Optimization of Assignment of Tasks to Teams using Multi-objective Metaheuristics

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ABSTRACT

A highly interesting but not thoroughly addressed optimization problem is a variation of the Assignment Problem (AP) where tasks are assigned to groups of collaborating agents (teams). In this paper, we address this class of AP as a bi-objective optimization problem, in which the cost is minimized and the quality is maximized. To solve the model, we adopt Non-dominated Sorting Genetic Algorithm-II (NSGA-II) and Strength Pareto Evolutionary Algorithm 2 (SPEA2). We conduct several experiments on problems with varying sizes to compare the NSGA-II and SPEA2 algorithms.

Categories and Subject Descriptors

I.2.8 [Artificial Intelligence]: Problem Solving, Control Methods, and Search—*Heuristic Methods*; I.6.3 [Computing Methodologies]: Simulation and Modeling—*Applications*

Keywords

Multi-objective optimization; combinatorial optimization; metaheuristics; assignment problem

1. INTRODUCTION

Most of the real-world optimization problems, have more than one conflicting objectives. In these optimization problems, there is no single optimal solution, instead there is a set of Pareto-optimal solutions. The Assignment Problem (AP), which is concerned with how to match agents and tasks, is a combinatorial optimization problem. Many variations of the AP are multi-objective optimization problems. For example, the problem of assigning software developers to projects, employees to departments, soldiers to military operations and crew members to different flights often involve multiple (possibly conflicting) criteria (cost, time, security, etc.).

In this paper, we propose a model for the assignment of collaborating teams to tasks, in which two competing objectives: (i) quality and (ii) cost are considered. The suggested model deals with the assignment of collaborating agents (team of resources) to tasks so as to maximize the quality and minimize the cost of the underlying project. To solve the model, we use Non-dominated Sorting Genetic Algorithm-II (NSGA-II) [1] and Strength Pareto Evolutionary Algorithm 2 (SPEA2) [4]. To test the accuracy of NSGA-II and SPEA2

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for our application, we consider a small problem which can be solved using an enumerative algorithm. We further conduct a set of experiments on problems with varying sizes and identify the algorithm with better results.

2. MULTI-OBJECTIVE MODEL FOR COL-LABORATING TEAMS ASSIGNMENT

The problem discussed in this paper is an extension of the problem defined in [3]. It deals with multi-objective optimal assignment of tasks to teams of collaborating agents. Let $\mathcal{A} = \{a_i | i = 1, \ldots, m\}$ be the set of m agents and let $\mathcal{T} = \{t_j | j = 1, \ldots, n\}$ be the set of n tasks, where $m \geq n$. Each agent $a_i \in \mathcal{A}$ has a set of p attributes $c_i = \{c_{ik} | k = 1, \ldots, p\}$. Similarly for each task $t_j \in \mathcal{T}$, a weight is associated to each capability and it defines capability weight vector $w_j = \{w_{kj} | k = 1, \ldots, p\}$. Assume that each task t_j requires a team with a fixed number b_j of agents.

We also assume that the total completion time for each task t_j is $time_j$. Assuming each agent a_i participating in $team_j$ works for equal $\left(\frac{time_j}{b_j}\right)$ number of hours on task t_j and has a salary $Salary_i$, which is a function of its capabilities and certain weights (values) for those capabilities $h(c_{ik}, v_k | k = 1, ..., p)$, where v_k defines relative value for capability k for calculating salary. Within each team, agents collaborate with each other which improve the quality of the task performed. The goal is to optimally assign all tasks to teams of agents such that quality is maximized and cost is minimized. The mathematical formulation is summarized as follows:

$$Maximize \ \mathcal{Z}_1 = \sum_{j=1}^n \sum_{i=1}^m \sum_{k=1}^p (c_{ik} + \frac{c_{ik}(c_k^{\max} - c_{ik})}{c_k^{\max}}) w_{kj} x_{ij}, \ (1)$$

Minimize
$$Z_2 = \sum_{j=1}^{n} (\frac{time_j}{b_j} \sum_{i=1}^{m} \sum_{k=1}^{p} c_{ik} v_k x_{ij}),$$
 (2)

subject to constraints

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad \forall a_i \in \mathcal{A}$$
(3)

$$\sum_{i=1}^{m} x_{ij} = b_j, \ \forall t_j \in \mathcal{T}; \quad \sum_{j=1}^{n} b_j \le m,$$

$$(4)$$

where $c_k^{\max} = \max_{1 \le i \le m} \{c_{ik} x_{ij}\}$ and $x_{ij} \in \{0, 1\}$ $(x_{ij} = 1$ if task t_j is assigned to agent a_i and 0 otherwise).

