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CALIBRATION OF A DECISION-MAKING PROCESS IN A SIMULATION MODEL BY A BICRITERIA OPTIMIZATION PROBLEM

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ABSTRACT

In a previous paper, we developed an accurate simulation model of an Intensive Care Unit to study bed occupancy level (BOL). By means of accurate statistical analysis we were able to fit models to arrivals and length-of-stay of patients. We model doctors' patient discharge decisions and define a set of rules to determine the conditions for earlier or delayed discharge of certain patients, according to BOL. For the calibration of the rule parameters, we proposed a nonlinear stochastic optimization problem aimed at matching the model outputs with the real system outputs. In this paper, we improve the calibration of the rule parameters by including the principle of "minimum medical intervention" as a second objective function. We replace the previous objective function with a satisficing matching, in order to gain more degrees of freedom in the search for better rules according to the new objective.

1 INTRODUCTION

Simulation has been widely used to analyze health-care system management problems, which are characterized by a stochastic environment and limited human and material resources. Reviews and discussion papers dealing with the application of simulation modelling in health care can be found in Brailsford et al. (2009), Eldabi et al. (2007), Günal and Pidd (2010) and Katsaliaki and Mustafee (2011). Many studies use simulation to analyze hospital capacity and bed allocation, but only a few deal specifically with ICUs. Worth noting are Kim et al. (1999, 2000), in which ICU admission and discharge processes are analyzed through simulation and several rules for bed allocation are evaluated; Litvack et al. (2008) and Ridge et al. (1998) and Costa et al. (2003), which analyze the problem of ICU capacity; and Kolker (2009), in which an ICU simulation model is used to establish a quantitative link between the daily load levelling of elective surgeries and ICU diversion.

The construction of an ICU simulation model involves finding the appropriate statistical models for its stochastic elements: arrival patterns, patient's personal and medical characteristics and length of stay (LoS) in the ICU. An overview of LoS and patient flow modelling techniques can be found in Marshall et al. (2005). All these simulation studies assume that the LoS is independent of the ICU workload and bed occupancy level. However, recent research Mallor and Azcárate (2012) has shown that the LoS of some patients can be influenced by the ICU bed occupancy level. Doctors may discharge patients earlier when the number of occupied beds threatens the unit's capacity to accommodate new incoming patients and, conversely, when the ICU bed occupancy is low, patients may be allowed to complete their recovery in

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the ICU. Thus a valid simulation model should include doctors' patient discharge decisions. The modeling task is hampered, however, by the lack of any written decision protocol that could be implemented in the simulation model.

We address the problem of modelling doctors' decision-making by defining a set of rules dependent on a set of parameters. Model calibration is the process of determining the values of unobservable parameters by constraining model output to replicate observed data. Research papers dealing with the calibration of simulation models are not numerous. There are some that deal with this topic in the field of traffic simulation, e.g. in Park and Qi (2005) which proposes a general procedure for the calibration of microscopic simulation models. A discussion on the computational complexity of model calibration appears in Hofmann (2005); and remarks on calibration with respect to validity can be found in Bayarri et al. (2004). To estimate the rule parameters, we formulated a nonlinear stochastic optimization problem with the aim of matching the bed occupancy outputs from the simulation model to those of the actual system.

The aim of this paper is to present an improved version of the calibration process designed to take into account not only the match between the simulation output and the system output but also the opinions of doctors. We consider a bicriteria optimization problem in which the principle of "minimum medical intervention" is included as a second objective function. We replace the previous objective function with a satisficing matching in order to gain more degrees of freedom in the search for better rules according to the new objective.

In section 2, we present the elements included in the ICU simulation model. Section 3 discusses a way to represent doctors' decisions by means of a set of rules. The optimization problems defined to calibrate the parameters of this set of rules are described in Section 4. In Section 5, we present the results of the calibration process applied to the simulation of the ICU at the Hospital of Navarre. The final section presents the conclusions.

