# Advances on Many-objective Evolutionary Optimization

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http://www.sigevo.org/gecco-2013/


*GECCO'13 Companion,* July 6–10, 2013, Amsterdam, The Netherlands.
ACM 978-1-4503-1964-5/13/07.

## Outline

- Basic concepts
  - Multi-, many-objective optimization
- Scalability issues and fundamentals of many-objective optimization
- Various approaches to many-objective optimization
- Conclusions

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## Multi-objective Optimization

- Find vectors of decision variables,

$$x = (x_1, x_2, \cdots, x_n) \in S$$ **Decision space**

- Subject to certain constraints,

$$x \in F, F \subseteq S$$ **Feasible solutions**

- Simultaneously optimizing two or more performance criteria expressed as a vector of objective functions.

**Objective space**

$$\text{Maximize } f(x) = (f_1(x), f_2(x), \cdots, f_m(x))$$

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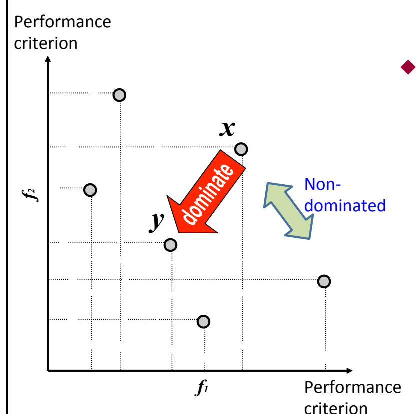
## Pareto Dominance




- Dominance
  $m \in \{1, 2, \cdots M\}$
  $\boldsymbol{x}$ dominates $\boldsymbol{y}$ ( $f(\boldsymbol{x}) \succeq f(\boldsymbol{y})$ ) *iff*
  $\forall m \ f_m(x) \geq f_m(y) \ \wedge$
  $\exists m \ f_m(x) > f_m(y)$

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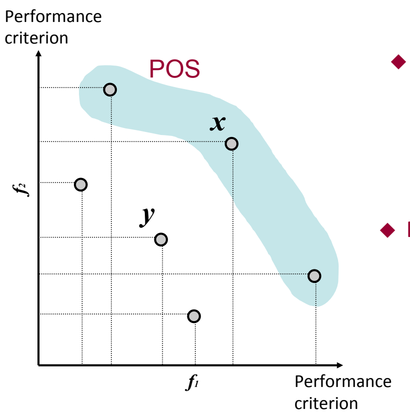
# Pareto Optimum Solutions (POS)




- Dominance
  $m \in \{1, 2, \cdots M\}$
  $x$ dominates $y$ ( $f(x) \succeq f(y)$ ) *iff*
  $\forall m \; f_m(x) \geq f_m(y) \quad \wedge$
  $\exists m \; f_m(x) > f_m(y)$
- Pareto-Optimal Solutions (POS)

$$\mathcal{P} = \{x \in \mathcal{F} \mid \neg \exists y \in \mathcal{F} : f(y) \succeq f(x)\}$$

non-dominated solutions

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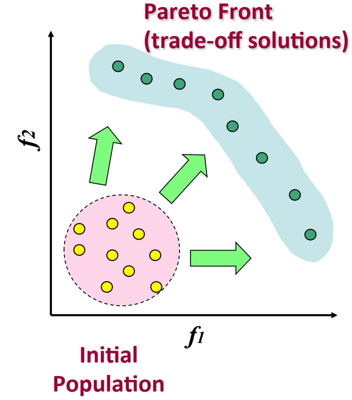
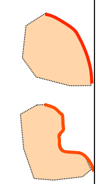
# Evolutionary Multi-objective Optimization




Find a good approximation of the Pareto Optimal Set in a single run!

- Convergence
- Spread
- Distribution

Good for convex, concave, and discontinuous Pareto fronts



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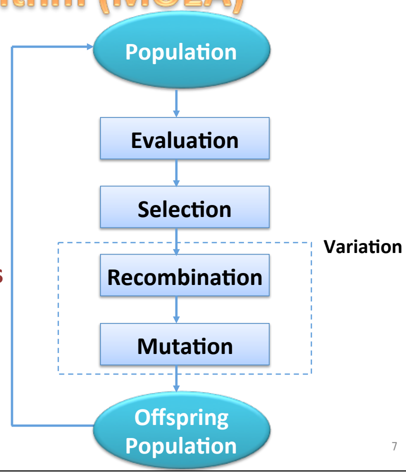
# Multi-objective Evolutionary Algorithm (MOEA)

Problem

- Genetic Representation
- Fitness
- Variation Operators

**Multi-objective Optimization problems**

$$f(x) = (f_1(x), f_2(x), \cdots, f_m(x))$$

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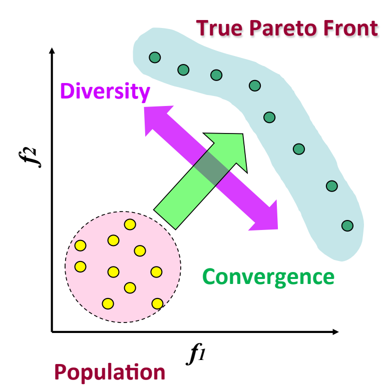
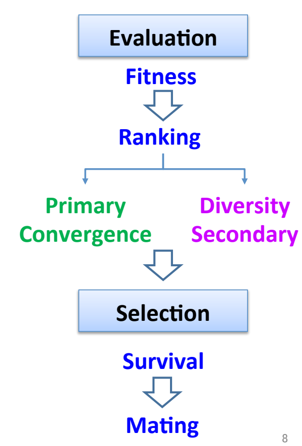
# Primary and Secondary Rankings

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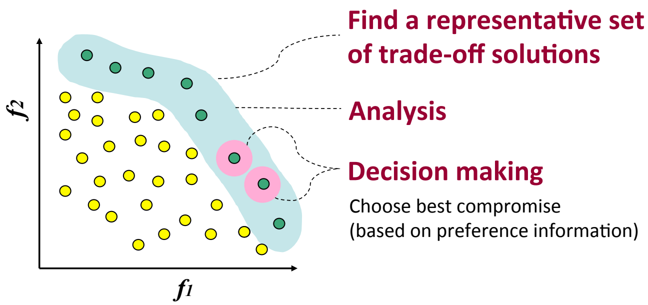
## Optimization, Analysis, and Decision Making




- Decision making before search: define a single objective
- Decision making after search: find/approximate Pareto set first
- Decision making during search: guide search interactively

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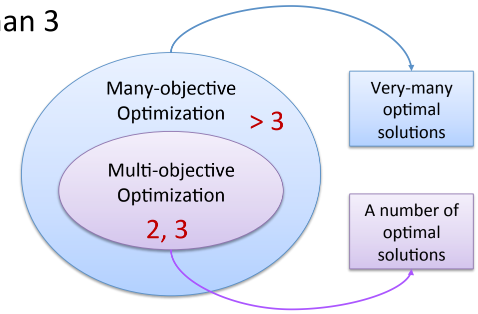
## Many-objective Evolutionary Optimization

- Evolutionary multi-objective optimization where the number of objectives to optimize is more than 3

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## From Multi- to Many-objective Evolutionary Optimization

- Evolutionary algorithms are quite effective for 2 and 3 objective-problems
- MOEAs do not scale up well on many-objective problems

**Reasons?**
**What do we need to fix?**

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## Real World Design Problems

- Multiple objective functions
  - Nonlinear, multimodal, discontinuous
  - Objective function evaluation is very expensive
- Extraction of useful design knowledge is important
  - Design knowledge is more appreciated than "exact" optimal design parameter values

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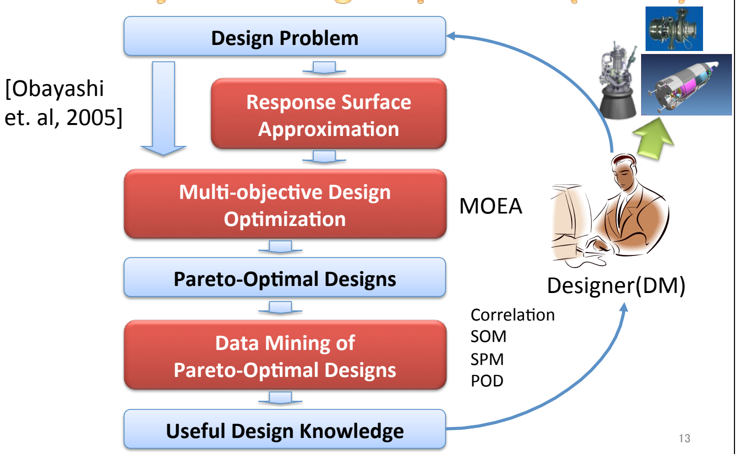
## Real-World Design Innovation with Multi-objective Design Exploration (MODE)

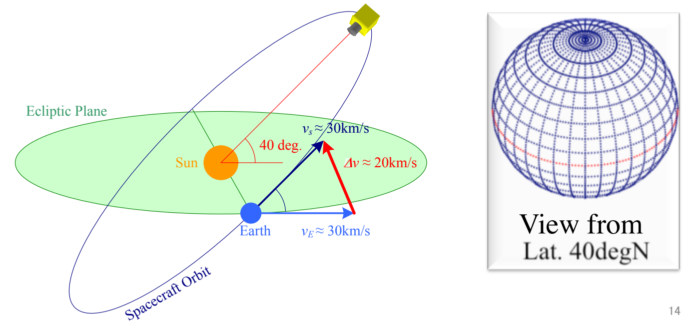





## Mission: Observation of Polar Region of the Sun

[Oyama, 2011]

- Orbit largely inclines with the ecliptic plane (40 deg.)
- Large delta-v (20km/s) is required

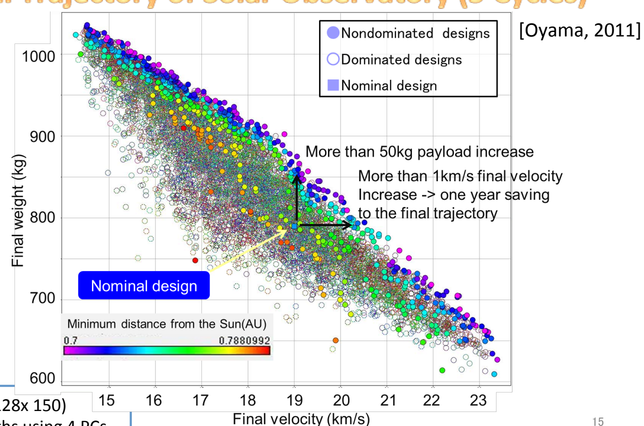





## MODE of Pareto Front

### Full Trajectory of Solar Observatory (5 Cycles)

[Oyama, 2011]




NSGA-II (128x 150)
Two months using 4 PCs



## MODE for Large Scale Many-Objective Problems

- MODE has been used successfully for 2 and 3 objective problems
- Demands for MODE applicable to large scale many-objective problems

