Partially Connected Topologies for Particle Swarm

Carlos M. Fernandes[1,2]
Juan Julián Merelo
[1]Department of Computer Architecture
University of Granada, Granada, Spain
cfernandes@laseeb.org,
jmerelo@geneura.ugr.es

Juan L.J. Laredo
University of Luxembourg, Luxembourg
juan.jimenez@uni.lu

Carlos Cotta
Dept. Lenguages y Ciencias de la Computación,
University of Málaga, Spain
ccottap@lcc.uma.es

Agostinho C. Rosa
[2]Dep. of Electrotechnical Engineering
Laseeb-ISR-IST, Technical Univ.of Lisbon
Lisbon, Portugal
acrosa@laseeb.org

ABSTRACT
The effects of dynamic and partially connected 2-dimensional topologies on the particle swarm are studied. The particles are positioned on 2-dimensional grids of nodes, where they move according to a simple rule. The von Neumann neighborhood is used to decide which particles influence each individual. Structures with growing size are tested on a classical benchmark. The partially connected grids with von Neumann neighborhood structure perform more consistently than other strategies.

Categories and Subject Descriptors
H.4 [Information Systems Applications]: Miscellaneous

General Terms
Algorithms, Theory.

Keywords: Particle Swarm, Population Structure.

1. INTRODUCTION
The Particle Swarm Optimization (PSO) algorithm [1] is a population-based meta-heuristic for binary and real-valued function optimization inspired by the social behavior of bird flocks and fish schools. The population consists of a group of solutions that travels through the search space according to a set of rules that favor their movement towards optimal regions of the space. The algorithm is described by a simple set of equations that define the velocity and position of each particle. The position vector of the *i-th* particle is given by $\vec{X}_i = (x_{i,1}, x_{i,2}, \dots x_{1,D})$, where D is the dimension of the search space. The velocity is given by $\vec{V}_i = (v_{i,1}, v_{i,2}, \dots v_{1,D})$. The particles are evaluated with a fitness function $f(\vec{X}_i)$ in each time step and then their positions and velocities are updated by:

$$v_{i,d}(t) = \omega . v_{i,d}(t-1) + c_1 r_1 \big(p_{i,d} - x_{i,d}(t-1)\big) + c_2 r_2 \big(p_{g,d} - x_{i,d}(t-1)\big) \quad (1)$$

$$x_{i,d}(t) = x_{i,d}(t-1) + v_{i,d}(t) \quad (2)$$

were p_i is the best solution found so far by particle i and p_g is the best solution found so far by the neighborhood. Parameters r_1 and r_2 are random numbers uniformly distributed in the range [0,1] and c_1 and c_2 are acceleration coefficients that tune the relative influence of each term of the formula.

PSO has been applied with success to a number of problems and motivated several lines of research that investigate its main working mechanisms. One of these research lines deals with the population topology, which is the structure that defines the connections between the particles. In their turn, these connections guide the flow of information through the swarm and therefore they deeply affect the convergence skills of the algorithm. In 2002, Kennedy and Mendes [2] published an exhaustive study on population structures for PSO. They tested several types of structures, including the traditional *lbest*, *gbest* and von Neumann configuration. They also tested populations arranged in graphs that were randomly generated and optimized to meet some criteria. Amongst the large set of graphs tested in [2], the von Neumann configuration performed more consistently, and in the conclusions the authors recommend its use.

This paper extends the concept of von Neumann configuration and investigates the behavior of a partially connected topologies with von Neumann neighborhood. The particles are distributed on a grid of nodes. The size of the grid is set so that the number of nodes is larger than the number of particles. The particles are placed randomly on the grid and a simple set of rules guide their movements through the nodes during the run. The population structure is defined by the von Neumann neighborhood between the nodes, which means that the degree of connectivity of each particle varies between 1 and 5 during the run. Preliminary tests are conducted with local neighborhood random structures, that is, the particles move randomly through the grid, choosing between free adjacent nodes. The PSO with partially connected 2-dimensional structures is summarized in Table 1.

