

Expressive Genetic Programming

Tutorial

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Instructor

Lee Spector is a Professor of Computer Science in the School of Cognitive Science at Hampshire College in Amherst, Massachusetts, and an adjunct professor in the Department of Computer Science at the University of Massachusetts, Amherst. He received a B.A. in Philosophy from Oberlin College in 1984 and a Ph.D. from the Department of Computer Science at the University of Maryland in 1992. His areas of teaching and research include genetic and evolutionary computation, quantum computation, and a variety of intersections between computer science, cognitive science, evolutionary biology, and the arts. He is the Editor-in-Chief of the journal *Genetic Programming and Evolvable Machines* (published by Springer) and a member of the editorial board of *Evolutionary Computation* (published by MIT Press). He is also a member of the SIGEVO executive committee and he was named a Fellow of the International Society for Genetic and Evolutionary Computation.

Tutorial Description (1)

The language in which evolving programs are expressed can have significant impacts on the problem-solving capabilities of a genetic programming system. These impacts stem both from the absolute computational power of the languages that are used, as elucidated by formal language theory, and from the ease with which various computational structures can be produced by random code generation and by the action of genetic operators. Highly expressive languages can facilitate the evolution of programs for any computable function using, when appropriate, multiple data types, evolved subroutines, evolved control structures, evolved data structures, and evolved modular program and data architectures. In some cases expressive languages can even support the evolution of programs that express methods for their own reproduction and variation (and hence for the evolution of their offspring).

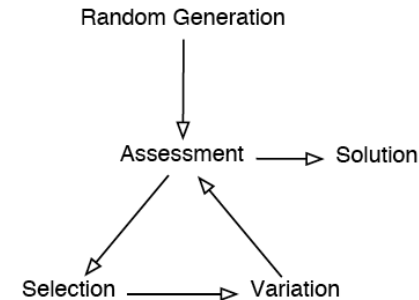
Tutorial Description (2)

This tutorial will begin with a comparative survey of approaches to the evolution of programs in expressive programming languages ranging from machine code to graphical and grammatical representations. Within this context it will then provide a detailed introduction to the Push programming language, which was designed specifically for expressiveness and specifically for use in genetic programming systems. Push programs are syntactically unconstrained but can nonetheless make use of multiple data types and express arbitrary control structures, supporting the evolution of complex, modular programs in a particularly simple and flexible way. The Push language will be described and ten years of Push-based research, including the production of human-competitive results, will be briefly surveyed. The tutorial will conclude with a discussion of recent enhancements to Push that are intended to support the evolution of complex and robust software systems.

Course Agenda

- Genetic Programming refresher
- Why evolve programs in expressive languages?
- Expressivity and evolvability
- Expressive trees, bits, graphs, grammars, stacks
- **Push**
- Expressing the future

Evolutionary Computation



Evolution, the Designer

“Darwinian evolution is itself a designer worthy of significant respect, if not religious devotion.” *Boston Globe* OpEd, Aug 29, 2005

WHAT WOULD DARWIN SAY? | LEE SPECTOR

And now, digital evolution

The Boston Globe

By Lee Spector | August 29, 2005

RECENT developments in computer science provide new perspective on "intelligent design," the view that life's complexity could only have arisen through the hand of an intelligent designer. These developments show that complex and useful designs can indeed emerge from random Darwinian processes.

Genetic Programming (GP)

- Evolutionary computing to produce executable computer programs
- Programs are assessed by executing them
- Automatic programming; producing software
- Potential (?): evolve software at all scales, including and surpassing the most ambitious and successful products of human software engineering

Program Representations

- Lisp-style symbolic expressions (Koza, ...).
- Purely functional/lambda expressions (Walsh, Yu, ...).
- Linear sequences of machine/byte code (Nordin et al., ...).
- Artificial assembly-like languages (Ray, Adami, ...).
- Stack-based languages (Perkis, Spector, Stoffel, Tchernev, ...).
- Graph-structured programs (Teller, Globus, ...).
- Object hierarchies (Bruce, Abbott, Schmutter, Lucas, ...).
- Fuzzy rule systems (Tunstel, Jamshidi, ...).
- Logic programs (Osborn, Charif, Lamas, Dubossarsky, ...).
- Strings, grammar-mapped to arbitrary languages (O'Neill, Ryan, ...).