2 MODELING AN INTENSIVE CARE UNIT

An ICU is mathematically modelled as a queuing system in which the patients are the clients, the beds are the servers and there is no waiting room. We consider that any patient arriving when the ICU is full is transferred to an alternative ICU (in a neighbouring region). The simulation model is therefore structurally quite simple and can be run using appropriate statistical models for the patients' input pattern and for their LoS in the ICU.

Many studies (see for example, Ridge et al. 1998; Kim et al. 2000; Litvack et al 2008; Oddoye et al. 2009) use a Poisson Process as a statistical model for patient arrival to a health care centre. This model holds for outer patients arriving at the ICU on an individual basis, whose arrival times are not influenced by prior patient arrivals and are not coordinated to fit in with any pre-arranged schedule. However, the Poisson Process does not apply for patients coming from operating theatres. These results were confirmed in Mallor and Azcárate (2012), where we found that a Poisson Process fits outer patient arrivals very well but does not fit those of the pre-scheduled patients group, which required the use of empirical distributions. The latter group also presented a different arrival pattern for holiday periods.

One common characteristic of LoS data is the presence of a high percentage of *extreme* values, that is, values that are far enough from the mean to be considered outliers (see Vasilakis and Marshall 2005). In these cases, the distributions commonly used to represent service times in the health care context do not give a good fit to real LoS data. This problem of poor fit to the original data has been addressed in the literature in different ways. In our previous paper, we addressed this statistical fitting problem by developing non-normal regression models including variables with the power to explain some of the LoS variability, such as the Apache index.

3 MODELING THE MEDICAL DECISION MAKING

One of our main achievements in studying ICU simulation models has been to show that simulation models that fail to incorporate the management decisions made by clinical staff can hardly be considered valid. We reached this conclusion after comparing the bed occupancy outputs from the simulation model with those of the ICU (Figure 1). A visual inspection of Figure 1 suggests that management policies for patient admission and discharge affect the bed occupancy distribution. The ICU staff confirmed that some discharge decisions are made with view to keeping bed occupancy from becoming too high (thus compromising the admission of new patients) or too low (thus "wasting" valuable resources). ICU discharge decisions, therefore, can sometimes be driven by current occupancy rates and bed demand, provided the patient's welfare is not compromised.

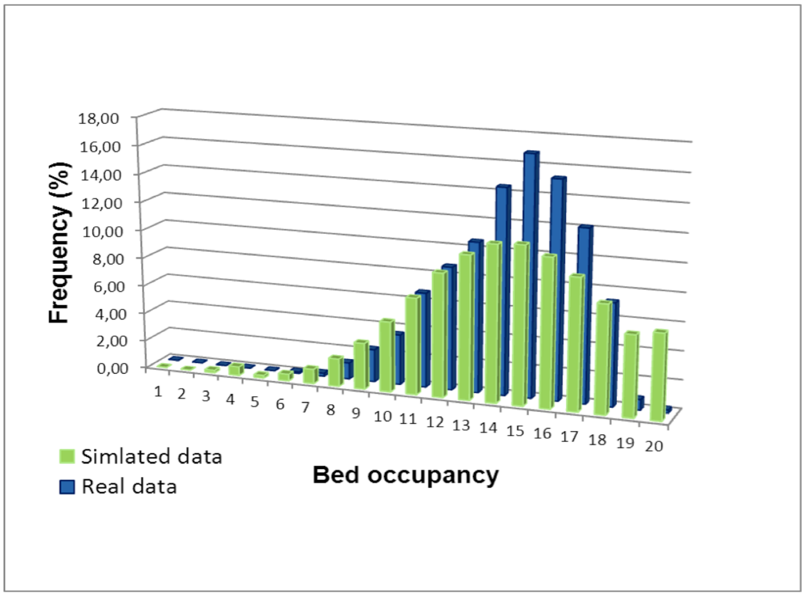

Figure 1: Real vs. simulated bed occupancy frequencies in a simulation model without managerial decision modelling.

