Pareto-front is relatively small, it will be adequately found. The use of a classical MOEA could fail to ensure that such an optimal concept will be found [6]. Even under the simple case where convergence characteristics of all concepts are the same, the use of a classical evolutionary strategy-based MOEA, with no distinction among solutions of different concepts may occasionally fail to produce the s-Pareto. This is especially because of the possible existence of "overlapping" regions in the s-Pareto-front where solutions from several concepts are mapped into the same or similar performances in the front. Such a phenomenon may become profound under a situation with a local Pareto.

In summary, different concepts are associated with different decision spaces, or with different regions within a mutual decision space. This may lead to the possibility of a local Pareto within a concept. In a sequential search approach any MOEA that can overcome local Pareto would be sufficient, since that the sequential approach does not involve a simultaneous search within several concepts. In a simultaneous search approach, the existence of a local Pareto-front, within any of the concepts, could be detrimental, as it can cause an improper balance among the search resources given to each concept.

4 The Proposed Algorithm

4.1 Tailoring Requirements

Generally, any state-of-the-art MOEA can be adapted to suit a simultaneous search for the s-Pareto. The main features of the required modifications are: 1. The division of the population to subsets according to the concepts; 2. The restrictions imposed namely no crossover among individuals of different concepts; and 3. The mechanism for resource distribution among the concepts.

A tailored algorithm, termed C-ε-MOEA is introduced below. C-ε-MOEA is a variant of the ε-MOEA algorithm of [7] with some modifications to handle concepts. The following refers to meeting the tailoring requirements by the proposed algorithm. The first two issues are explicitly dealt with as follows. In the proposed C-ε-MOEA the population is divided into sub-populations; each of them represents a different concept. The recombination operator allows recombination only among members of the same sub-population. In contrast, the third requirement concerning resources is only implicitly involved such that a concept that has better performance compared to another concept will be allocated more resources than the second according to the proposed selection process.

4.2 C-ε-MOEA

4.2.1 Main Steps of the Algorithm

The suggested procedure is described as follows:

1 Randomly initialize a population $P(0)$ with equally sized subpopulations for each concept. The concept-based ε-non-dominated solutions of $P(0)$, over the entire population, are copied to an archive population $A(0)$ (as detailed in section 4.2.2). Set the iteration counter $t = 0$.

2 One solution p is chosen from the population $P(t)$ using the “pop selection method" (detailed in section 4.2.3).

3 One solution a is chosen from the archive population $A(t)$ using the “concept archive selection method" (detailed in section 4.2.4).

4 One offspring solution c is created using p and a.

5 Solution c is included in $P(t)$ using the “concept pop acceptance method" (detailed in section 4.2.5).

6 Solution c is included in $A(t)$ using the “concept archive acceptance method" (detailed in section 4.2.6).

7 If termination criterion is not satisfied, set $t = t + 1$ and go to Step 2, else report $A(t)$.

4.2.2 Concept-Based Population and Archiving

Similar to the original algorithm of [7], C-ε-MOEA uses two co-evolving populations including a population $P(t)$ and an archive population $A(t)$ (where t is the iteration counter). The proposed MOEA begins with an initial population $P(0)$, which is composed of cs subsets of p solutions each. To meet the first tailoring requirement, as detailed in section 4.2.1, the $A(t)$ and $P(t)$ are maintained such that:

$$A(t) = \cup_{i=1}^{cs} A_i(t) \tag{1}$$

$$P(t) = \cup_{i=1}^{cs} P_i(t) \tag{2}$$

Where the sub-archive $A_i(t)$ and sub-population $P_i(t)$ contains individuals associated only with the i-th concept, and t is the iteration counter.

The archive population $A(0)$ is assigned with the concept-based ε-non-dominated solutions of $P(0)$. The concept-based ε-non-dominated solutions are obtained ("defined") as follows: for each hyper-box, which has at least one non-dominated solution from the entire P associated with it, we keep neither one such solution nor all. Rather, for each such hyper-box and for each concept i, which has one or more solutions with performances in that hyper-box, we save one solution which is selected randomly from the non-dominated solutions of the concept within that hyper-box.

4.2.3 Pop Selection Method ([7])

This procedure repeats the procedure in [7]. To choose a solution p from $P(t)$, two population members from $P(t)$ are picked up at random, regardless of their concept association, and a domination check is made. If one solution dominates the other, the former is chosen. Otherwise, the event indicates that these two solutions are non-dominated to each other and in such a case we simply choose one of them at random.