Mutating Lisp

```
(+ (* X Y)
   (+ 4 (- Z 23)))
```

```
(+ (* X Y)
   (+ 4 (- Z 23)))
```

```
(+ (- (+ 2 2) Z)
   (+ 4 (- Z 23)))
```

Recombining Lisp

```
Parent 1: (+ (* X Y)
              (+ 4 (- Z 23)))
Parent 2: (- (* 17 (+ 2 X))
              (* (- (* 2 Z) 1)
                 (+ 14 (/ Y X))))
```

```
Child 1: (+ (- (* 2 Z) 1)
            (+ 4 (- Z 23)))
Child 2: (- (* 17 (+ 2 X))
            (* (* X Y)
               (+ 14 (/ Y X))))
```

Symbolic Regression

A simple example

Given a set of data points, evolve a program that produces y from x .

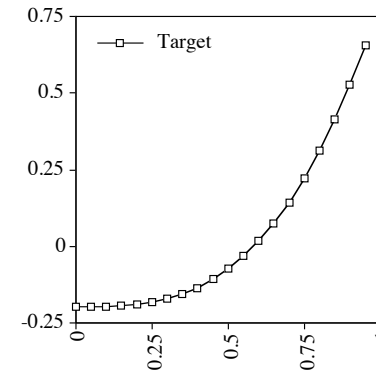
Primordial ooze: +, -, *, %, x, 0.1

Fitness = error (smaller is better)

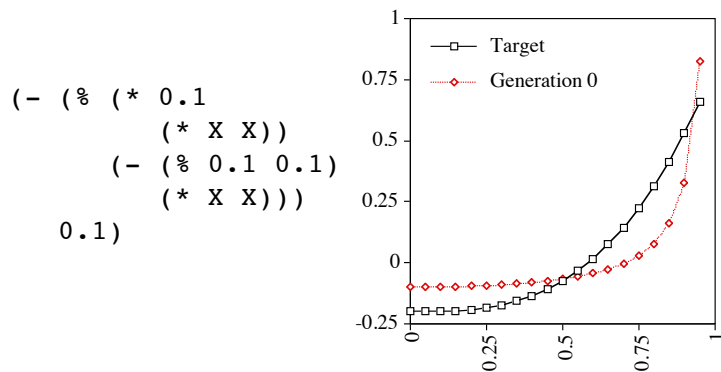
GP Parameters

Maximum number of Generations: 51
 Size of Population: 1000
 Maximum depth of new individuals: 6
 Maximum depth of new subtrees for mutants: 4
 Maximum depth of individuals after crossover: 17
 Fitness-proportionate reproduction fraction: 0.1
 Crossover at any point fraction: 0.3
 Crossover at function points fraction: 0.5
 Selection method: FITNESS-PROPORTIONATE
 Generation method: RAMPED-HALF-AND-HALF
 Randomizer seed: 1.2

