Preference-Inspired Co-Evolutionary Algorithm Using Weights for Many-objective Optimization

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ABSTRACT

Decomposition based approaches are known to perform well on many-objective problems when a suitable set of weights is provided. However, providing a suitable set of weights *a priori* is difficult. This study proposes a novel algorithm: preference-inspired co-evolutionary algorithm using weights (PICEA-w), which co-evolves a set of weights with the usual population of candidate solutions during the search process. The co-evolution enables suitable sets of weights to be constructed along the optimization process, thus guiding the candidate solutions toward the Pareto optimal front. Experimental results show PICEA-w performs better than algorithms embedded with random or uniform weights.

Categories and Subject Descriptors

I.2.8 [**Heuristic methods**]: [multi-objective evolutionary algorithms]

Keywords

Evolutionary algorithms,co-evolution, multi-objective optimization

1. INTRODUCTION

Many-objective optimization problems (MaOPs) remain challenging in terms of obtaining a good approximation of the whole Pareto optimal front (POF). Decomposition based multi-objective evolutionary algorithm decomposes a MaOP into several independent subproblems by means of scalarizing functions. This approach performs well on MaOPs when suitable weights are provided for the decomposition. However, the choice of weights is problem-dependent and therefore is difficult to be defined *a priori* if no information about the problem is known beforehand. For example, evenly distributed weights are good for problems having a linear POF (e.g. see Figure 1(a)), however, they are not suitable for problems having disconnected Pareto fronts (e.g. see Figure 1(b)).

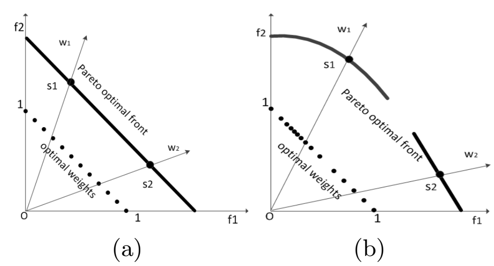

Figure 1: Illustration of a good distribution of weights for different Pareto fronts (using Chebyshef scalarizing function)

Our interest remains in *a posteriori* decision-making, that is, providing decision-makers with both a proximal and diverse representation of the entire Pareto front. This study proposes a new method to adaptively modify the weights during the search and so obtain a good approximation of the POF. The new method is based on the preference-inspired co-evolutionary concept [1], denoted as PICEA-w. Specifically, a set of weights are co-evolved with a population of candidate solutions during the search. The co-evolution enables (1) the candidate solutions to be guided toward the POF, and (2) the weights to be adaptively modified according to the shape of the Pareto front, constructing a set of suitable weights on the fly.

2. PICEA-W

The pseudo-code of PICEA-w is presented in Algorithm 1. Function `coevolveS` is executed as follows. For each $\mathbf{w} \in W$, we first identify its neighbouring candidate solutions. The neighbourhood is calculated by the angle between an $\mathbf{s}$ and a $\mathbf{w}$. If the angle is smaller than a pre-defined value θ, then $\mathbf{s}$ and $\mathbf{w}$ are defined as neighbours. Then we rank these neighbouring candidate solutions based on their performance measured by the corresponding weighted Chebyshev scalarizing function. The best solution is ranked 1. The rank values for solutions that are not neighbours of the $\mathbf{w}$ are set as inf. These rank results are stored in a matrix, R. The fitness, $Fit_{\mathbf{s}_i}$, of a candidate solution, $\mathbf{s}_i$ is then defined as $Fit_{\mathbf{s}_i} = \sum_{j=1}^{N_w} \frac{1}{R_{ij}}$. The best N solutions are selected as

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new S based on their fitness. Function `coevolveW` selects a suitable weight for each solution in S. First, solutions in S are ranked by each weight in $JointW$. Second, for each $\mathbf{s}_i \in S$, we find all the neighbouring weights on which $\mathbf{s}_i$ performs the best. If more than one weight vector is found, then the weight that has the largest angle with $\mathbf{s}_i$ is selected. To avoid multiple selections of a weight, once the weight $\mathbf{w}_i$ is selected for the solution $\mathbf{s}_i$ the i-th row of the matrix is set as inf.