## Real-World Design Innovation: Problems Characteristics – Algorithm's Issues

| Problem Characteristics | Algorithm's implications |
|---|---|
| Various formulations:<br>- Multi-, many-objectives<br>- Few, many variables | Handle within a single framework: Any-objective optimizer<br>- Scalability in objective space<br>- Scalability in decision space |
| Extract<br>design knowledge | - Good approx. of the Pareto optimal set: convergence, spread, distribution<br>- A large number of solutions |
| Expensive<br>fitness evaluation | - Few generations<br>- Parallelization<br>- One shot (one run): Adaptive, low standard deviations |

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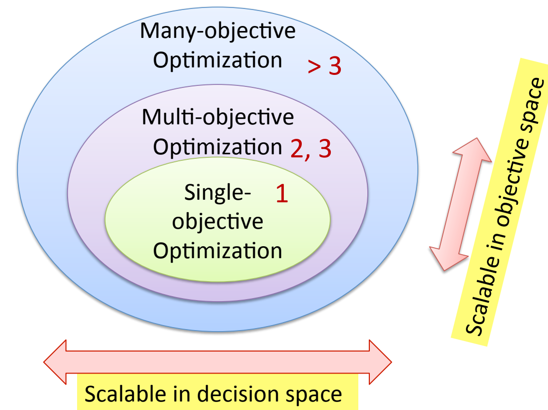
## Any-objective Optimization

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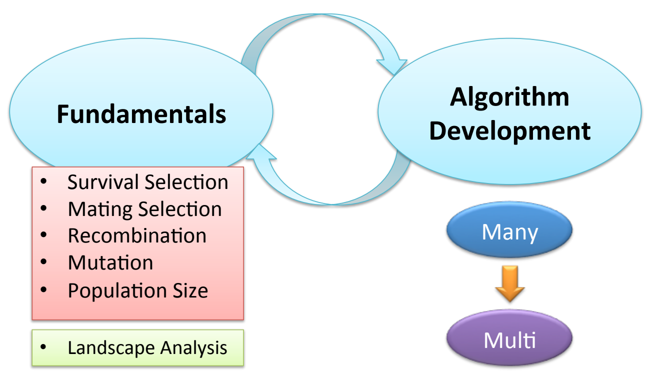
## Approach to Effective Any-objective Optimization

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# MNK-Landscapes

$$f(\cdot) = (\, f_1(\cdot), \dots, f_M(\cdot)\,) : B^N \to \Re^M$$

*M*, number of objectives

*N*, number of bits

*B*={0,1}

Fitness of string $x$ in $i$-th objective [Kauffman, 1993]

$$f_i(x) = \frac{1}{N}\sum_{j=1}^{N} f_{i,j}\left(x_j, z_1^{(i,j)}, z_2^{(i,j)}, \dots, z_{K_i}^{(i,j)}\right)$$

$K_i$ bits interacting with $x_j$

$f_{i,j} : B^{K_i+1} \to \Re$

Fitness contribution of bit $x_j$

Random number [0.0,1.0]

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# MNK-Landscapes: Example

M=2, N=8, K=2

$x_3$

$x$: 0 1 **0** 0 1 1 0 0

Epistasis $f_1$

$z_1^{(1,3)}$ $x_3$ $z_2^{(1,3)}$

$x$: 0 **1** **0** **0** 1 1 0 0

| $z_1 x_3 z_2$ | $f_{1,3}$ |
|---|---|
| 000 | 0.83 |
| 001 | 0.34 |
| 010 | 0.68 |
| 011 | 0.10 |
| **100** | **0.67** |
| **101** | 0.24 |
| 110 | 0.60 |
| 111 | 0.64 |

Fitness contribution of bit $x_j$

Random number [0.0,1.0]

Epistasis $f_2$

$z_1^{(2,3)}$ $x_3$ $z_2^{(2,3)}$

$x$: **0** 1 **0** 0 1 **1** 0 0

| $z_1 x_3 z_2$ | $f_{2,3}$ |
|---|---|
| 000 | 0.21 |
| **001** | **0.48** |
| 010 | 0.81 |
| 011 | 0.23 |
| 100 | 0.19 |
| 101 | 0.44 |
| 110 | 0.30 |
| 111 | 0.72 |

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# Number of Interacting Bits $K_i$

$$f_i(x) = \frac{1}{N}\sum_{j=1}^{N} f_{i,j}\left(x_j, z_1^{(i,j)}, z_2^{(i,j)}, \dots, z_{K_i}^{(i,j)}\right)$$

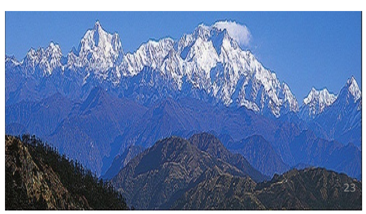
$K_i$ bits interacting with $x_j$

**$K_i$ = 0**

- One peaked, smooth, highly correlated *i*-th landscape

**Varying $K_i$ from 1 to N-1**

- Increasingly rugged multi-peaked *i*-th landscape



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# Features of MNK-Landscapes by Enumeration

- Enumerate MNK-landscapes
  - M: 2-10 objectives
  - N: 10, 15, 20 bits
  - K: 0, 5, 15, 25, 35, 50 (%N) bits
- Rank solutions by non-dominated sorting
- Average on 30 landscapes: L1-L30

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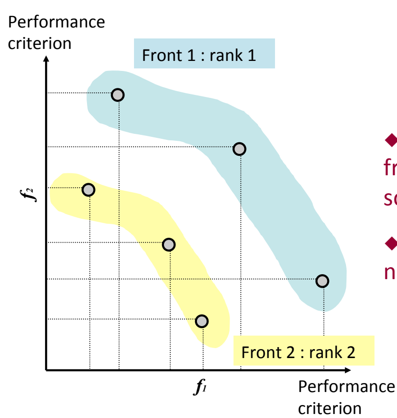
# Non-dominated Sorting




Goldberg
- NSGA-II, Deb et.al

◆ Classifies a population in fronts of non-dominated solutions

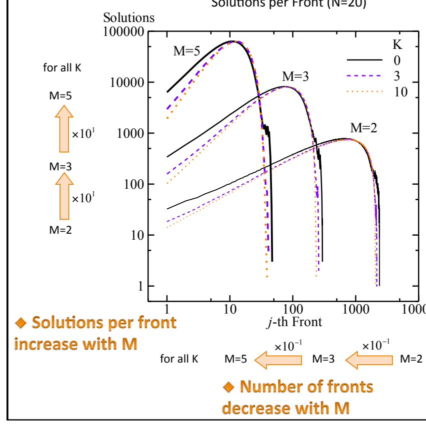
◆ Ranks solutions by their non-domination front



# Effects of Number of Objectives

## Number of Non-Dominated Solutions and Fronts

[Aguirre and Tanaka, MNK-Landscapes, 2004]

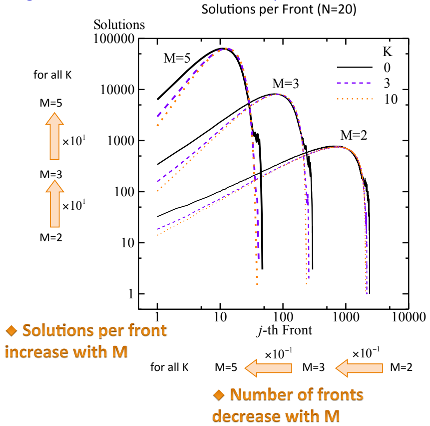





# Effects of Number of Objectives

## Number of Non-Dominated Solutions and Fronts

[Aguirre and Tanaka, MNK-Landscapes, 2004]




**Implications?**

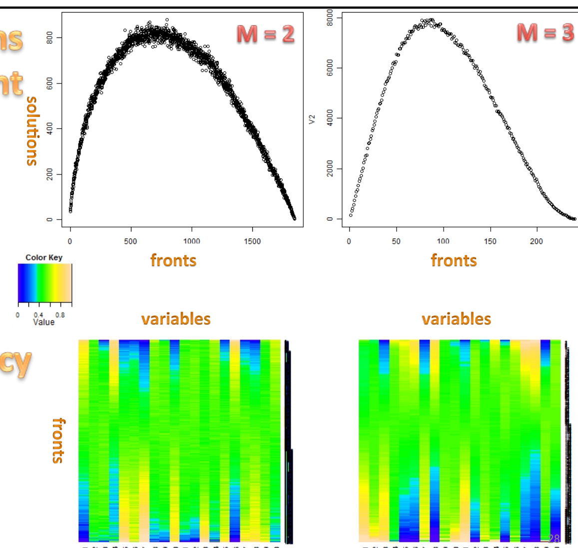
**Objective Space**
- **Selection**
  - **dominance**
  - **diversity estimation**