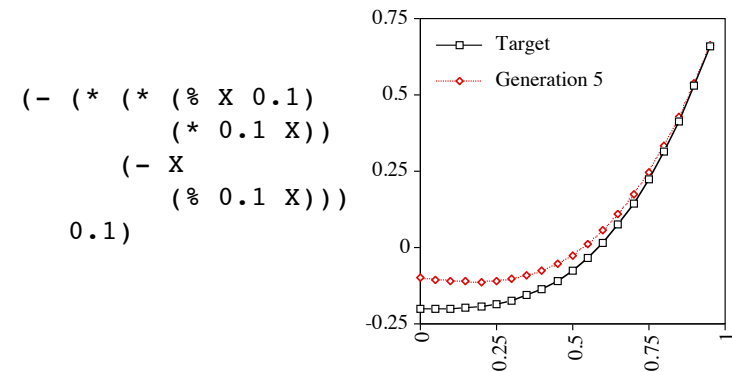
Evolving $y = x^3 - 0.2$



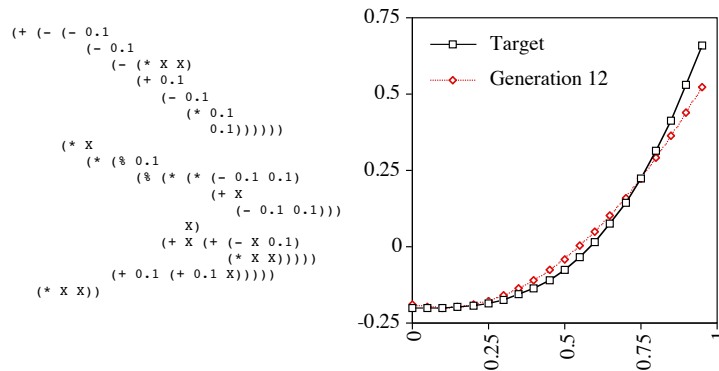
Best Program, Gen 0



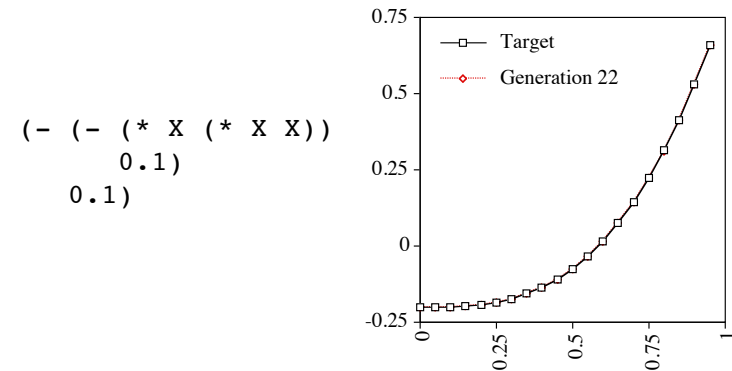
Best Program, Gen 5



Best Program, Gen 12



Best Program, Gen 22



Expressiveness

- Turing machine tables
- Lambda calculus expressions
- Register machine programs
- Partial recursive functions
- etc.

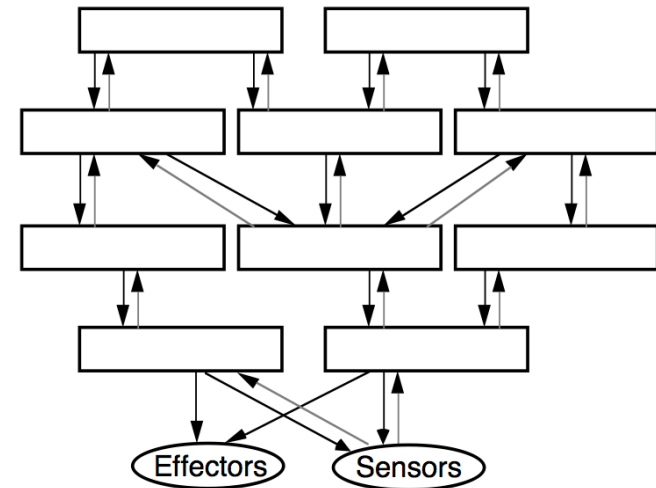
Evolvability

The fact that a computation *can* be expressed in a formalism does not imply that a correct expression can be produced in that formalism by a human programmer or by an evolutionary process.

Modularity

- Cars, airplanes, and other complex engineered artifacts...
 - Evolved biological organisms...
 - Large-scale software systems...
- ... are each composed of millions of specialized parts, chosen, in each case, from a portfolio of domain-specialized components and processes.

Modularity is Everywhere



Modularity in Software

- Pervasive and widely acknowledged to be essential
- Modules may be functions, procedures, methods, classes, data structures, interfaces, etc.
- Modularity measures include coupling, cohesion, encapsulation, composability, etc.

Data/Control Structure

- Data abstraction and organization
Data types, variables, name spaces, data structures, ...
- Control abstraction and organization
Conditionals, loops, modules, threads, ...