Table 1: Specification of the Collaborating Teams Assignment Problem Instances

1			_		
	Prob#	Total Agents	Required Agents	Number of Tasks	Teams (# of agents assigned to the tasks)
Ì	1	10	10	4	[2, 3, 2, 3]
	2	50	20	4	[3, 4, 6, 7]
	3	20	20	8	[3, 2, 4, 3, 2, 2, 2, 2]
	4	30	30	12	[2, 2, 3, 3, 1, 4, 5, 2, 3, 2, 2, 1]
	5	80	80	20	[5, 4, 8, 2, 3, 3, 4, 3, 4, 7, 1, 6, 3, 6, 2, 6, 3, 1, 5, 4]
	6	96	96	30	[5, 4, 8, 2, 3, 2, 2, 3, 1, 4, 7, 1, 5, 3, 2, 6, 2, 6, 3, 1, 5, 4, 2, 2, 4, 2, 1, 2, 3, 1]
	7	400	400	100	100 teams (each team has 4 agents)

3. MULTI-OBJECTIVE ALGORITHMS

In order to solve our proposed multi-objective problem, we focus and compare two widely used evolutionary algorithms: (i) NSGA-II [1], and (ii) SPEA2 [4]. We use the algorithms provided by jMetal framework [2] and modify the implementations for maintaining non-duplicate solutions. We use binary tournament selection, two-point crossover (probability = 0.95) and swap mutation (probability = 0.20) operators.

4. EXPERIMENTAL RESULTS

In this section, a set of experiments and their results using NSGA-II and SPEA2 are presented. The aim is to identify the most effective algorithm for finding the Pareto front approximation. In order to make the analysis unbiased, we consider several instances of the problem with different sizes as shown in Table 1. In all problem instances, each agent a_i has 10 capabilities and each task t_j has certain weights for these capabilities. The data for these capabilities and their weights are integer values between 0 and 4 inclusive. The completion time for each task is generated with uniform distribution in [1000, 1500].

The performance of multi-objective algorithms is measured by assessing the quality of the obtained Pareto front approximation. The quality of the solutions depends on convergence to the Pareto front and diversity of the solutions. In the literature, a number of quality indicators have been proposed to measure these two properties. Hypervolume (HV) is considered to be the most widely used. The HV calculates the volume in the objective space, which is covered by the solutions of the obtained Pareto front approximation.

In order to test the accuracy of the results, we consider a small problem with 4 teams and 10 agents (problem #1in Table 1). Due to smaller solution space size, this problem can be solved using an enumerative algorithm. Both algorithms find all true Pareto-optimal solutions and the execution time of these two algorithms is also smaller than the enumerative algorithm.

To compare and discuss the performance of NSGA-II and SPEA2, a set of computational experiments are performed with different problem sizes as shown in Table 1. We conduct 50 replications of each algorithm on five different initial population sizes and each algorithm terminates after 100,000 function evaluations. We calculate the mean and standard deviation of HV of these 50 replications for each population size. For small population sizes, SPEA2 performs better, while in case of larger initial population sizes, NSGA-II outperforms SPEA2. Further for smaller problems, the difference in performance is small for both algorithms. For larger problem sizes (Problems 5 to 7), the difference in performance is significant. For larger problems, NSGA-II gives better results on larger population sizes as shown in Figure 1.



Figure 1: NSGA-II and SPEA2 Comparison

In a sense, NSGA-II is more robust as far as initial population size (number of solutions maintained in each generation) is concerned.

5. CONCLUSIONS

In this paper, we propose a multi-objective model for a specific class of the AP, where agents performing a task collaborate with each other and work as a team. The goal of the multi-objective optimization is to maximize the quality and minimize the cost. In order to solve the optimization model, we adopt two widely used multi-objective evolutionary algorithms, NSGA-II and SPEA2. We verify the accuracy of the algorithms in this context by comparing the results of the algorithms with results provided by an enumerative method on a small problem. The comparison shows that both algorithms provide all Pareto-optimal solutions. We also conduct several experiments on problems with varying sizes, which show that for larger problem instances, generally, NSGA-II provides better Pareto-optimal solutions with respect to both convergence and spread, that is NSGA-II has higher HV values.

6. **REFERENCES**

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