**Variable Space**
- **Variation Operators**
  - **crossover**
  - **mutation**

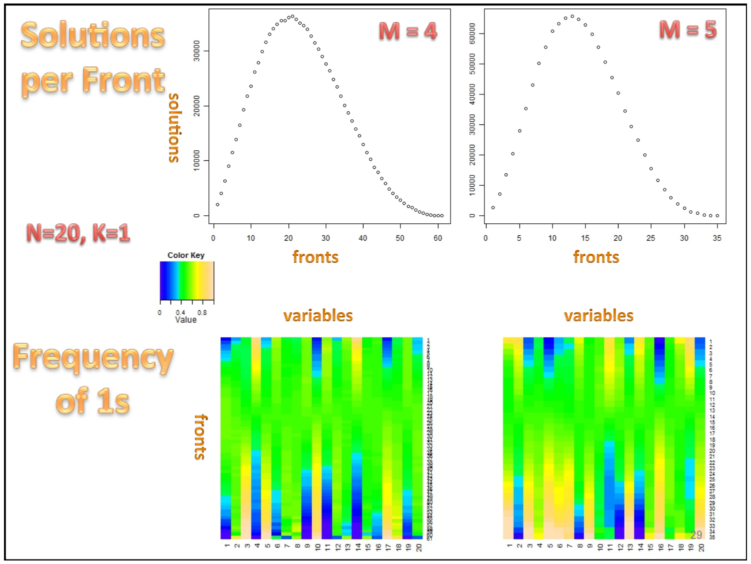


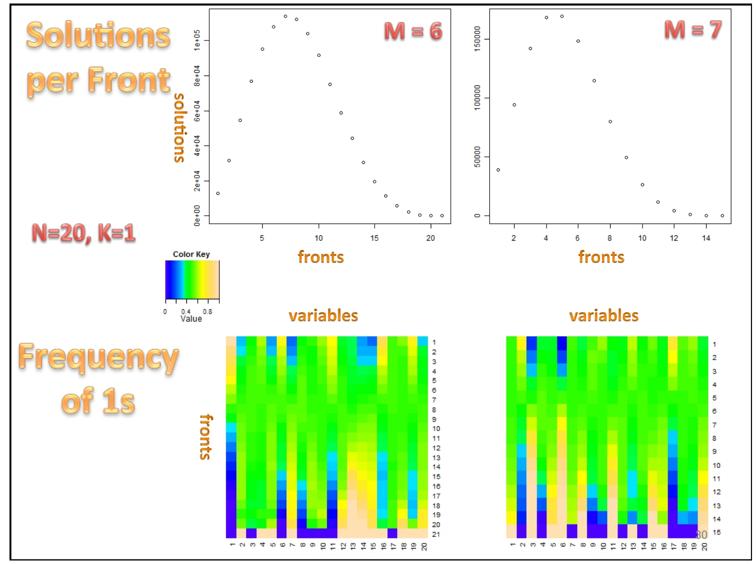
# Solutions per Front

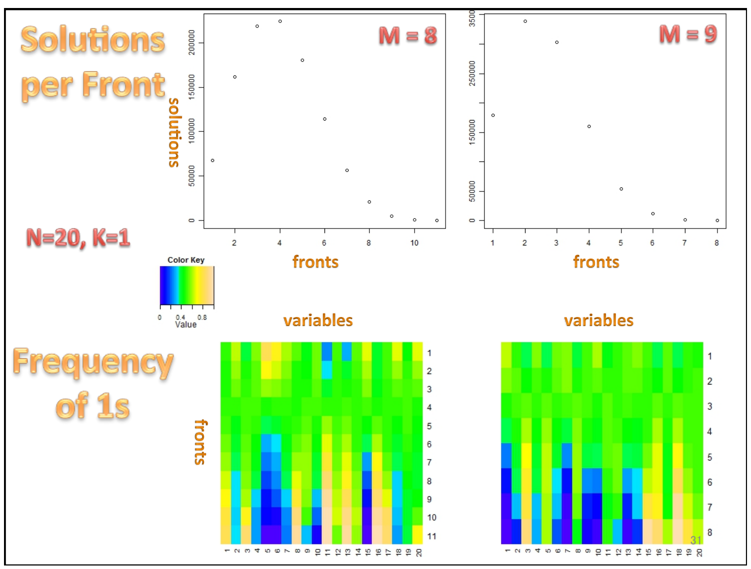
N=20, K=1

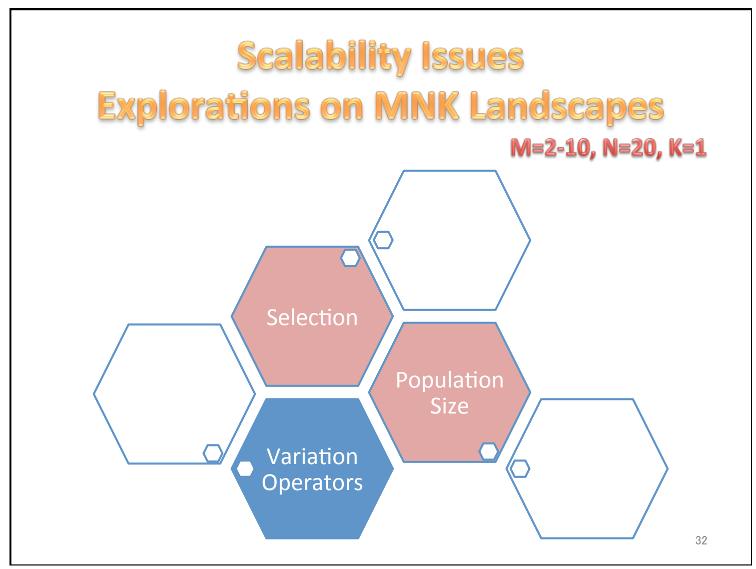



# Frequency of 1s


Solutions per Front
N=20, K=1
M = 4
M = 5
solutions
fronts
fronts
Color Key
Value
Frequency of 1s
variables
variables
fronts
29


Solutions per Front
N=20, K=1
M = 6
M = 7
solutions
fronts
fronts
Color Key
Value
Frequency of 1s
variables
variables
fronts
30


Solutions per Front
N=20, K=1
M = 8
M = 9
solutions
fronts
fronts
Color Key
Value
Frequency of 1s
variables
variables
fronts
31


Scalability Issues
Explorations on MNK Landscapes
M=2-10, N=20, K=1
Selection
Population Size
Variation Operators
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# Scalability Issues : Selection

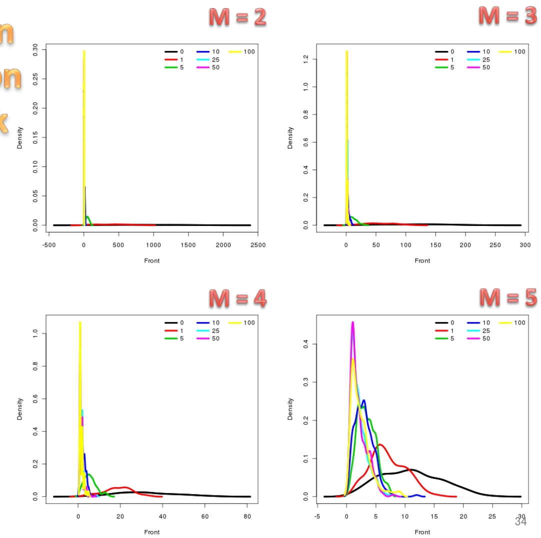
- Population Distribution by True Rank
- Non-dominated and True Pareto Optimal Solutions



## Population Distribution True Rank

N=20, K=1

PopSize = 200

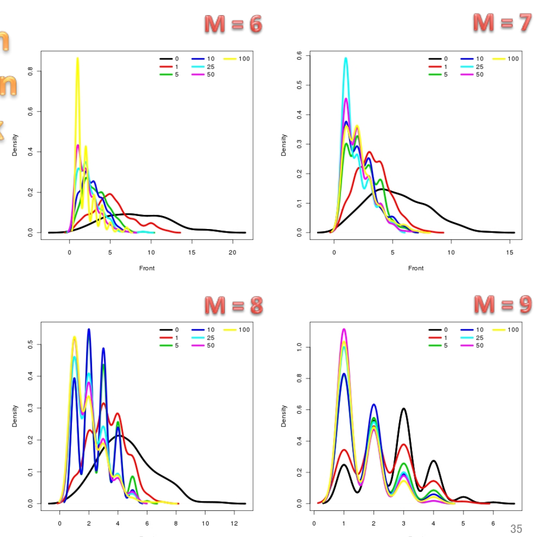





## Population Distribution True Rank

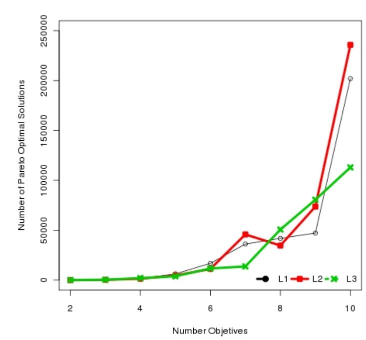
N=20, K=1

PopSize = 200

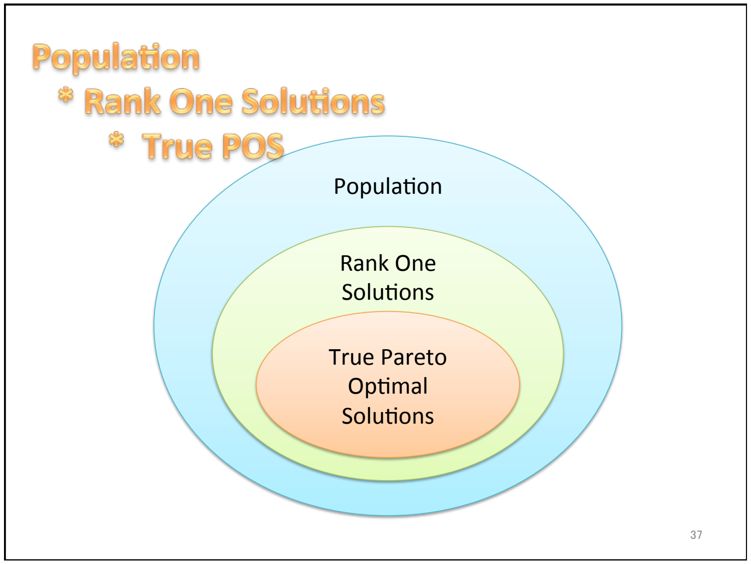





# Number of True Pareto Optimal Solutions




| M | L 1 | L 2 | L 3 |
|---|---|---|---|
| 2 | 26 | 18 | 56 |
| 3 | 152 | 330 | 376 |
| 4 | 1,554 | 1,349 | 2,014 |
| 5 | 6,265 | 4,701 | 3,846 |
| 6 | 16,845 | 11,177 | 11,673 |
| 7 | 36,142 | 45,806 | 13,714 |
| 8 | 41,836 | 34,655 | 50,646 |
| 9 | 47,203 | 73,691 | 80,539 |
| 10 | 202,001 | 235,812 | 112,945 |


Population
* Rank One Solutions
* True POS
Population
Rank One Solutions
True Pareto Optimal Solutions
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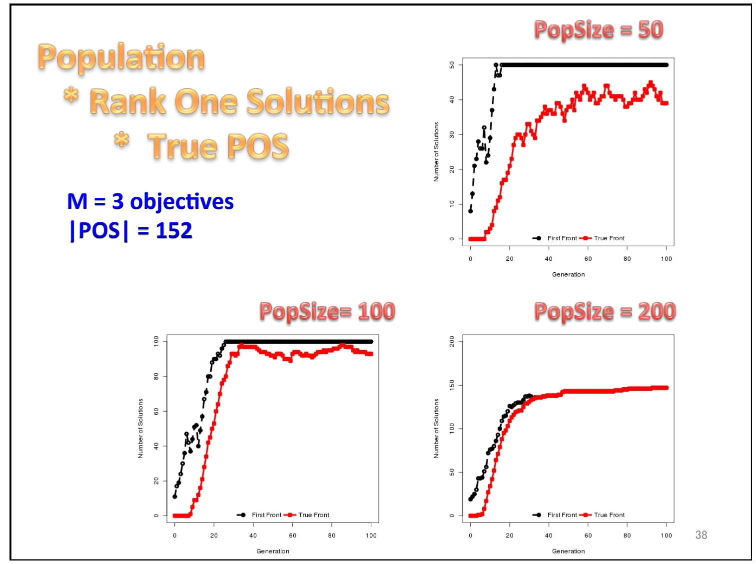
Population
* Rank One Solutions
* True POS
M = 3 objectives
|POS| = 152
PopSize = 50
PopSize= 100
PopSize = 200
Number of Solutions
Generation
First Front
True Front
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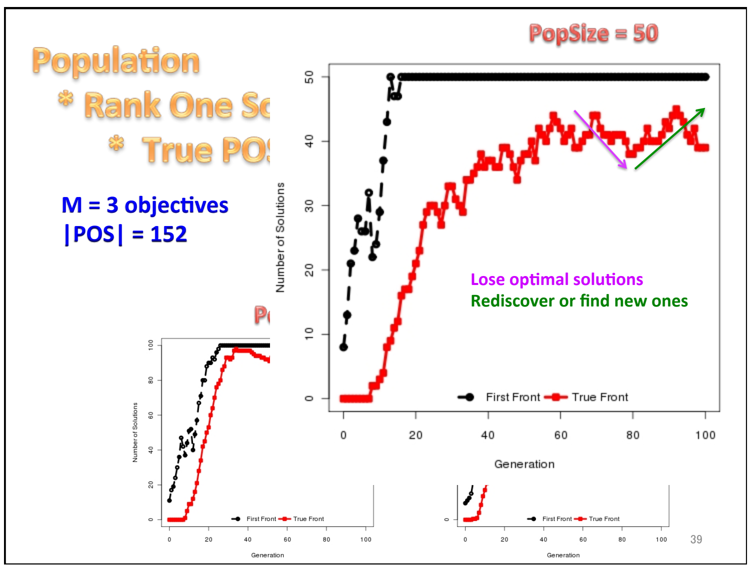
Population
* Rank One Sc
* True PO
M = 3 objectives
|POS| = 152
PopSize = 50
Number of Solutions
Generation
Lose optimal solutions
Rediscover or find new ones
First Front
True Front
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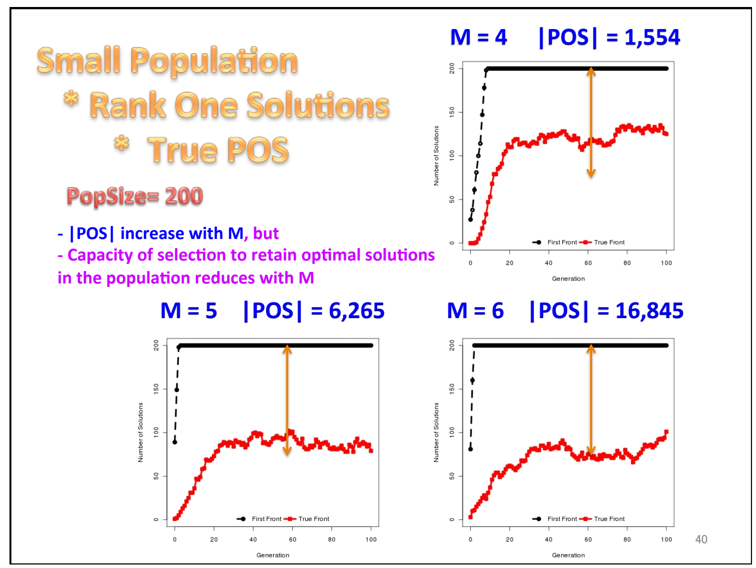
Small Population
* Rank One Solutions
* True POS
PopSize= 200
- |POS| increase with M, but
- Capacity of selection to retain optimal solutions in the population reduces with M
M = 4 |POS| = 1,554
M = 5 |POS| = 6,265
M = 6 |POS| = 16,845
Number of Solutions
Generation
First Front
True Front
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## Scalability Issues : Selection Lapse