Thus, to obtain a valid simulation model, it is necessary to include clinical decision-making. There is no written protocol for managers to determine patient admission and discharge automatically; these decisions are subject to the judgment of the intensive care consultant. To model these human decisions concerning patient LoS depending on the bed occupancy level, we consider two kinds of management rules:

- if the bed occupancy level i is *high* and certain conditions are satisfied *(the decision must not involve a terminal patient and the estimated remaining LoS of the one selected for discharge must be less than* $\%PR_i$ *and less than* DR_i *days),* one patient (the one in the best state of health) leaves the ICU prior to completing treatment.
- if the bed occupancy level i is *low,* then the LoS of a patient may be increased by one day with certain probability, PI_i. There is a maximum number of days DE_i by which the LoS of a patient can be extended.

These rules can be identically defined for all types of patients or can be made to vary for different groups of patients. For example, we might distinguish the group of programmed-surgery patients, whose stay tends to be short and consider that it can be shortened by only one day with certain probability PC_i when bed occupancy is i.

These rules link patients' LoS to bed occupancy levels. ICU discharge time is a value belonging to a set of admissible values defined by the rule parameters (see Figure 2).

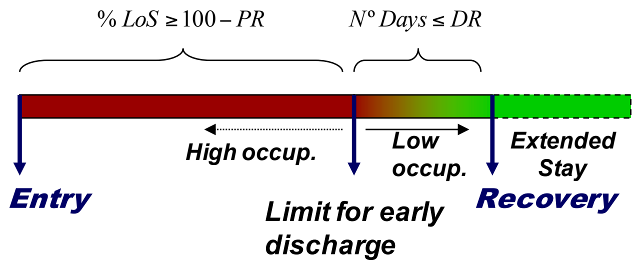

Figure 2: Patient recovery is a continuous process leading to ICU discharge in a period of time with a value that belongs to an interval of admissible values.

Observe that the set of rules defines infinite management policies for the ICU: one for each set of parameter values. The simulation model allows us to assess the performance of only one set of parameters in each simulation run. By running the simulation model for some reasonable values of the rule parameters, it would be possible to assess their influence on the distribution of the bed occupancy frequencies and the degree to which these simulated results approximate ICU observed data. This brings us to the question of how to find the best set of parameter values, that is, how to calibrate the simulation model. We address this question in the next section by defining different optimization problems.

4 OPTIMIZATION PROBLEMS FOR THE CALIBRATION OF THE SIMULATION MODEL

Model calibration is the process by which we determine simulation parameters values not observed in the system, such that model output replicates empirical data. In our case, we need to determine the rule parameter values that produce a bed occupancy output similar to that observed in a real ICU.

In a first approach, we formulate an optimization problem aimed at matching the simulation model output as closely as possible to the ICU historical data. The decision variables are the parameters *PR, DR, PI, DE* and *PC* above defined. Constraints represent realistic monotonous relationships into each set of parameters and upper bounds for their values (*uPR, uDR, uPI, uDE* and *uPC*). We set as our objective function to minimize the squared differences of real and simulated frequency (absolute value of differences or maximum difference also can be used).

$$
\begin{aligned}
& Min \quad \sum_{i=0}^{n} \left(real\_freq(i) - simul\_freq(i)\right)^2 \\
& subject\ to \\
& \begin{cases}
uDR \geq DR_n \geq DR_{n-1} \geq \ldots \geq DR_{n-k2} \geq 0 \\
uPR \geq PR_n \geq PR_{n-1} \geq \ldots \geq PR_{n-k2} \geq 0 \\
uDE \geq DE_1 \geq \ldots.. \geq DE_{k1} \geq 0 \\
uPI \geq PI_1 \geq \ldots.. \geq PI_{k1} \geq 0 \\
uPC \geq PC_n \geq PC_{n-1} \geq \ldots \geq PC_{n-k2} \geq 0 \\
DR_i, DI_j \quad \text{integer}\ i = n-k2,\ldots,n \quad j = 1,\ldots,k1,\ k1 \leq k2
\end{cases}
\end{aligned}
\tag{1}
$$

To solve this problem, we incorporate these decision rules into the simulation model and combine optimization and simulation (ARENA-OptQuest). This gives us a bed occupancy distribution that closely matches the observed one, as shown in Figure 3.

