

# Evolutionary Hyperheuristic for Capacitated Vehicle Routing Problem\*

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## ABSTRACT

In this paper a novel constructive hyperheuristic for CVRP is proposed. This hyperheuristic, called HyperPOEMS, is based on an evolutionary-based iterative local search algorithm. Its inherent characteristics make it capable of autonomously searching a structured space of low-level domain specific heuristics for their suitable combinations that produce good solutions to particular problem instance. HyperPOEMS was tested on standard benchmarks and compared to two existing constructive hyperheuristic, HHC-VRP and EHH-VRP. The results show that HyperPOEMS outperforms both compared hyperheuristics and produces solutions competitive to solutions obtained by specialized metaheuristics designed for CVRP.

## Categories and Subject Descriptors

I.2.8 [Artificial Intelligence]: Problem Solving, Control Methods, and Search—*heuristic methods*

## Keywords

Hyperheuristics; Vehicle Routing Problem; Combinatorial Optimization; Evolutionary Algorithms

## 1. INTRODUCTION

Vehicle routing problem (VRP) and its variants belong to classical problems in operations research. They are of great practical importance, especially in the logistics and the distribution of goods where efficient solution algorithms bring large transportation cost savings. VRP is very hard combinatorial problem. The largest solvable instances of the VRP are two orders of magnitude smaller than those of the TSP [5]. Hence, either approximation or (meta)heuristic algorithms are often used to find if not optimal solutions then at least solutions of good quality in limited time.

Recently, hyperheuristics<sup>1</sup> described as "heuristics to choose heuristics", that are search methods or learning mechanisms for selecting or generating heuristics to solve computational

search problems were proposed. This paper focuses on *heuristic selection* hyperheuristic approaches that for a given problem instance and a set of problem-specific low-level heuristics (LLHs) select and apply the most suitable LLH at each stage of the problem solving process. The selection hyperheuristics can be either *constructive* or *local search* depending on the type of used LLHs. Constructive LLHs add a new component to partially developed solution whilst local search LLHs try to improve current solution by rearranging its existing components.

To the best of our knowledge there are just two constructive selection hyperheuristics for VRP, namely the Hill-climbing based hyperheuristic (HHC-VRP) [1] and the Evolutionary hyperheuristic (EHH-VRP) [2].

In this work, a novel evolutionary-based *constructive selection hyperheuristic* for the CVRP is proposed. The hyperheuristic, called HyperPOEMS, is based on the evolutionary iterative local search POEMS algorithm [3]. The POEMS based approach has already been successfully adopted as *local search selection hyperheuristic* [4].

## 2. PROPOSED HYPERPOEMS APPROACH

POEMS is an optimization algorithm that operates on a single candidate solution called a *prototype* and tries to improve it in an iterative process. In each iteration, it runs an evolutionary algorithm (EA) that seeks for the most valuable modification to the prototype. The modifications are represented as fixed length sequences of actions, i.e. sequences of problem-specific variation operators. When the EA has finished, the best evolved action sequence is checked for its effect on the current prototype. If an improvement is achieved or the modified prototype is at least as good as the current one, then the modified prototype is considered a new prototype for the next iteration. Otherwise, the current prototype remains unchanged. The iterative process stops after a specified number of iterations.

**Prototype representation.** Here, the prototype is represented as an ordered sequence of units (a concept similar to the one used in HHC-VRP) that produces a solution to the problem. Each unit  $j$  has assigned its *order*, *constructive*, *single-route improvement* and *two-route improvement* heuristic. Each unit has also assigned  $n_j$ , the number of customers that is to be added to the partial solution by the respective constructive heuristic. The sum of  $n_j$  over all units has always to be equal to the total number of customers,  $N$ .

**Actions.** HyperPOEMS uses 10 types of actions from

\*A full version of this paper is available at  
[http://labe.felk.cvut.cz/~kubalik/cvrp\\_full.pdf](http://labe.felk.cvut.cz/~kubalik/cvrp_full.pdf)

<sup>1</sup>Comprehensive bibliography of Hyperheuristics:  
<http://www.cs.nott.ac.uk/~gxo/hhbibliography.html>  
<http://allserv.kahosl.be/~mustafa.misir/hh.html>