- Ranking
  - Based on current population
  - Does not completely capture true order in objective space
- Lost of optimal solutions

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## Population Size

- Many more solutions are required to represent the Pareto front
- Larger populations can support better the evolutionary process

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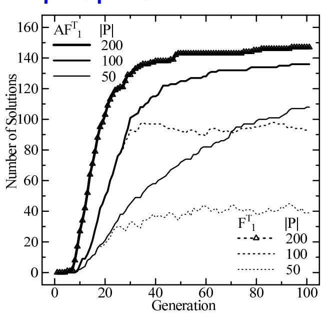
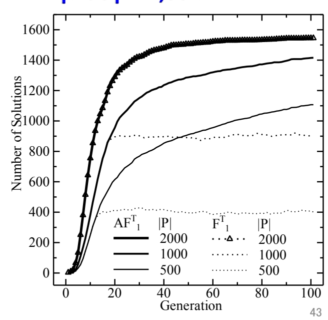
## Large Population : 1/3, 2/3, 4/3 |POS|
### * Rank One Solutions
#### * Accumulated True POS

**M = 3 objectives**
**|POS| = 152**




**M = 4 objectives**
**|POS| = 1,554**

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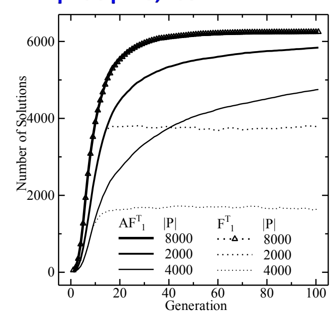
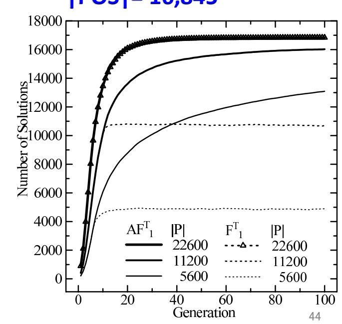
## Large Population : 1/3, 2/3, 4/3 |POS|
### * Rank One Solutions
#### * Accumulated True POS

**M = 5 objectives**
**|POS| = 6,265**




**M = 6 objectives**
**|POS|= 16,845**

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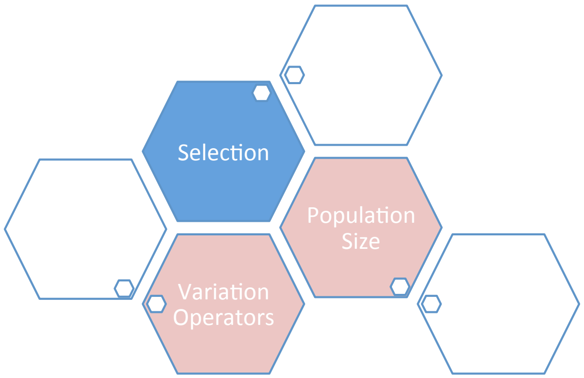
## Scalability Issues
## Explorations on DTLZ Test Functions

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## DTLZ test functions

**DTLZ2**

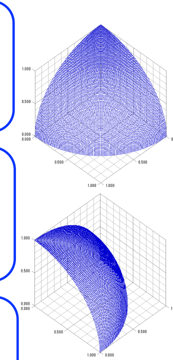
$$f_1 = (1+g)\prod_{i=1}^{M-1} cos(y_i\pi/2)$$
$$f_{m=2:M-1} = (1+g)\prod_{i=1}^{M-1} cos(y_i\pi/2)sin(y_{M-m+1}\pi/2)$$
$$f_M = (1+g)sin(y_1\pi/2)$$
$$g = \sum_{i=1}^{k}(z_i - 0.5)^2$$

**DTLZ3**

$$f_1 = (1+g)\prod_{i=1}^{M-1} cos(y_i\pi/2)$$
$$f_{m=2:M-1} = (1+g)\prod_{i=1}^{M-1} cos(y_i\pi/2)sin(y_{M-m+1}\pi/2)$$
$$f_M = (1+g)sin(y_1\pi/2)$$
$$g = 100\left[k + \sum_{i=1}^{k}((z_i - 0.5)^2 - cos(20\pi(z_i - 0.5)))\right]$$

**DTLZ4**

$$f_1 = (1+g)\prod_{i=1}^{M-1} cos(y_i^{\alpha}\pi/2)$$
$$f_{m=2:M-1} = (1+g)\prod_{i=1}^{M-1} cos(y_i^{\alpha}\pi/2)sin(y_{M-m+1}\pi/2)$$
$$f_M = (1+g)sin(y_1^{\alpha}\pi/2)$$
$$g = \sum_{i=1}^{k}(z_i - 0.5)^2$$



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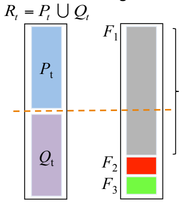
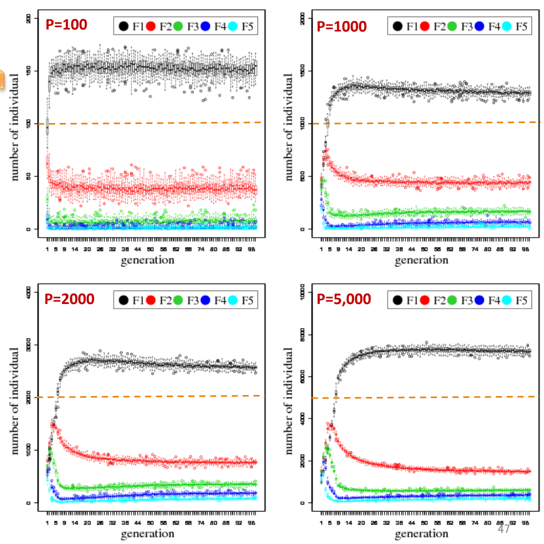
## Fronts Distribution (NSGA-II)

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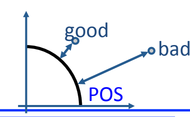
## Proximity Indicator

- Measures convergence of solutions
- Smaller values of $Ip$ indicate that the population $P$ is closer to the true Pareto front

**Convergence**




How close is the set obtained by the algorithms to POS ?

**Proximity Indicator**

$$I_p = \underset{\boldsymbol{x}\in P}{median}\left\{\left[\sum_{i=1}^{m}(f_i(\boldsymbol{x}))^2\right]^{\frac{1}{2}} - 1\right\}$$

$f_m$ : objective function

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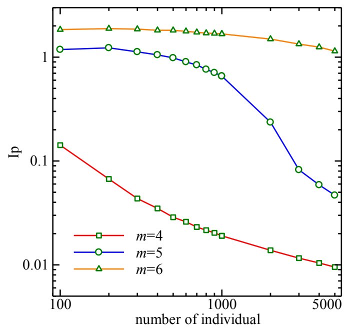
# Population Size: Effect on Convergence

**Variables: n= m + 9**

- NSGA-II (DTLZ2)

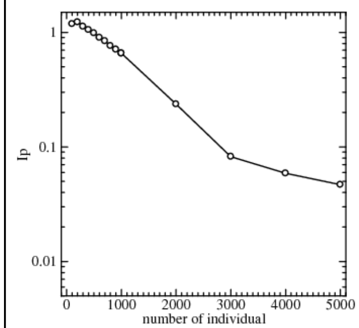


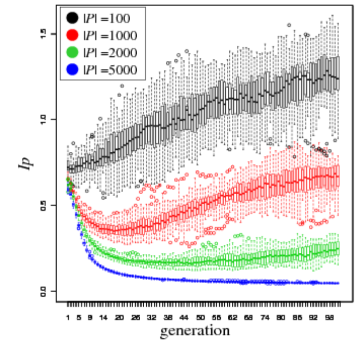

Good convergence

Large population



# Proximity Indicator Over the Generations (NSGA-II)

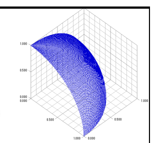


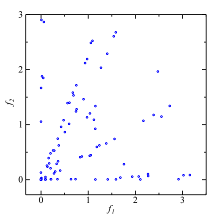




**Objectives: m= 5**

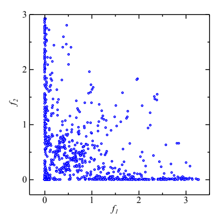


# Population Size: Effect on Distribution of Solutions

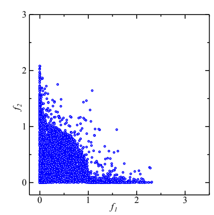




$m$=5,|$P$|=100



$m$=5,|$P$|=1000



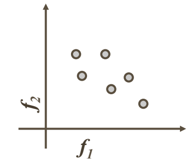
$m$=5,|$P$|=5000

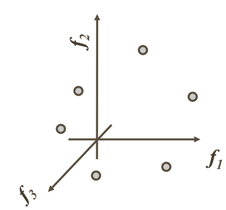
- Solutions tend to concentrate along the axis
- When the population increases the solutions tends to cluster towards the central regions of objective space



# Disruptive Recombination ?

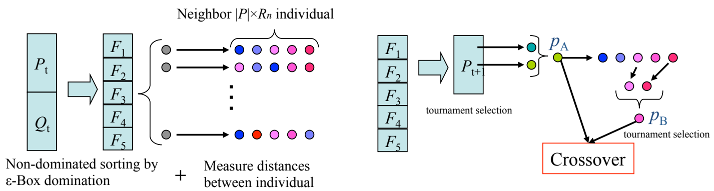
- Non-dominated solutions in many-objective problems cover a larger portion of objective and variable space
- Difference between individuals in the instantaneous population is expected to be larger.
- Recombining two very different individuals could be too disruptive.