Table 1. Dynamic PSO on a partially connected grid.

1. For each particle $1 \to n$:
    1.1. Initialize particle i
    1.2. Evaluate particle's position $\vec{x_i}$: $f(\vec{x_i})$
    1.3. Set $p_g(i) = p_i(i) = f(\vec{x_i})$
2. Set grid size: $X \times Y$
3. Place the particles randomly on the grid
4. For each particle $1 \to n$
    4.1. If the fitness of the best position found so far p_j by any of the particles j in the von Neumann neighborhood of particle i is better than $p_g(i)$, then $p_g(i) = p_j$
    4.2. Choose randomly a free node in the Moore neighborhood and move the particle to that node.
5. For each particle
    5.1. Update velocity and position using equations 2 and 3.
    5.2. Evaluate particle's position $\vec{x_i}$: $f(\vec{x_i})$
    5.2. If $f(\vec{x_i}) < f(p_i(i))$, then $p_i(i) = \vec{x_i}$
5. If stop criterion not met, go to 4

GECCO'13 Companion, July 6–10, 2013, Amsterdam, The Netherlands.
ACM 978-1-4503-1964-5/13/07.

Table 2. Best fitness averaged over 50 runs.

	f_1	f_2	f_3	f_4	f_5
VN	1.05e-35 ±1.06e-35	1.31e+01 ±2.16e+01	6.99e+01 ±1.83e+01	6.25e-03 ±8.23e-03	1.94e-04 ±1.37e-03
VN (9×9)	9.13e-37 ±2.10e-36	9.72e+00 ±1.88e+01	6.89e+01 ±1.71e+01	7.68e-03 ±9.56e-03	1.94e-04 ±1.37e-03
lbest	2.61e-25 ±4.33e-25	1.40e+01 ±3.53e+01	1.07e+02 ±2.23e+01	4.93e-04 ±1.99e-03	3.89e-04 ±1.92e-03
gbest	4.00e+03 ±6.06e+03	4.91e+00 ±1.26e+01	1.05e+02 ±2.89e+01	5.42e+01 ±6.82e+01	2.33e-03 ±4.19e-03

2. RESULTS AND CONCLUSIONS

Five benchmark functions were used for testing the algorithm: Sphere (f_1), Rosenbrock (f_2), Rastrigin (f_3), Griewank (f_4) and Schaffer (f_5). The optimum of all functions is located in the origin with fitness 0. The dimension of the search space is set to $D = 30$ (except Schaffer, with 2 dimensions). The population size n is set to 40. The acceleration coefficients were set to 1.494 and the inertia weight is 0.729. $Xmax$ is defined as usual by the domain's upper limit and $Vmax = Xmax$. A total of 50 runs for each experiment are conducted. *Asymmetrical initialization* is used.

Two types of experiments were conducted. In the first one, the algorithms were run for a limited amount of iterations (3000 for f_1 and f_5, 10000 for f_2, f_3 and f_4) and the fitness of the best solution found was averaged over the 50 runs. In the second set of experiments the algorithms were all run for 20000 iterations or until reaching a stop criterion. The criteria were taken from [2]. The number of iterations required to meet the criteria was recorded and averaged over the 50 runs. A success measure was defined as the number of runs in which an algorithm attains the fitness value established as the stop criterion. PSOs with *lbest*, *gbest* and standard von Neumann configurations were tested on the five benchmark problems. Partially connected structures with size 7×7, 8×8, 9×9 and 10×10 were also tested.

Table 2 (averaged best fitness) and Table 3 (averaged number of iterations to meet the criterion and number of runs in which the criterion is met) compare the PSOs with *lbest*, *gbest*, standard von Neumann configuration and partially connected von Neumann structure with size 9×9.