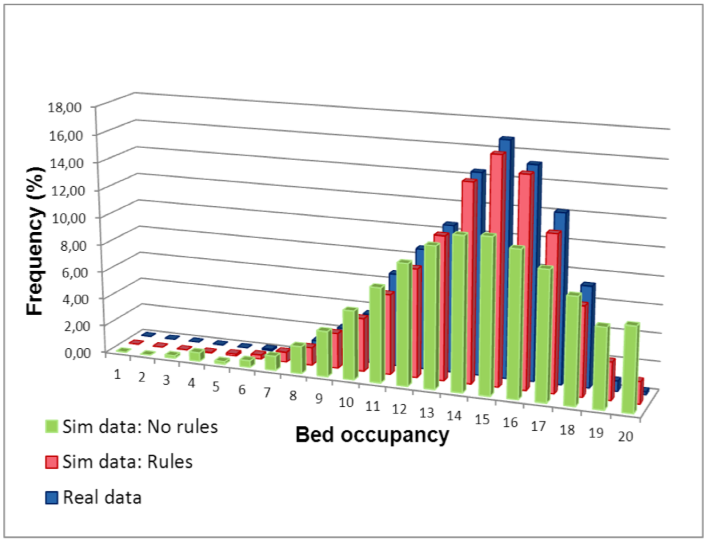

Figure 3: Real vs. simulated (with and without rules) bed occupancy frequencies.

Management policies obtained from this "historical data approach" can be too "aggressive" in the sense of allowing "extreme" shortening of some stays. An example of such management policies is given in the first column of table 1 and commented in next section.

Furthermore, the simulation model is only a representation of the real ICU, therefore the results of the simulated system do not have to be identical to those of the real system. *It is enough that they be similar*. Then we propose a new approach that takes into account not only the historical data but also the opinions of experts who stated that early discharges should be kept as low as possible. We interpret this idea as the principle of "minimum medical intervention" and consider it as our second objective function.

Observe that a zero value for all parameters would indicate a situation in which clinicians pay no heed to the bed occupancy level, but, as it rises, its impact on their decisions grows stronger. Similarly, if there is no situation in which patient LoS may be shortened or extended, then clinicians will again disregard the bed occupancy level, while the greater the number of days by which LoS is shortened or extended, the more the bed occupancy level influences clinicians' decisions. Following these ideas, we consider two different ways of measuring the "medical intervention level". From a decision-making point of view, we will try to minimize the rule parameter values to keep them as close to zero as possible*($INFL_1$)*. From the consequences of the decision-making process we will try to minimize the total number of days of LoS extension or reduction *($INFL_2$)*.

Specifically, we define the following measure of the influence of the bed occupancy level in ICU management:

$$INFL = w_1 INFL_1 + w_2 INFL_2$$

where,

$$INFL_1 = \frac{1}{k}\left(\sum_i PI_i + \sum_i PR_i + \sum_i PC_i + \sum_i DR_i^* + \sum_i DE_i^*\right)$$

$$DR_i^* = DR_i / uDR \quad \text{and} \quad DE_i^* = DE_i / uDE$$

and

$$INFL_2 = \%\,days'\,extension + \%\,days'\,reduction$$

Observe that, to make parameter values representing a percentage or probability comparable with those representing a number of days, we normalize the latter to a maximum number of days, *uDR* and *uDE*, set as admissible for an early discharge or extended stay. The value *k* is the total number of parameters involved in the expression of $INFL_1$. Then $INFL_1$ ranges from 0 to 1, where a 0 value indicates that the bed occupancy level has no impact on ICU management and increasing values of $INFL_1$ indicate an increasing impact. The objective $INFL_2$ is expressed in percentage terms by dividing the number of days of LoS extension or reduction by the total days LoS of all current patients.