# Neighborhood Recombination : Method

- Calculates the distance between individuals in objective space and keeps a record of the $|P| \times Rn$ closets neighbors of each individual
- We set the parameter $Rn$ to 2% 5% 10%

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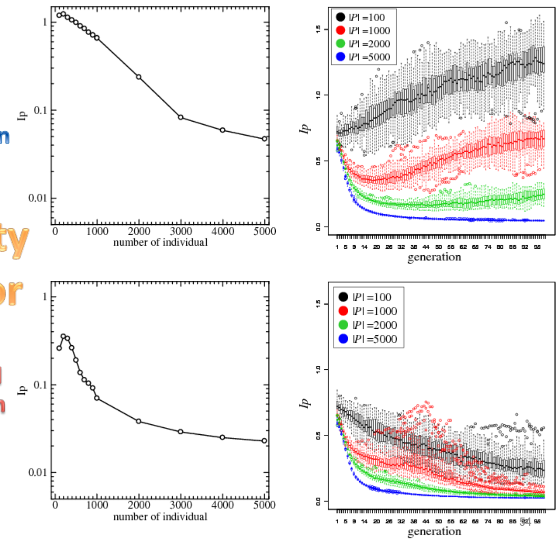
## Proximity Indicator

Unrestricted Recombination

Neighborhood Recombination

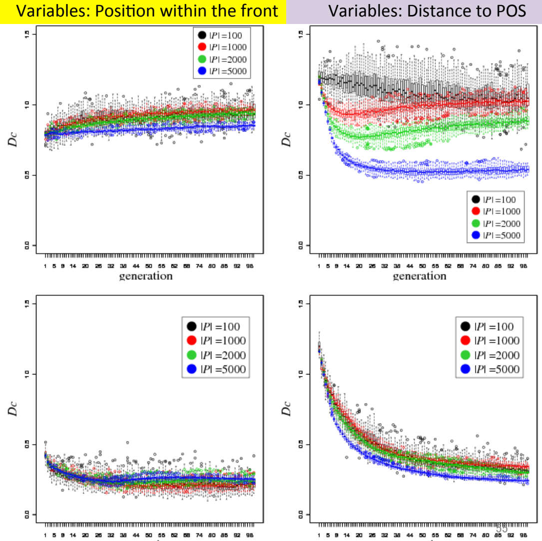



## Mates Distance

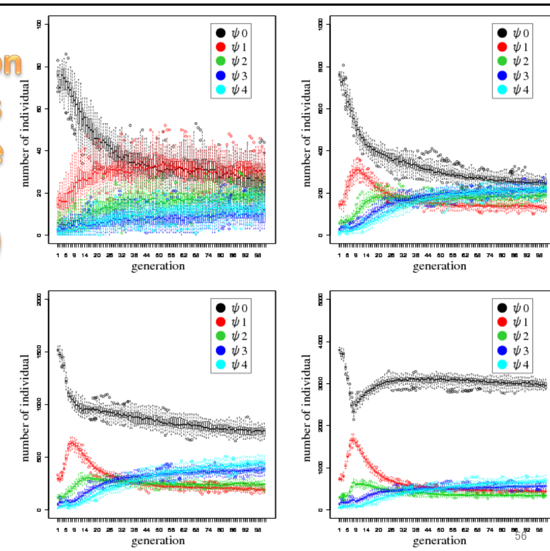
Unrestricted Recombination

Neighborhood Recombination

| Variables: Position within the front | Variables: Distance to POS |
| --- | --- |




## Distribution Solutions Objective Space (NSGA-II)

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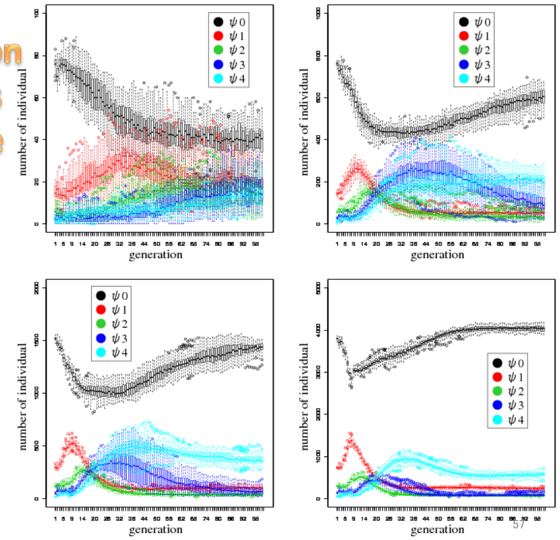
## Distribution Solutions Objective Space (NR)

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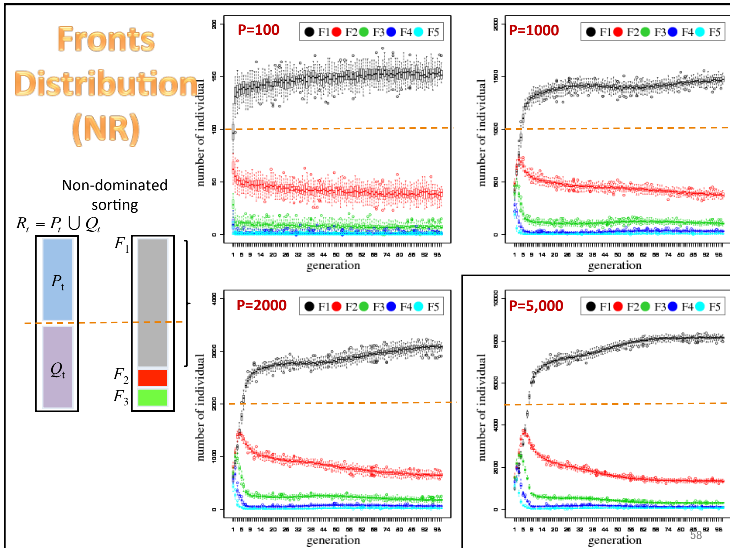
## Fronts Distribution (NR)

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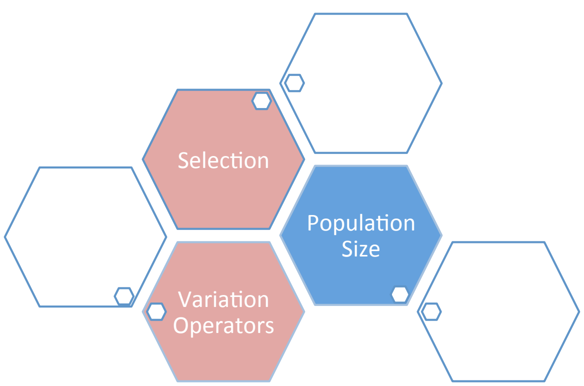
## Scalability Issues
## Explorations on DTLZ Test Functions

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## Distribution Search

- Find solutions with good convergence and well distributed, according to a desired distribution

**Selection** dominance, diversity estimation ⇨ **Adaptive ε-Ranking** Three linear mapping functions to bias selection

**Variation Operators** crossover, mutation ⇨ **Recombination rate** To properly guide the algorithm towards the distribution we seek to find

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## Pareto Dominance and ε-Dominance

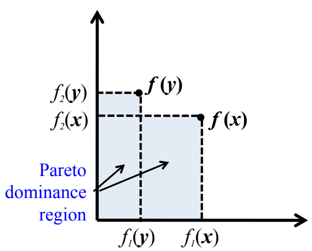
- **Dominance**
  $x$ dominates $y$ *iff*

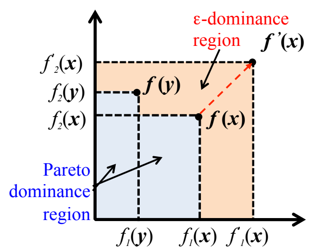
  $\forall i \; f_i(x) \geq f_i(y) \quad \wedge$
  $\exists i \; f_i(x) > f_i(y)$

  $i = 1, 2, \cdots m$

- **ε-Dominance**
  $x$ ε-dominates $y$ *iff*

  $f(x) \mapsto^{\varepsilon} f'(x)$
  $\forall i \; f_i'(x) \geq f_i(y) \quad \wedge$
  $\exists i \; f_i'(x) > f_i(y)$






## Mapping Functions for ε-Dominance

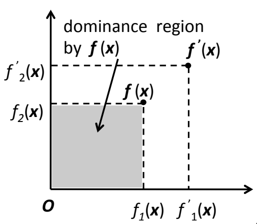
$$f(x) \mapsto^{\varepsilon} f'(x)$$

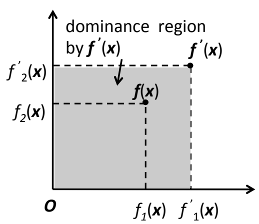
- Various functions have been used
  - ✓ Logarithmic [Leummans et al.] [Deb] [Aguirre, Tanaka]
  - ✓ Multiplicative [Leummans et al.] [Aguirre, Tanaka]
  - ✓ Additive [Leummans et al.]