Table 3. Iterations to a solution averaged over 50 runs and number of successful runs.

	f_1	f_2	f_3	f_4	f_5
Stand. VN	489.86 ±18.55 (50)	1443.24 ±1547.11 (50)	748.98 ±1706.20 (49)	458.36 ±29.10 (50)	454.56 ±659.27 (50)
VN (9×9)	474.96 ±22.60 (50)	1589.56 ±2137.00 (50)	314.43 ±81.37 (49)	450.56 ±54.45 (50)	264.80 ±395.90 (49)
lbest	662.30 ±21.81 (50)	1800.69 ±1650.07 (49)	2014.77 ±2331.92 (22)	618.22 ±31.87 (50)	708.08 ±849.52 (50)
gbest	489.86 ±18.55 (50)	891.42 ±1066.82 (50)	211.13 ±77.46 (23)	315.08 ±56.67 (24)	395.05 ±795.04 (40)

The von Neumann structure with size 9×9, improves the standard configuration fitness in functions f_1, f_2, f_3. In f_4 the standard structure is better, while in f_5 the result is the same. As for the average iterations to a solution, the 9×9 structure is faster in every function except f_2.

Non-parametric Mann–Whitney *U* statistical tests (with 0.05 level of significance) comparing the fitness values attained by each configuration in each function return the following results: the 9×9 structure is significantly better than the standard configuration on function f_1; in the remaining problems the two configurations are statistically equivalent. Applying the Mann–Whitney *U* tests to the iterations metrics, the conclusions are that the 9×9 structure is statistically better on f_1, f_3, f_4 and f_5. The algorithms are statistically equivalent in f_2. Therefore, the partially connected structure significantly improves the performance of the standard Von Neumann configuration in every function except f_2 (in which the algorithms were found to be statistically equivalent in both fitness and convergence speed).The proposed topology is able to improve *lbest* fitness values in f_1, f_2, f_3 and f_5; in f_1 and f_3 the differences are statistically significant. The differences in f_4 are also significant but in this case *lbest* is better. As for the average iterations for a solution, the partially structured von Neumann structure improves *lbest* in every function, with statistical differences between the results.

The differences between the best fitness values attained by *gbest* and 9×9 structure are statistically different for every function. Von Neumann 9×9 is better in f_1, f_3, f_4 and f_5, while *gbest* is better in f_2. Comparing the proposed structure with *gbest* is not trivial because *gbest* fails very often in meeting the stop criteria. It is faster in three functions (f_2, f_3, f_4) but in f_3 and f_4 the topology fails to meet the criteria in more 50% of the runs. Therefore, we may conclude that the von Neumann 9×9 performs more consistently than *gbest* throughout the test set.

A general evaluation of the four topologies according to fitness, speed and success results in the following ranking: 9×9 von Neumann (1.7), standard von Neumann (2.1), *lbest* (3.0) and *gbest* (3.2). The proposed structure ranks first.

Acknowledgements

Authors wish to thank FCT (Ministério da Ciência e Tecnologia) Fellowship SFRH/BPD/66876/2009; The work is supported by FCT (ISR/IST plurianual funding) through the PIDDAC Program funds, projects ANYSELF (TIN2011-28627-C04-01 and -02) and 83 CEI-Biotic 2013, Spanish Minitry of Science, and Luxembourg FNR Green@Cloud project (INTER/CNRS/11/03).

References

[1] Kennedy, J., Eberhart, R. 1995. Particle Swarm Optimization. In *Proceedings of IEEE International Conference on Neural Networks*, Vol.4, 1942–1948.

[2] Kennedy, J., Mendes, R. 2002. Population structure and particle swarm performance. In *Proceedings of the IEEE World Congress on Evolutionary Computation*, 1671–1676.

[3] Liang, J.J., Qin, A.K., Suganthan, P.N., Baskar, S. 2006. Comprehensive learning particle swarm optimizer for global optimization of multimodal functions. *IEEE Trans. Evolutionary Computation*, 10(3), 281–296.