4.2.4 Concept Archive Selection Method

In this method we randomly pick a solution a from $A(t)$, from the subset $A_i(t)$ which corresponds to the subset $P_i(t)$ that p was chosen from, namely

$$\{p \in P_i(t) \ \ and \ \ a \in A_j(t) \mid i = j\} \tag{3}$$

If $A_i(t)$ is empty, we select another solution a at random from $P_i(t)$. This ensures that in step 4 (see section 4.2.1) the mating is done meeting the second tailoring requirement as detailed in section 4.1

4.2.5 Concept Pop Acceptance Method

This method defines the decision criteria for an offspring c to replace any population member. We compare the offspring with all population members, regardless of their concept association. If any population member dominates the offspring, the offspring is not accepted. Otherwise, if the offspring dominates one or more population members, then the offspring replaces one of the dominated ones (chosen at random). This means that in such a case a change in the allocated resources occurs; no longer the concepts have equal resources (see section 4.1). When both the above tests fail (that is, the offspring is non-dominated by the population members), the offspring replaces a randomly chosen population member from its' own concept sub-population.

4.2.6 Concept Archive Acceptance Method

For the offspring c to be included in the archive population, the offspring is compared with each member of the archive, in the ε-dominance sense, as follows:

1. If the offspring is ε-dominated by a member of the archive – it is not accepted.
2. If the offspring ε-dominates a member of the archive – it replaces that member.
3. If none of the following exists then the offspring is ε-non-dominated with all archive members.
   a. If the offspring shares a hyper-box with an archive member, who is from the same concept as the offspring, then they are compared in the usual dominance sense – and the member which dominates is chosen. Otherwise they are non-dominated and the member which is closer to the B vector, as defined in [7], (in the Euclidean sense) is chosen. If they have the same distance – one is chosen at random.
   b. If none of the archive members, which are associated with the same concept, share the same hyper-box as the offspring, then the offspring is accepted.

It is interesting to note that the suggested procedure ensures that only one solution per concept may exist in each hyper-box.

4.2.7 Algorithm Properties

The following properties of the C-ε-MOEA procedure are derived from the basic ε-MOEA algorithm ([7]):

1. It is a steady-state MOEA.
2. It emphasizes concept-based non-dominated solutions, and by so emphasizes concepts with better performing solutions.

3. It maintains the diversity in the archive by allowing only one solution per concept to be present in each pre-assigned hyper-box on the Pareto-optimal front.
4. It is an elitist approach.
5. It solves for the s-Pareto front within the pre-defined resolution.

5 Results

Two tests are reported. In each of the tests two concepts are simultaneously evolved. Different test functions are used for the different concepts, where one concept involves a multi-modal behavior, and the other exhibits a single-modality behavior. The used functions include: the ZDT4 multi-modal function, the discrete function, and the SCH function. The definitions of the above can be found in [8]. In the first test, ZDTt4 and SCH are used, respectively, for each of the two concepts tested. In the second test, SCH is replaced by FON. The decision spaces are kept for each of these functions (per concept) as in [8]. All tests, which are described below, are done with a population size of 100 and for 250 generations. We use the real-parameter SBX recombination operator with probability of 0.9 and ηc=15 and a polynomial mutation operator with probability of $1/n$ (n is the number of decision variables) and ηm=20 [8]. The results of C_1-NSGA-II are taken after elite preserving operator is applied. Epsilon values were chosen after several trials to be $\varepsilon_1 = \varepsilon_2 = 0.05$. A too large epsilon will result in a low granulation front – small set of solutions found. A too small epsilon will not make the desired effect on diversity and convergence.

Figures 1 and 2 show typical results of the s-Pareto fronts for the two tests. Clearly in both cases, C-ε-MOEA overcame the numerical difficulty whereas C_1-NSGA-II failed to cope with it. Both tests were run 30 times each with random initial population. The statistics are included in Table 1. We use convergence and sparsity metrics [6] to compare between the two algorithms. It can be observed that while the sparsity metric is similar, there is a significant improvement in convergence when C-ε MOEA is used. Moreover, this is done with better efficiency as the time (measured in seconds) is also significantly decreased.

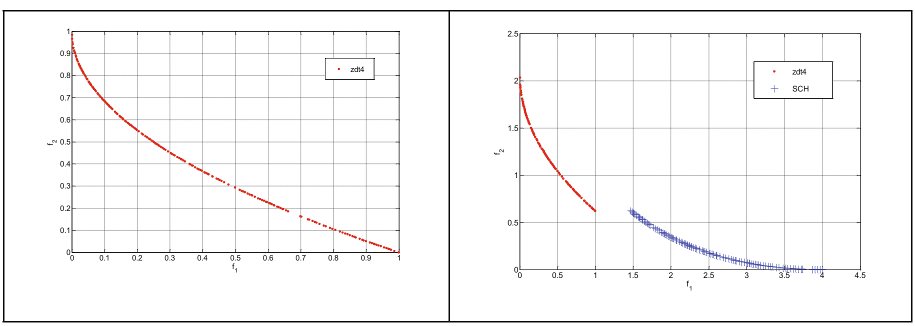

Fig. 1. ZDT4 & SCH
Left and Right: Front by C-ε-MOEA and by C_1-NSGA-II respectively
(ZDT4 designated by dots and SCH by pluses)