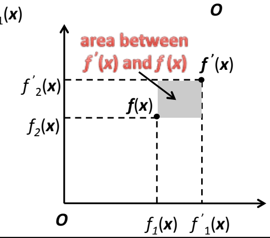
- In this work we study three additive linear functions
  - Additive Constant
  - Expansion from the Center
  - Contraction from the Center



## Dominance Regions and Area between f(x) and f′(x)



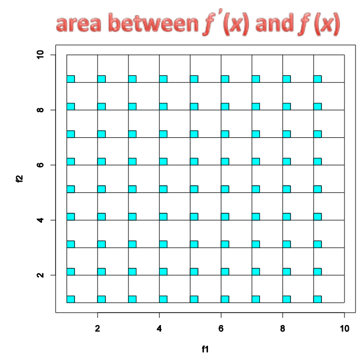






## Additive Constant

$$f_i'(x) = f_i(x) + \varepsilon, \quad i = 1, \ldots, m$$




distributions of solutions evenly spaced

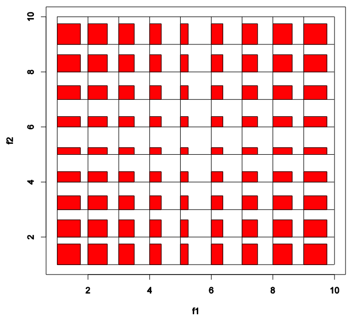
# Expansion from the Center

$$f_i'(x) = f_i(x) + \varepsilon + \delta_i,$$
$$i = 1, \ldots, m$$

$$\delta_i = \gamma \frac{\left| f_i(x) - \overline{f_i} \right|}{\frac{1}{2}\left| f_i^{\max} - f_i^{\min} \right|}$$

$$\overline{f_i} = \frac{1}{2}\left( f_i^{\max} + f_i^{\min} \right)$$

area between $f'(x)$ and $f(x)$




solutions spaced by a distance that increases
linearly from the center towards the extremes of objective space



# Contraction from the Center

$$f_i'(x) = f_i(x) + \varepsilon + \delta_i,$$
$$i = 1, \ldots, m$$

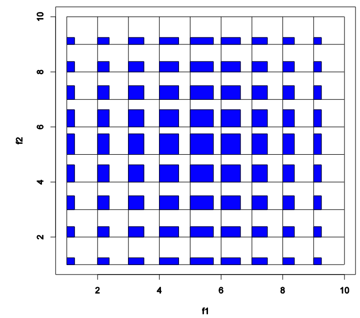
$$f_i(x) > \overline{f_i}$$

$$\delta_i = \gamma \frac{\left| f_i(x) - f_i^{\max} \right|}{\frac{1}{2}\left| f_i^{\max} - f_i^{\min} \right|}$$

Otherwise

$$\delta_i = \gamma \frac{\left| f_i(x) - f_i^{\min} \right|}{\frac{1}{2}\left| f_i^{\max} - f_i^{\min} \right|}$$

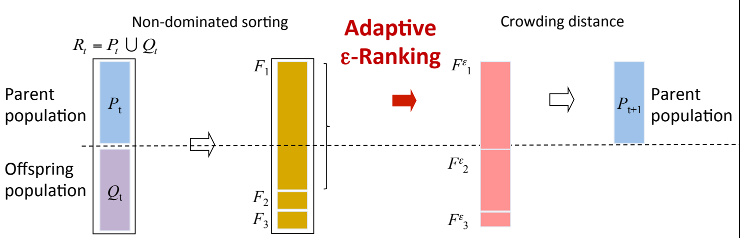
area between $f'(x)$ and $f(x)$




solutions spaced by a distance that decreases
linearly from the center towards the extremes

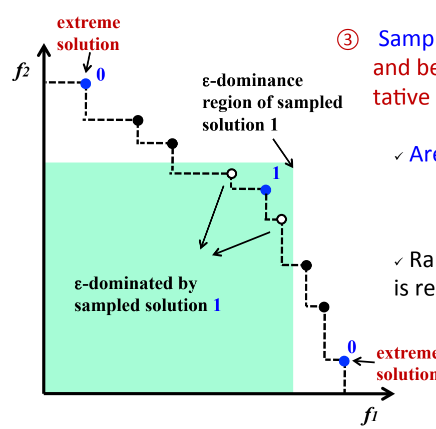


# Adaptive ε-Ranking






# ε-Sampling



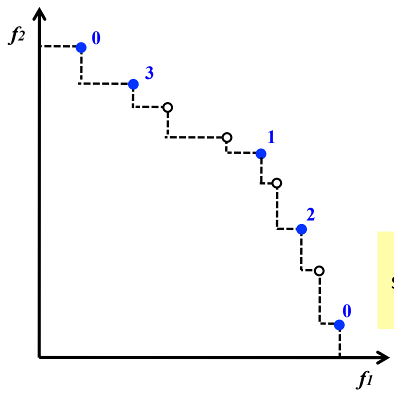

③ Sampled solution keep their rank and becomes the sole representative of their area of influence

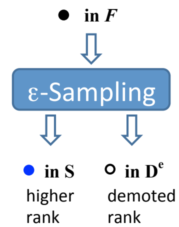
- Area of influence: ε-dominance

$$f(x) \mapsto^{\varepsilon} f'(x)$$

- Rank of **ε-dominated** solutions is reduced

# ε-Sampling (2)




Induce the distribution of solutions implicit in the mapping function used for ε-dominance

$$f(x) \mapsto^{\varepsilon} f'(x)$$

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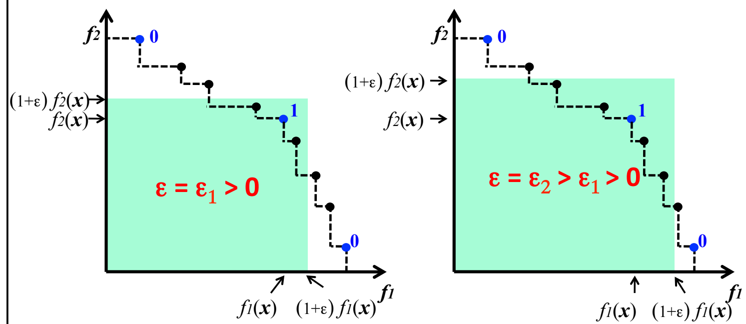
# Appropriate value of ε ?




Larger ε → fewer solutions in the sample
fewer solutions with rank 1

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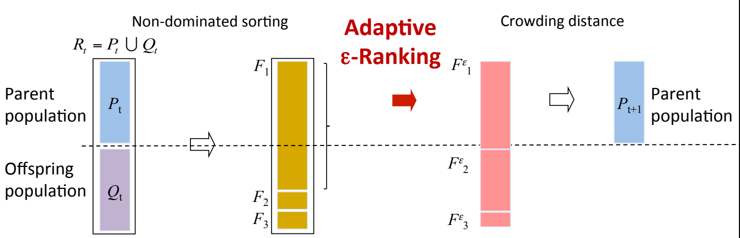
# Adaptation of ε




$\varepsilon \leftarrow 0.0$

$\Delta$ *step of adaptation*

$\Delta \leftarrow [\Delta\min, \Delta\max]$

**Adaptation rule**

$$\begin{aligned}
&if\ |F_1^{\varepsilon}| > |P| \\
&\quad \Delta \leftarrow \Delta \times 2 \\
&\quad \varepsilon \leftarrow \varepsilon + \Delta \\
&else\ if\ |F_1^{\varepsilon}| < |P| \\
&\quad \Delta \leftarrow \Delta \div 2 \\
&\quad \varepsilon \leftarrow \varepsilon - \Delta
\end{aligned}$$

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# Test Problems and Parameters

- Problems
  - DTLZ2, DTLZ3, DTL4 functions
  - 6 objectives
  - Number of variables, nVar 10, 30, 50
- Algorithm Parameters
  - SBX Crossover, Polynomial Mutation
  - Crossover rate per variable, Pcv 0.5, 0.1
  - Three mapping functions for ε-dominance, Additive, Expansion, Contraction
  - Population size 300
  - Number of generations 500

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# Performance Measures

- Convergence
  - Generational Distance (GD) to True Pareto Front
- Distribution of Solutions
  - Scatter Plots
  - Hexagonal Binning Plots

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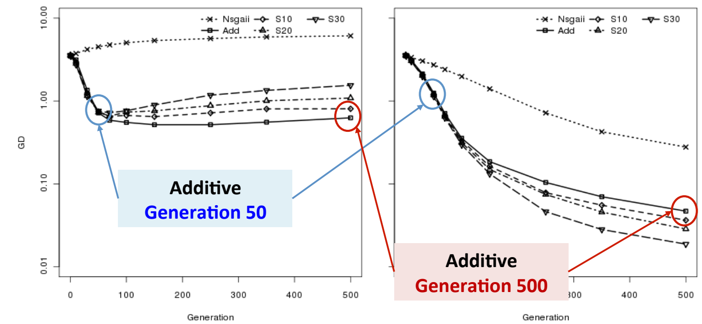
# GD: Additive

nVar = 50

**Pcv = 0.5** **Pcv = 0.1**

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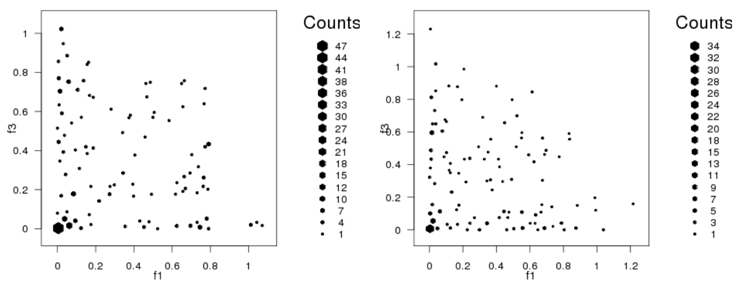
# Hexagonal Bins: Additive

**Generation 500**

**Pcv = 0.5** nVar = 50 **Pcv = 0.1**

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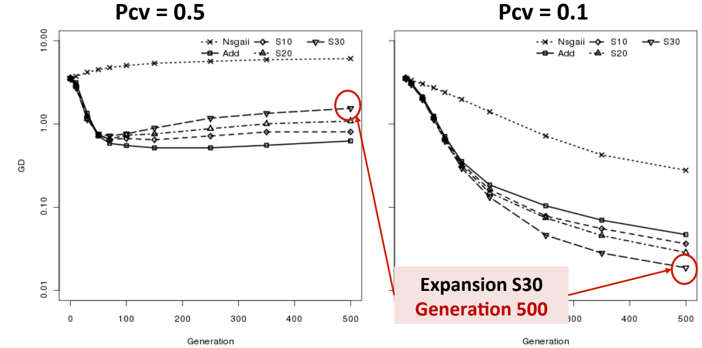
# GD: Expansion