**Table 1: Comparison of HyperPOEMS with other approaches.**

Fisher's instances										
instance	Ejection Chains	Minimum K-trees	AGES	HHC-VRP	EHH-VRP	HyperPOEMS	Avg	Min	Avg	Min
f71	2.30	<b>2.10</b>	<b>2.10</b>	2.23	<b>2.10</b>	2.66	<b>2.10</b>	2.19	<b>2.10</b>	
f134	1.32	0.14	<b>0.08</b>	1.0	0.27	0.77	0.53	0.17	<b>0.08</b>	
Average	1.81	1.12	<b>1.09</b>	1.61	1.18	1.72	1.31	1.18	<b>1.09</b>	
Taillard's instances										
instance	Ejection Chains	Rochat and Taillard	AGES	HHC-VRP	EHH-VRP	HyperPOEMS	Avg	Min	Avg	Min
tai75a	0.13	—	<b>0.0</b>	0.29	0.05	0.31	0.26	0.05	<b>0.0</b>	
tai75b	0.76	—	<b>0.0</b>	0.13	0.05	0.13	<b>0.0</b>	<b>0.0</b>	<b>0.0</b>	
tai75c	0.62	—	<b>0.0</b>	0.11	<b>0.0</b>	0.21	<b>0.0</b>	0.01	<b>0.0</b>	
tai75d	0.10	—	<b>0.0</b>	0.29	<b>0.0</b>	0.33	<b>0.0</b>	<b>0.0</b>	<b>0.0</b>	
tai100a	1.80	0.32	<b>0.0</b>	2.01	1.51	1.96	1.59	1.54	0.80	
tai100b	0.62	0.04	<b>0.0</b>	0.86	0.09	0.71	0.14	0.19	0.04	
tai100c	1.10	0.09	<b>0.0</b>	0.89	0.32	0.64	0.01	0.24	<b>0.0</b>	
tai100d	1.34	<b>0.0</b>	<b>0.0</b>	1.38	1.14	1.26	1.03	1.04	0.32	
tai150a	2.71	0.51	<b>0.0</b>	4.03	2.16	3.81	2.37	2.05	0.66	
tai150b	5.71	2.9	<b>2.68</b>	5.69	4.92	5.06	3.74	3.97	3.20	
tai150c	3.43	0.96	<b>0.05</b>	2.56	1.86	2.49	1.59	1.58	1.00	
tai150d	1.93	0.67	<b>0.0</b>	2.54	1.99	2.20	1.28	1.77	0.64	
Average	1.69	0.69	<b>0.23</b>	1.73	1.17	1.59	1.0	1.04	0.56	

which the evolved action sequences are composed. The actions can add, delete and modify units of the prototype.

### 3. EXPERIMENTS

**Test Data.** For the proof-of-concept experiments we used the same data that were used in [1] and [2]. These include 21 standard benchmarks of Christofides (7 instances,  $50 \leq n \leq 199$ ), Taillard (12 instances,  $75 \leq n \leq 150$ ) and Fisher (2 instances,  $n \in \{71, 134\}$ ).

**Compared Algorithms.** We compared HyperPOEMS with HHC-VRP and EHH-VRP hyperheuristics and also with several specialized state-of-the-art algorithms for CVRP.

**Results.** Results presented in Table 1 are calculated as the deviation percentage from the best known values. Due to a limited space the table contains only results for Fisher's and Taillard's instances.

The main observation is that the proposed HyperPOEMS outperforms the two hyperheuristics HHC-VRP and EHH-VRP in terms of the average deviation percentage from the best known value on all three data sets. In particular, HyperPOEMS outperforms HHC-VRP and EHH-VRP on 4 instances of Christofides data set, while it gets outperformed on just one instance. On both Fisher's instances has HyperPOEMS better average statistic than HHC-VRP and EHH-VRP, however the differences are rather small on instance f71. Finally, on Taillard's instances the differences between compared hyperheuristics are already more substantial as the HyperPOEMS clearly outperforms both HHC-VRP and EHH-VRP.

HyperPOEMS also produces solutions competitive to those obtained by specialized metaheuristics. This holds especially for the Ejection chains and Minimum K-means methods.

Last but not least, it was observed consistently across all test instances that the best evolved action sequences were in vast majority of cases composed of multiple active actions. This justifies the concept, adopted from POEMS, of search-

ing a neighborhood structure defined by fixed-length action sequences instead of just a single action in order to improve the prototype.

### 4. ACKNOWLEDGMENT

This work was supported by the Grant Agency of the CTU in Prague, grant No. SGS12/145/OHK3/2T/13 and by the research program No. MSM 6840770038 "Decision Making and Control for Manufacturing III" of the CTU in Prague.

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