Structure via GP (I)

- Specialize GP techniques to directly support human programming language abstractions
- Strongly typed genetic programming
- Module acquisition/encapsulation systems
- Automatically defined functions
- Automatically defined macros
- Architecture altering operations

Evolving Modular Programs

With “automatically defined functions”

- All programs in the population have the same, pre-specified architecture
- Genetic operators respect that architecture
- Significant implementation costs
- Significant pre-specification
- Architecture-altering operations: more power and higher costs

ADMs

- Macros implement control structures
- ADMs can be implemented via small tweaks to any system that supports ADFs
- Similar pros and cons to ADFs, but provide additional expressive power

Control Structures (I)

Multiple evaluation

```
(defmacro do-twice (code)
  `(progn ,code ,code))

(do-twice (incf x))
```

Control Structures (2)

Conditional evaluation

```
(defmacro numeric-if (exp neg zero pos)
  `(if (< ,exp 0)
      ,neg
      (if (< 0 ,exp) ,pos ,zero)))

(numeric-if (foo) (bar) (baz) (bix))
```

Structure via GP (2)

- Specialize GP techniques to **indirectly** support human programming language abstractions
- Constrain genetic change, or repair after genetic change, to satisfy abstraction syntax
- Map from unstructured genomes to programs in languages that support abstraction (e.g. via grammars)

Structure via GP (3)

- Develop new program encodings, represented most generally as graphs
- Develop abstraction mechanisms for these representations
- Specialize GP techniques to directly or indirectly support abstraction in these new program encodings

Structure via GP (4)

- Evolve programs in a minimal-syntax language that nonetheless supports a full range of data and control abstractions
- For example: orchestrate data flows via stacks, not via syntax
- **Push**

Push

- Stack-based postfix language with one stack per type
- Types include: integer, float, Boolean, name, **code**, **exec**, vector, matrix, quantum gate, [add more as needed]
- Missing argument? NOOP
- Minimal syntax:
program \rightarrow instruction | literal | (program*)

Why Push?

- Highly expressive: data types, data structures, variables, conditionals, loops, recursion, modules, ...
- Elegant: minimal syntax and a simple, stack-based execution architecture
- Evolvable
- Extensible
- Supports several forms of meta-evolution