nVar = 50

**Pcv = 0.5** **Pcv = 0.1**

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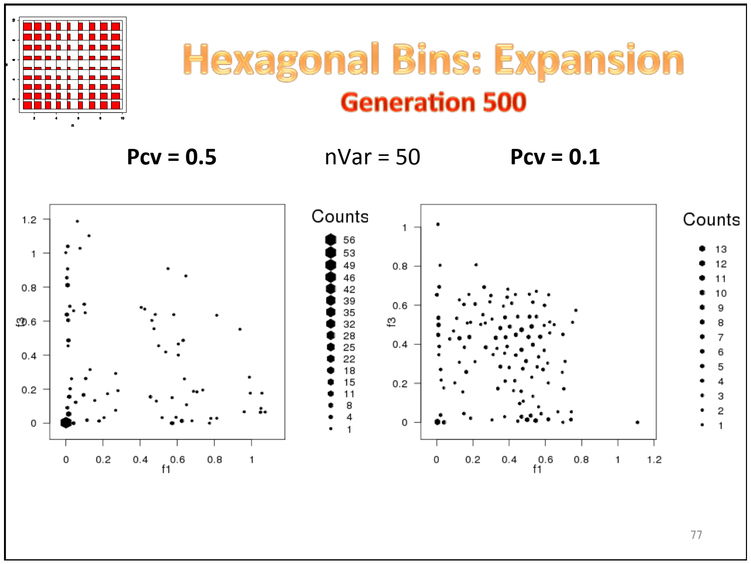
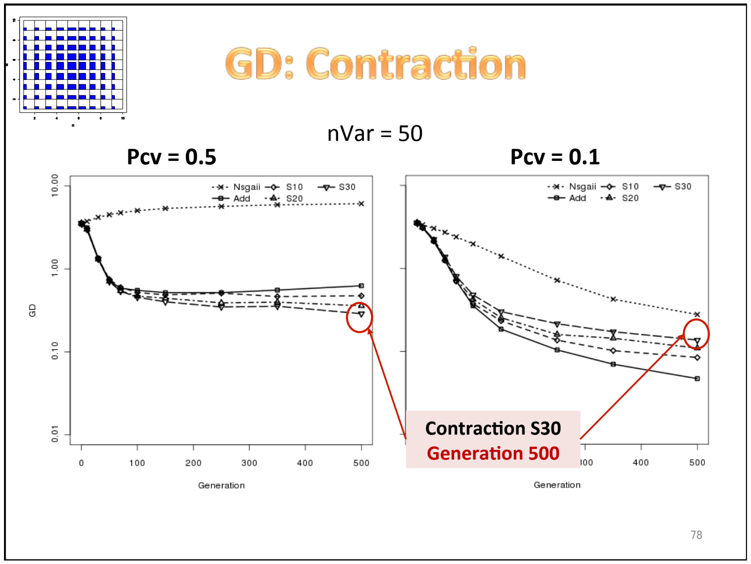
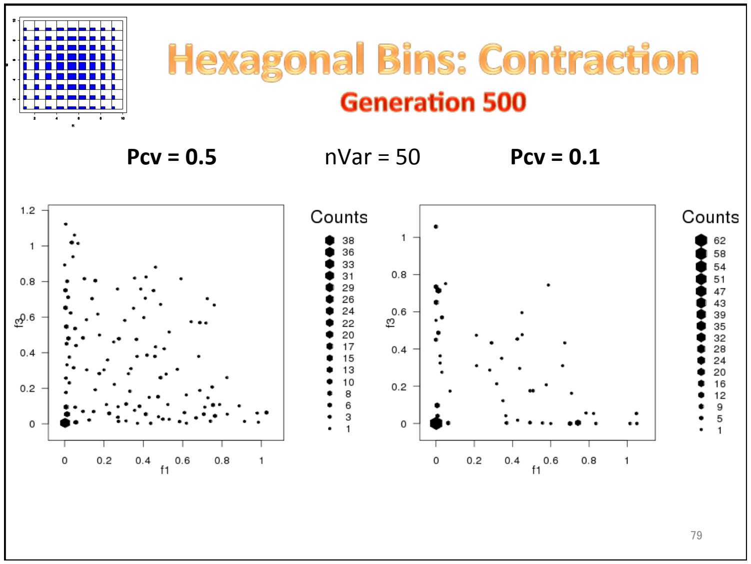
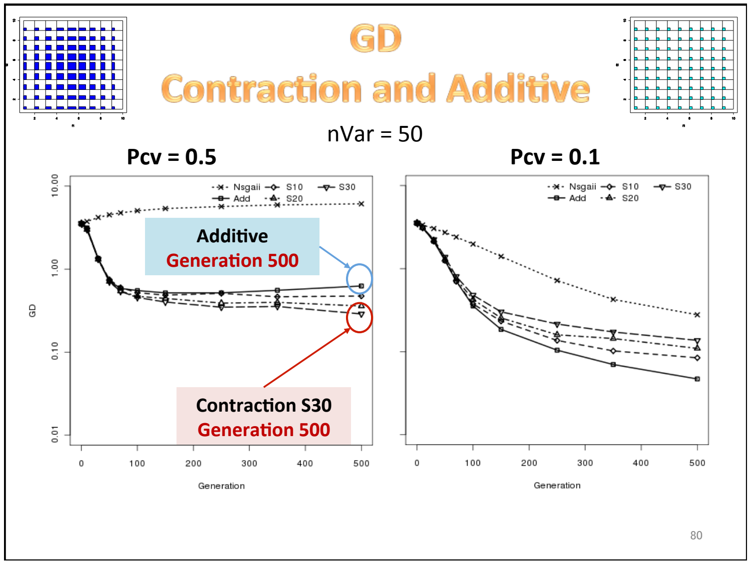
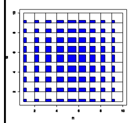

Hexagonal Bins: Expansion
Generation 500
Pcv = 0.5
nVar = 50
Pcv = 0.1
Counts
f1
f3
77

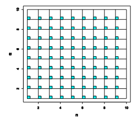

GD: Contraction
nVar = 50
Pcv = 0.5
Pcv = 0.1
Nsgaii
S10
S30
Add
S20
GD
Generation
Contraction S30
Generation 500
78


Hexagonal Bins: Contraction
Generation 500
Pcv = 0.5
nVar = 50
Pcv = 0.1
Counts
f1
f3
79


GD
Contraction and Additive
nVar = 50
Pcv = 0.5
Pcv = 0.1
Nsgaii
S10
S30
Add
S20
GD
Generation
Additive
Generation 500
Contraction S30
Generation 500
80

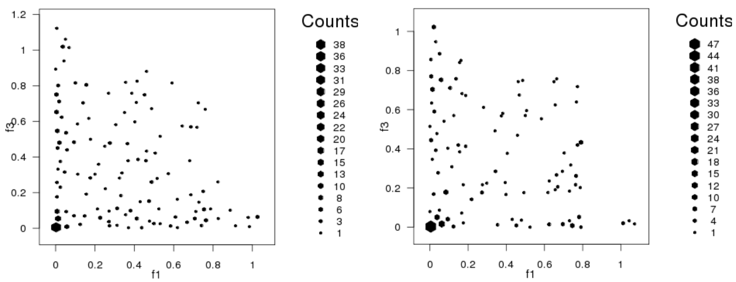


# Hexagonal Bins: Contraction and Additive



**Generation 500**

**Contraction Pcv = 0.5** nVar = 50 **Additive Pcv = 0.5**






# Distribution Search: Conclusions

- In many-objective continuous problems a smaller recombination rate per variable can increase substantially convergence of solutions.
- Selection alone without considering a proper recombination rate cannot induce the distribution we seek to achieve.



# Outline

- Basic concepts
  - Multi-, many-objective optimization
- Scalability issues and fundamentals of many-objective optimization
- Various approaches to many-objective optimization
- Conclusions



# Various Approaches to Many-objective Optimization

- Pareto Dominance Extensions
- Scalarazing Functions
  - Decomposition
  - Performance Indicators
- Objective space partitioning
- Dimensionality reduction

## The Adaptive ε-Sampling and ε-Hood EMyO (AεSεH)

- Survival Selection
  - Eliminate inferior solutions: Dominance
  - Get a well distributed sample: Adaptive ε-Sampling
  - Reduces dropping of optimal solutions: Improves selection
- Mating Selection
  - Create ε-Neighborhoods: Adaptive ε-Hood
  - Balance search effort: More reproductive opportunities to under-represented regions
- Recombination
  - Within the ε-Neighborhood
  - Improves effectiveness of recombination

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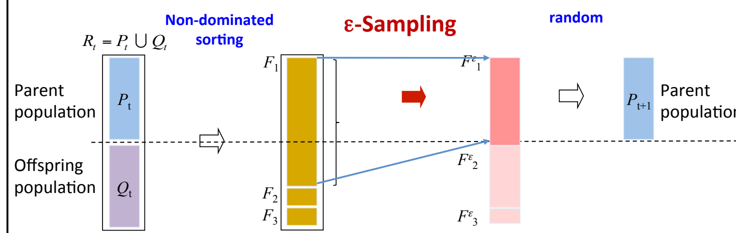
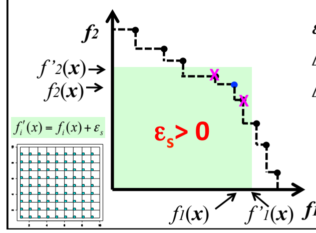
## Adaptive ε-Sampling (AεS)




$\varepsilon \leftarrow 0.0$

$\Delta$ *step of adaptation*

$\Delta \leftarrow [\Delta \min, \Delta \max]$

**Adaptation rule**

$if\ |F_1^{\varepsilon}| > |P|$
$\quad \Delta \leftarrow \Delta \times 2$
$\quad \varepsilon \leftarrow \varepsilon + \Delta$
$else\ if\ |F_1^{\varepsilon}| < |P|$
$\quad \Delta \leftarrow \Delta \div 2$
$\quad \varepsilon \leftarrow \varepsilon - \Delta$

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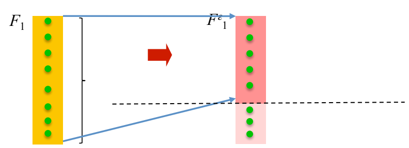
## Adaptive ε-Sampling : Why ?

- Non-dominated in population

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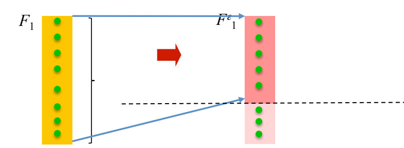
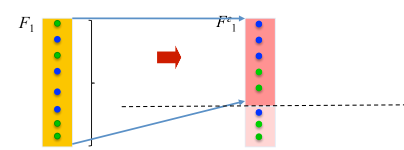
## Adaptive ε-Sampling : Why ?

- Non-dominated in population




- Non-dominated in population
- Superior in the landscape




**ε-Sampling reduces dropping of superior solutions**

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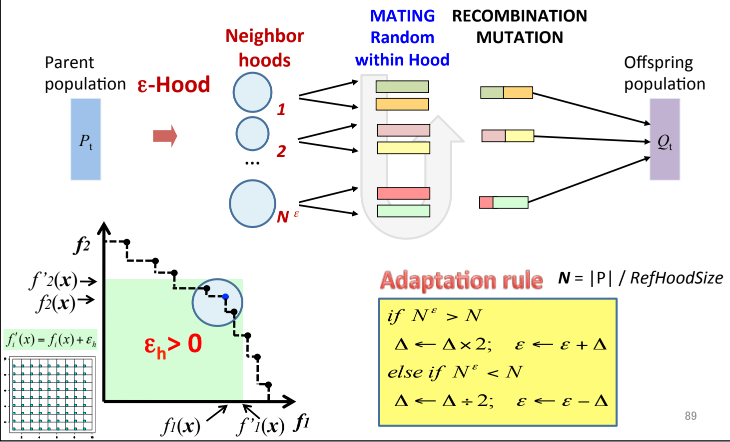
# Adaptive ε-Hood (AεH) & Reproduction

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# Parameters and Problems

EMyO :
- Adaptive ε-Sampling & Adaptive ε-Hood (AεSεH)
- Adaptive ε-Box with Neighborhood Recombination (Aε-Box NR) [LION 06]

Problems:
- DTLZ2, DTLZ3, DTLZ4

- Performance measures
  - Generational Distance GD
  - Scatter Plots

| | |
|---|---|
| Objectives | $m$ = 4 - 6 |
| Variables | $n = m + 9$ |
| Population size<br><br>Ref Hood Size<br>(Num Hoods) | $P$ = **100-5000**<br>**(10000, 20000)**<br>$H$=20<br>$(N=P/H)$ |
| Crossover | SBX, $\eta_c = 15$ |
| Crossover rate | $p_c = 1.0$ |
| Crossover rate per variable | $p_e = 0.5$ |
| Mutation | Polynomial, $\eta_c = 20$ |
| Mutation rate | $p_m = 1.0 / n$ |
| **Generations** | **100, 500** |
| Runs | 30 |