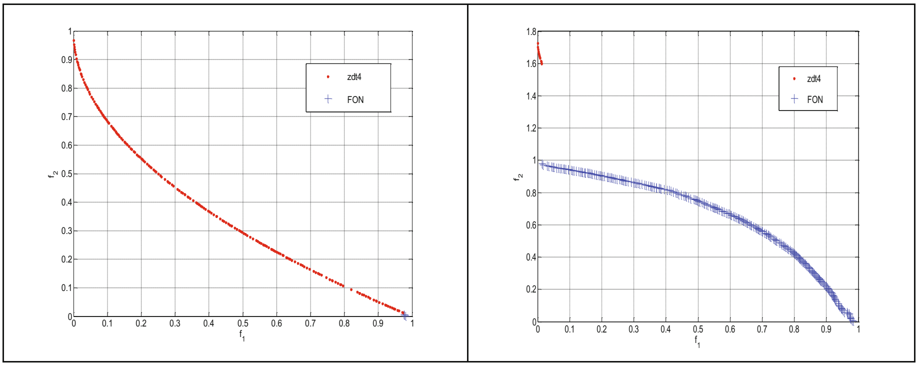

Fig. 2. ZDT4 & FON
Left and right– Resulted front using C-ε-MOEA and C_1-NSGA-II respectively
(ZDT4 designated by dots and FON by pluses)

Table 1. Statistics of the runs

MOEA	Convergence		Sparsity		Time	
	Avg.	SD	Avg.	SD	Avg.	SD
ZDT4 & SCH						
C1-	2.8113	1.8815	0.945	0.0321	1190	115
C-ε-	0.01016	2.43e-03	0.914	0.0710	32	0.8
ZDT4 & FON						
C1-	0.07395	0.00692	0.931	0.0560	1719	123
C-ε-	0.00361	1.92e-04	0.901	0.0462	42	1.7

6 Conclusions and Future Work

Although C1-NSGA-II has been shown to produce good results for many test cases involving SBCs, [6], it is shown here that it often fails to converge in the case of multi-modal concept-based problems. Solving such problems can result in sub-optimal front and may lead to undesired results. This can restrict the application of C1-NSGA-II to real-world problems. In this paper a tailored algorithm, C-ε-MOEA, is proposed, based on [7], in order to deal with the premature convergence difficulty, which is expected in concept-based problems. The experimental results show that C-ε-MOEA is able to obtain the s-Pareto front on hard multi-modal test cases, where C_1-NSGA-II fails to do so.

It should be noted that the current study, which focuses on the s-Pareto approach, is likely to also be most relevant for the future extension of this work to support concept selection by other SBCs methods (e.g. [3]). This is expected since that, in the context of SBCs, the problem of concept-based premature convergence is a generic one.

Future work should include an expansion of the tests done here, and sensitivity analysis to different epsilons. It may also be beneficial to compare the proposed algorithm with others that could be developed based on newer algorithms such as in [9].

References

1. Moshaiov, A., Avigad, G.: Concept-based Multi-objective Problems and Their Solution by EC. In: Proc. of the GECCO 2007 Workshop on User Centric Evolutionary Computation, GECCO, London (2007)
2. Avigad, G., Moshaiov, A.: Set-based Concept Selection in Multi-objective Problems: Optimality versus Variability Approach. J. of Eng. Design 20(3), 217–242 (2009)
3. Denenberg, E., Moshaiov, A.: Evolutionary Search of Optimal Concepts using a Relaxed-Pareto-optimality Approach. In: Proc. of the IEEE Congress on Evolutionary Computations, CEC 2009, Trondheim, Norway, pp. 1093–1100 (2009)
4. Mattson, C.A., Messac, A.: Pareto Frontier based Concept Selection under Uncertainty with Visualization. Optimization and Engineering 6, 85–115 (2005)
5. Mattson, C.A., Mullur, A.A., Messac, A.: Case Studies in Concept Exploration and Selection with s-Pareto Frontiers. Int. J. of Product Development, Special Issue on Space Exploration and Design Optimization 9(1/2/3), 32–59 (2009)
6. Avigad, G., Moshaiov, A.: Simultaneous Concept based Evolutionary Multiobjective Optimization. Applied Soft Computing 11(1), 193–207 (2011)
7. Deb, K., Mohan, M., Mishra, S.: Evaluating the ε-domination based Multi-objective Evolutionary Algorithm for a Quick Computation of Pareto-optimal Solutions. Evolutionary Computation 16(3), 355–384 (2005)
8. Deb, K., Pratap, A., Agarwal, S., Meyarivan, T.: A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II. IEEE Tran. on Evolutionary Computation 6, 182–197 (2002)
9. Deb, K., Tiwari, S.: Omni-optimizer: A Generic Evolutionary Algorithm for Single and Multi-objective Optimization. European J. of Operational Research 185(3), 1062–1087 (2008)