

## **A SIMULATION APPROCH FOR AN $(R,Q)$ INVENTORY MODEL WITH A DETERIORATING ITEM, POISSON DEMAND AND STOCHASTIC LEAD TIME**

S. Golshid Sharifnia  
S. Mehdi Sajadifar

Science and Culture University  
Department of Industrial Engineering  
Tehran, 13145-871, IRAN

Mohammadmahdi Alizadeh

Tarbiat Modares University  
School of Industrial Engineering  
Tehran, 14117-13116, IRAN

### **ABSTRACT**

In this study, we consider an inventory system with a modified  $(R,Q)$  ordering policy and stochastic lead time for a single product with continuous decaying. Each customer demands only one unit of the product and the system faces independent Poisson demand. An order with size of  $Q(1+\alpha)$  is ordered immediately after the inventory position reaches  $R$ . Each order is considered to be as a separate package. Shortage is allowed and completely backordered. The objective is to minimize the long-run total expected cost. At first step, the mathematical model is developed for deterministic lead time. Since stochastic lead time leads to extremely complex model we apply simulation modeling approach. Then we validate the simulation model by comparing the outputs of simulation model with the mathematical model in the case of deterministic lead time. Finally, a number of numerical examples are optimized by applying the optimizer module of the applied software.

### **DEFINITION OF STUDY**

In this system ,there is a retailer warehouse with deteriorating items. These items are decayed continuously with constant deterioration rate. Customers enter the system according to Poisson distribution and demand one unit. If demand occurs while there is no inventory on hand, it would be backordered so when units are again available, customers are served according to first come first served policy. In this system ordering policy is assumed to be modified  $(R,Q)$  continuous review. It means whenever the inventory position declines to reorder point  $R$ , a batch quantity of size  $(1+\alpha)Q$  is ordered. The order is containing  $Q$  packages and each package has  $(1+\alpha)$  units. We modify  $(R,Q)$  policy to be applicable for the case that the deterioration goods are like alcohol, flavoring or hydrogen peroxide. In this case ordering  $(1+\alpha)Q$  means that  $Q$  containers are ordered where each of them contains  $(1+\alpha)$  units (unit could be ml, liter etc.). Therefore, each container contains  $\alpha$  unit more than customers demand (one unit) and this extra amount will increase the probability of giving complete unit to customers. In addition, we consider a penalty cost for those demands which do not completely satisfied. Also some other costs are defined in this system. For instance, Shortage is incurred if a customer enters the system before his assigned demand. While if a customer entrance happened after delivery of his assigned demand to the retailer's warehouse, holding cost is incurred. We also have deterioration cost because of decaying during the shipment and storage. We defined these costs in two different cases where the lead time is stochastic and deterministic .And also prove that in the case of stochastic lead time the functions become very complicated and it is not easy to analyze the convexity of the model. Therefore, to overcome this difficulty a simulation model is developed.

To validate the simulation model, for deterministic lead time the output results of the simulation model and the results of the mathematical model are calculated for a number of experiments chosen randomly.

For 10 different experiments, different cost elements are calculated and one way Analysis of Variance (ANOVA) is used to test whether there are significant differences between total expected holding, shortage, deterioration and penalty costs of the simulation and the mathematical models. The simulation model was run for 20 replications where each replication takes 421 days. Note that the number of replications is defined in a way that for consecutive numbers of replications the differences Erlang between performance measures become negligible. Furthermore, long replication length is needed since the objective function is long-run expected cost. Thus, we choose the replication length long enough to obtain reliable output results. The results show that the differences between the output results of the simulation model and the mathematical one is not considerable. Therefore, the model with stochastic lead time can be used.

The simulation model is developed by using *Arena* software. In addition, by applying the optimizer module of *Arena*, *OptQuest* it is possible to find near optimal solutions.

Then, some numerical examples are presented. In all of them, the same values for parameters are used and two different distributions for the lead time, Exponential and Erlang are considered. We can see that, while it is difficult to analyze the model analytically, using simulation optimization makes the analysis of the model very easy.

Moreover, a number of numerical examples are presented with different values of demand rate( $\lambda$ ), deterioration rate( $\theta$ ) and ordering quantity( $Q$ ) for both the exponential and the Erlang lead times to examine the effect of changes in these parameters.

We can see that, when the parameter  $\lambda$  increases and the other parameters ( $\theta$  and  $Q$ ) remain unchanged, reorder point increases to satisfy customers' demands. It is clear that when the ordering quantity ( $Q$ ) remains unchanged, any increase in demand intensity leads to an increase in inventory levels at the retailer's warehouse. Furthermore, when inventory level at the warehouse increases, the total expected cost increases as a result of the growth in holding, deterioration and penalty costs.

Also we can conclude that, when the parameter  $\theta$  increases and the other parameters ( $\lambda$  and  $Q$ ) remain unchanged, optimum value of extra amount and total expected cost increase as a result of the growth in the reduction of the products. It is obvious that by increasing in the deterioration rate the amount of the products which will deteriorate increases and consequently the optimum value of total expected cost increases as well. For larger deterioration rate, the best value for the extra amount of the product ordered by the retailer, will increase to prevent the penalty cost by delivering one unit of the product to customers. Moreover, increase in deterioration rate,  $\theta$ , does not cause change in optimum value of reorder point.

Using simulation modelling approach and simulation optimization help us to model and solve an extremely complicated problem. It should be noted that in real world cases which follow the assumptions of the proposed model, simulation optimization can be used as a decision support tool which causes reduction in the total cost. The results of the presented examples show that the proposed model has the capacity to work as a reliable decision support tool in similar cases.

## SOME REFERENCES

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