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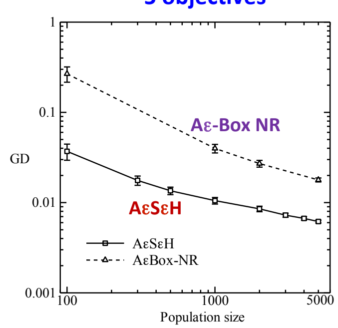
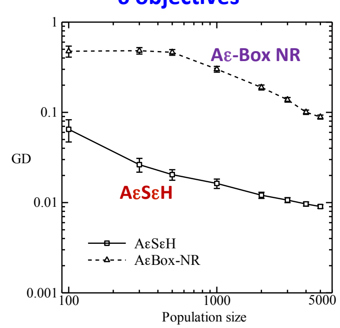
# GD - Convergence (DTLZ2)

### 5 objectives




### 6 objectives




100 Generations

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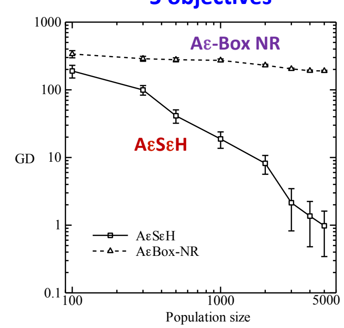
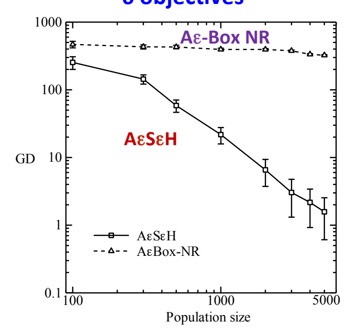
# GD - Convergence (DTLZ3)

### 5 objectives

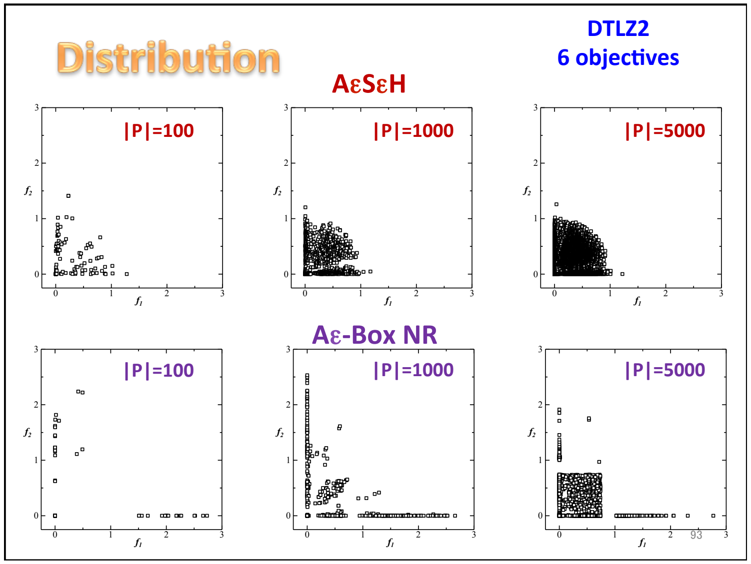


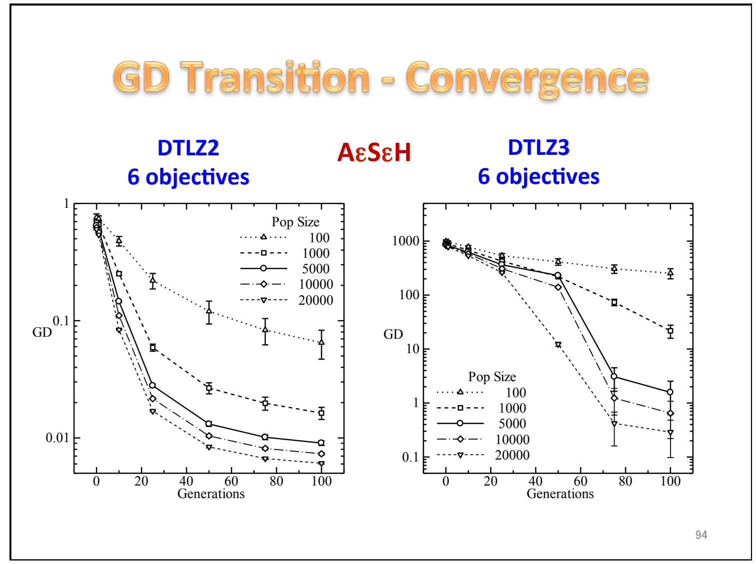

### 6 objectives

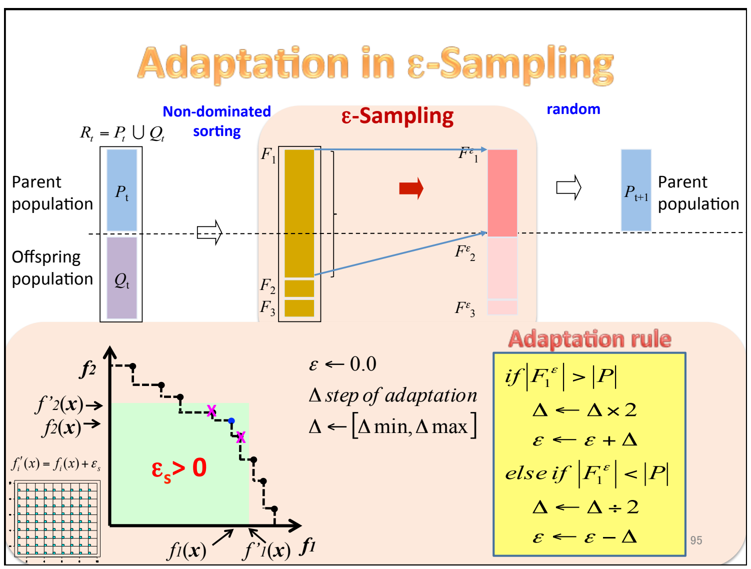


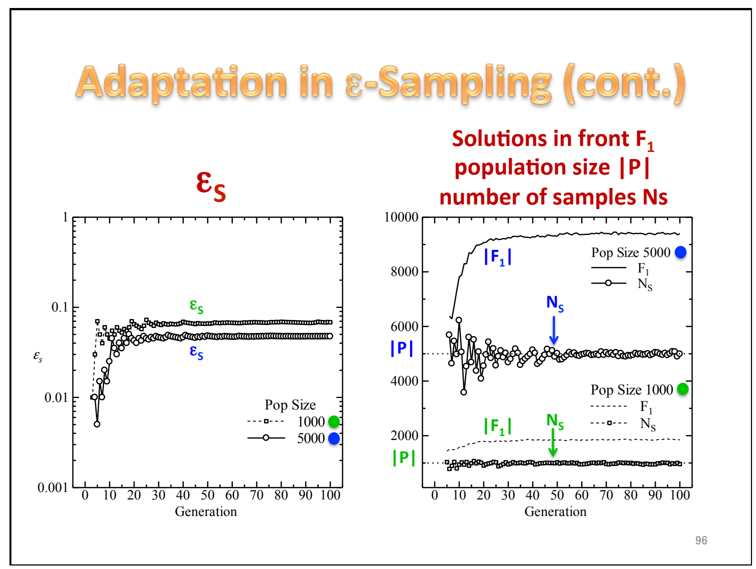

100 Generations

92

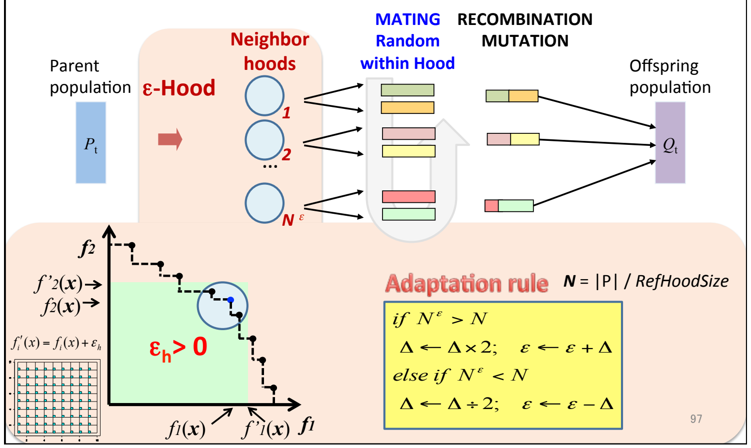

Distribution
DTLZ2
6 objectives
AεSεH
|P|=100
|P|=1000
|P|=5000
Aε-Box NR
|P|=100
|P|=1000
|P|=5000
93


GD Transition - Convergence
DTLZ2
6 objectives
AεSεH
DTLZ3
6 objectives
Pop Size
100
1000
5000
10000
20000
GD
Generations
94


Adaptation in ε-Sampling
Non-dominated sorting
ε-Sampling
random
Parent population
Offspring population
Parent population
Adaptation rule
ε ← 0.0
Δ step of adaptation
Δ ← [Δ min, Δ max]
if |F1ε| > |P|
Δ ← Δ × 2
ε ← ε + Δ
else if |F1ε| < |P|
Δ ← Δ ÷ 2
ε ← ε − Δ
εs > 0
95


Adaptation in ε-Sampling (cont.)
εS
Solutions in front F1
population size |P|
number of samples Ns
Pop Size
1000
5000
Generation
Pop Size 5000
Pop Size 1000
|F1|
|P|
NS
96

## Adaptation in ε-Hood

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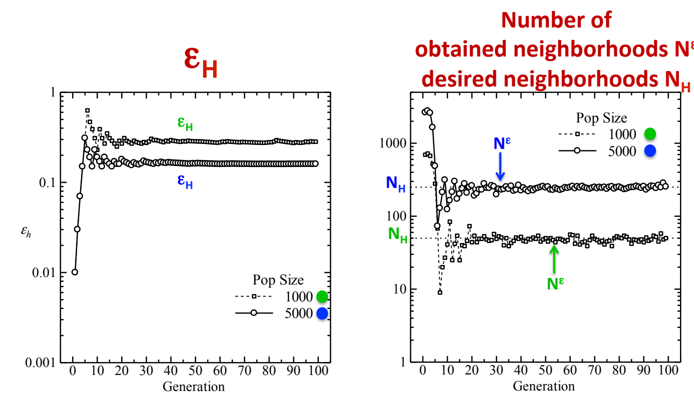
## Adaptation in ε-Hood (cont.)

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## Conclusions

- Have a better understanding of the problems associated to many-objective optimization
- Progress on the fundamentals for many-objective optimization
- A number of algorithms that are considerably better than conventional multi-objective optimizers
- Many-objective optimizers are being used as a practical tool
- Still a lot of challenges ahead

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Thank you for listening

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