5 CASE STUDY AND COMPUTATIONAL RESULTS

We have developed a simulation model that includes a representation of the discharge decisions made in the ICU of the Hospital of Navarre, in Spain. The Hospital of Navarra is a general public hospital with reference specialties in the Community of Navarra (Neurosurgery, cardiac surgery, vascular surgery, oncology, infectious diseases, etc.). It has 483 beds, 2015 members of staff and 10 operating theatres. The ICU of this hospital has 20 beds and 86 physicians and nurses. It receives patients from 3 sources (emergency, operating theatres and wards).The data were recorded and provided by the Hospital administration. We have three files: a patient file, a bed occupancy file and an arrivals file, with 9 years of data, from 1/1/2000 to 31/12/2008. The patient file contains a record of all patients admitted to ICU during that period. The known patient variables are as follows: age, arrival date, illness group (8 groups were considered), discharge date, APACHE (illness severity), ICU infections, and exitus (recovered or deceased). The bed occupancy file is a record of bed occupancy noted daily at 16h. These data are used to validate and calibrate the simulation model. The arrivals file is a record of the number of patients admitted to the ICU each day.

We consider the following bicriteria optimization problem, including both the medical intervention objective function, as described in Section 4, and a measure of the difference between the bed occupancy distribution in the simulation output and in the historical data (observe that this objective function is the statistic used by the Kolgomorov-Smirnov test):

$$\begin{aligned} &Min \quad INFL \\ &Min \quad \underset{i}{Max} \ \left| F\_real\,(i)\text{-}\, F\_simul(i) \right| \\ &subject\ to \quad \text{constraints} \quad \text{in}\,(1) \end{aligned}$$

We estimate the Pareto frontier by using the ε-constraint method:

$$\begin{aligned} &Min \quad INFL \\ &subject\ to \quad \begin{cases} \underset{i}{Max} \ \left| F\_real\,(i)\text{-}\, F\_simul(i) \right| \le \varepsilon_j \\ \text{constraints} \quad \text{in}\,(1) \end{cases} \end{aligned} \tag{2}$$

Figure 4 shows the Pareto frontier obtained by considering different ε_j-values for the “data matching” objective, ranging from 10% to 3%.

Representative optimal solutions of problem (2) are shown in Table 1. As an example, let us consider the solution for ε_j=3.5. $PR_{17}=45\%$ and $DR_{17}=4$ days mean that a patient´s LoS can be reduced a maximum of 4 days, whenever these 4 days represent less than 45% of the patient´s LoS to complete recovery, when occupancy level is 17 beds.

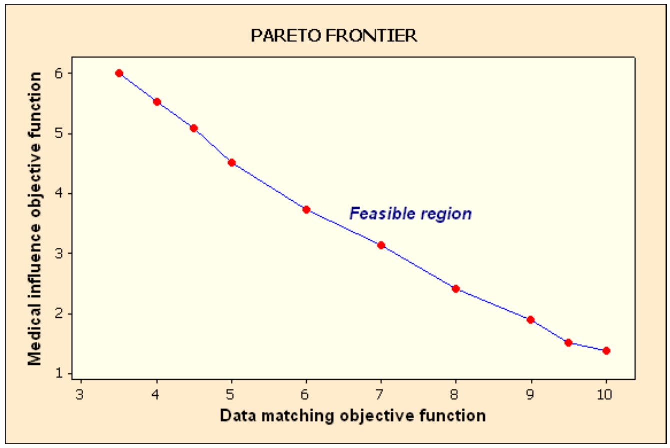

Figure 4: Pareto frontier for the data-matching and medical influence objective functions.

Figure 5 plots the bed occupancy distributions for real data (thick black line), simulated data without discharge rules (thick red line), and simulated data with different discharge rules scenarios. Dark blue dotted line represents the simulated bed occupancy distribution with the discharge rules obtained from the optimization problem in (1), that is by only optimizing the matching objective function. The set of thin lines, with colours ranging from red to blue, represent bed occupancy distributions from simulation models including the discharge rules obtained as solutions of optimization problems (2) (one line for each ε_j -value considered in Table 1). We observe that the closest distribution to the historical bed occupancy distribution is provided by the solution of the optimization problem (1). Nevertheless, the values for the rule parameters of this management policy are "too aggressive" as mentioned in the previous section: under this policy it is allowed to advance the discharge of a patient up to 5 days, shortening its LoS up to 50%. Setting the approach to the historical data as a constraint and minimizing the medical influence the distributions move gradually away from the historical one as the value of ε_j increases. On the other hand the increase of ε_j leads to softening the medical intervention. We see that bounding the difference to historical data distribution to a maximum of 10% all LoS are shortened in no more than one day.