Algorithm 1: PICEA-w

Input: initial candidate solutions, S of size N, initial weight vectors, W of size N_w
Output: S, W, offline archive $BestS$

1 initialize the S and W;
2 **while** *NOT stopped* **do**
3 | generate offspring Sc and merge $S \cup S_c$ as $JointS$;
4 | $S \leftarrow$`coevolveS`($JointS$,W,θ);
5 | generate weights W_c and merge $W \cup W_c$ as $JointW$;
6 | $W \leftarrow$`coevolveW`($JointW$,S,θ);
7 | update the offline archive, $BestS$;
8 **end**
9 return $S, W, BestS$

3. EXPERIMENT

In this section we compare PICEA-w with PICEA-w-r, MSOPS [2] and MOEA/D [3] on 2- and 7-objective WFG problems (from 2 to 9) [4]. PICEA-w-r replaces line 6 in Algorithm 1 with a set of randomly generated weights and is included so as to identify the benefits of using adaptive weights. The neighbourhood size, θ used in PICEA-w and PICEA-w-r is set as $\pi/18$ and $\pi/3$ for 2- and 7-objective problems respectively. Weights used in MSOPS and MOEA/D are set uniformly [2]. 100 weights are used for all the algorithms. 25000 function evaluations are accomplished for each of 30 independent runs. Generational distance GD, spread metric Δ and hypervolume metric HV are used as performance metrics.

Due to the limited space, we only plot the HV results for 7-objective problems (see Figure 2) [5]. For most of the 2-objective problems, PICEA-w has the best diversity performance. MOEA/D has the best convergence performance. In terms of HV metric, PICEA-w performs the best, followed by MOEA/D, and then MSOPS and PICEA-w-r. For the 7-objective problems, PICEA-w tends to perform the best in the round, across the three metrics. MSOPS perform the second best in terms of Δ and HV, followed by PICEA-w-r, and then MOEA/D. MOEA/D performs the second best in terms of GD.

Additionally, we also plot the non-dominated solutions and the co-evolved weights found in the last generation of PICEA-w on WFG4A-2 (a modified 2-objective WFG4). WFG4A-2 has a sharper POF than WFG4-2. From Figure 3, we observe that the distribution of the co-evolved weights is not even, but is dense in the middle while sparse in the edge as desired for a geometry of this type.

4. CONCLUSION

Defining good weights for decomposition based approaches *a priori* is difficult. This study proposes a new algorithm PICEA-w which co-evolves candidate solutions with weights during the search. The algorithm performs better than other MOEAs using random weights or uniform weights. In future, PICEA-w will be compared with other emerging MOEAs using adaptive weights. Also, the sensitivity of neighbourhood size θ will be investigated.

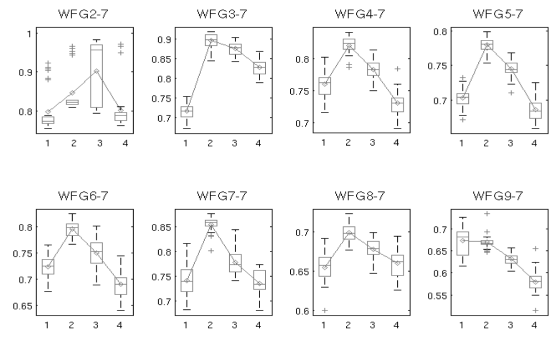

Figure 2: Box-plots of the HV results for 7-objective WFG problems. 1: PICEA-w-r, 2: PICEA-w, 3: MSOPS, 4: MOEA/D

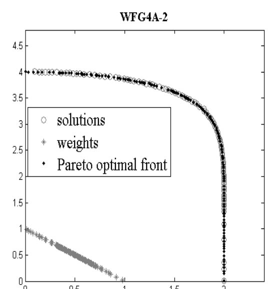

Figure 3: The obtained Pareto front and weights

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