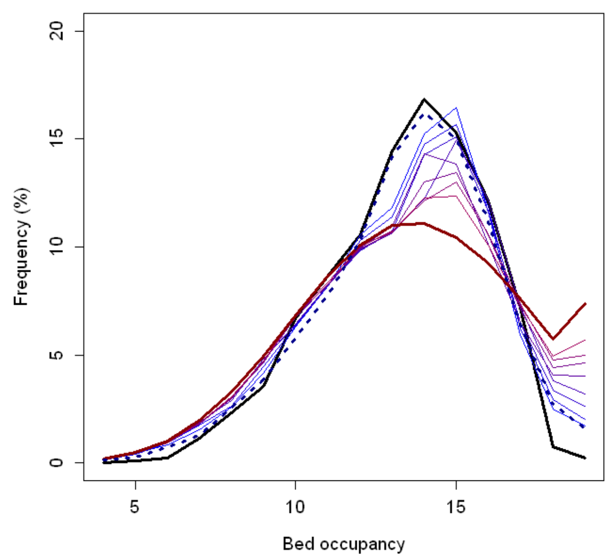

Figure 5. Bed occupancy distributions for the ICU: real data and simulated data with different discharge policies.

Table 1: Optimal solutions of optimization problems (1) and (2)

	Solution of optimization problem (1)	Solutions of optimization problem (2) for different ε_j-values							
		3.5%	4%	5%	6%	7%	8%	9%	10%
PI_{high}	25	15	10	0	0	0	0	0	0
PI_{low}	25	15	10	0	0	0	0	0	0
PR_{15}	15	0	0	0	0	0	0	0	0
PR_{16}	40	30	15	25	5	20	15	10	10
PR_{17}	50	45	40	35	35	20	20	15	15
PR_{18}	50	45	40	35	35	20	20	15	15
PR_{19}	50	45	40	35	35	20	20	15	15
PR_{20}	50	45	40	35	35	20	20	15	15
PC_{15}	5	0	0	0	0	0	0	0	0
PC_{16}	5	0	0	0	0	0	0	0	0
PC_{17}	90	0	5	0	0	0	0	0	0
PC_{18}	90	10	5	5	0	5	0	0	0
PC_{19}	100	10	5	10	10	5	5	5	5
PC_{20}	100	10	5	15	15	15	15	5	5
DR_{15}	1	0	0	0	0	0	0	0	0
DR_{16}	1	1	2	1	2	2	1	1	1
DR_{17}	2	4	4	3	3	3	2	2	1
DR_{18}	5	5	5	4	3	3	2	2	1
DR_{19}	5	5	5	4	3	3	2	2	1
DR_{20}	5	5	5	4	3	3	2	2	1

6 CONCLUSIONS

In this paper, we have shown that, in order to obtain valid ICU simulation models, it is necessary not only to ensure accurate statistical modelling of stochastic elements (arrival process and LoS) but also to include clinicians' discharge decisions. We have proposed a set of rules to model this human decision process.

Rule parameters are estimated by means of a calibration process that uses bed occupancy historical data. However, the search for a perfect matching does not prove fully satisfactory. To improve the calibration, therefore, we have incorporated a second objective, minimum medical intervention, to complement the classical data-matching approach and then formulated a bicriteria optimization problem.

One of the main problems of the Spanish public health care system is the length of waiting lists for some specialist surgical procedures. To reduce waiting lists and improve service quality, the hospital is extending some operating theatre hours, which increases the number of patients from elective surgery. In the past, the lack of beds caused to postpone surgeries. Nowadays, no surgery is cancelled due to the lack of operating rooms, and the patients are also transferred to other health facilities in the region, if necessary. Then the simulation model would benefit from the inclusion of the programmed surgeries. Unfortunately we have no access to the necessary information to model them.

More research is needed to select a compromise solution. We are exploring the possibility of interpreting the data-matching objective function as the statistic of a distribution fitting test. In that way, its admissible values are those belonging to the acceptance region.

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