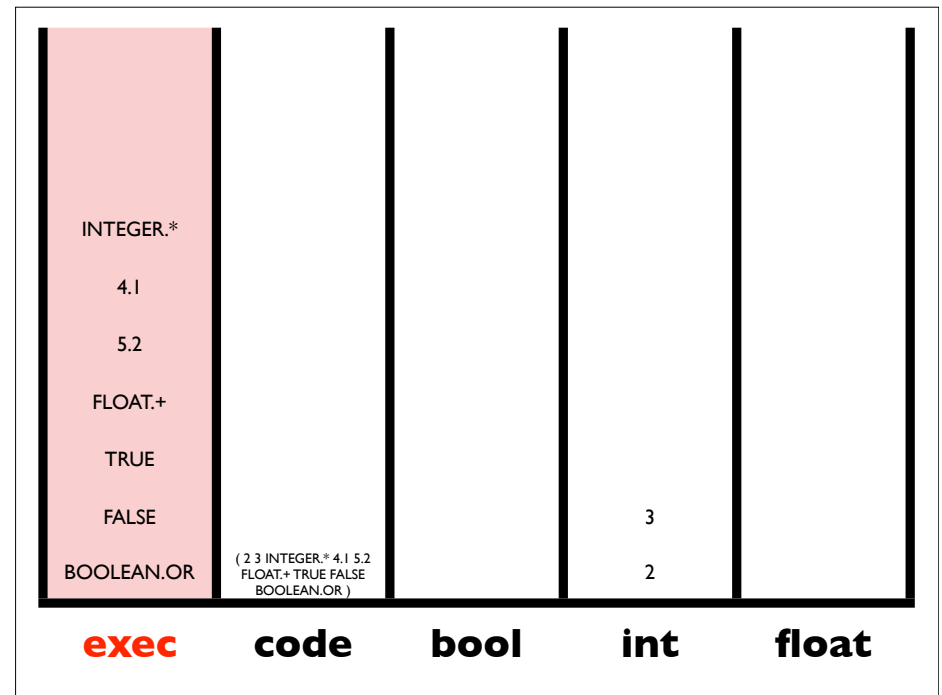
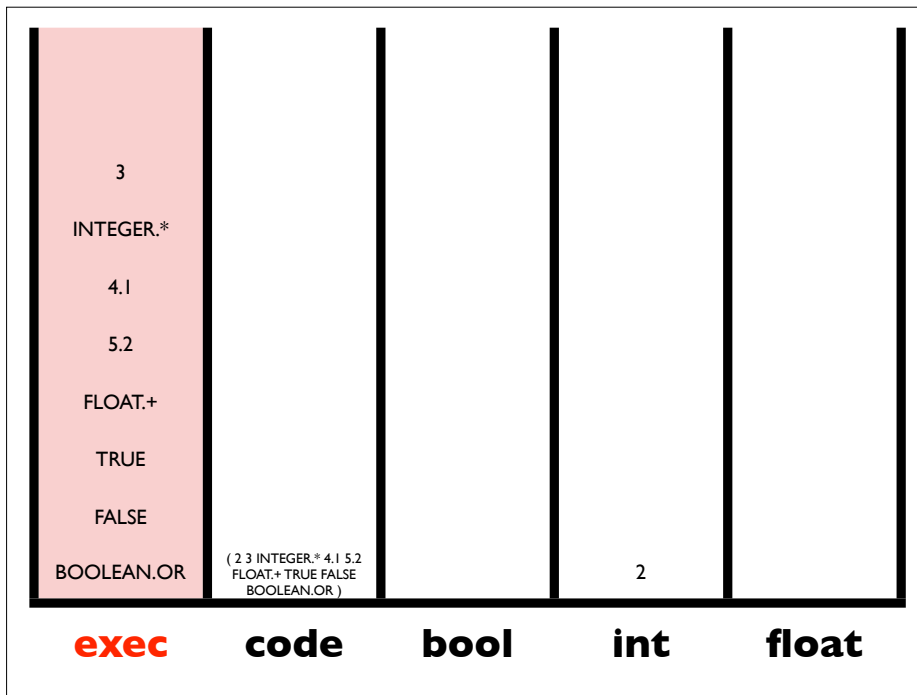
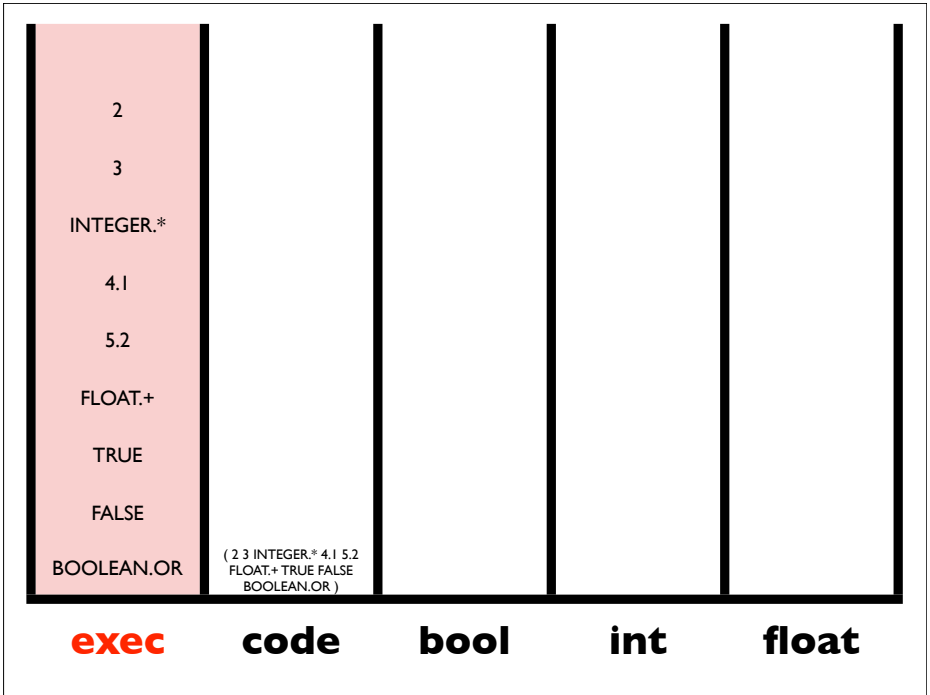
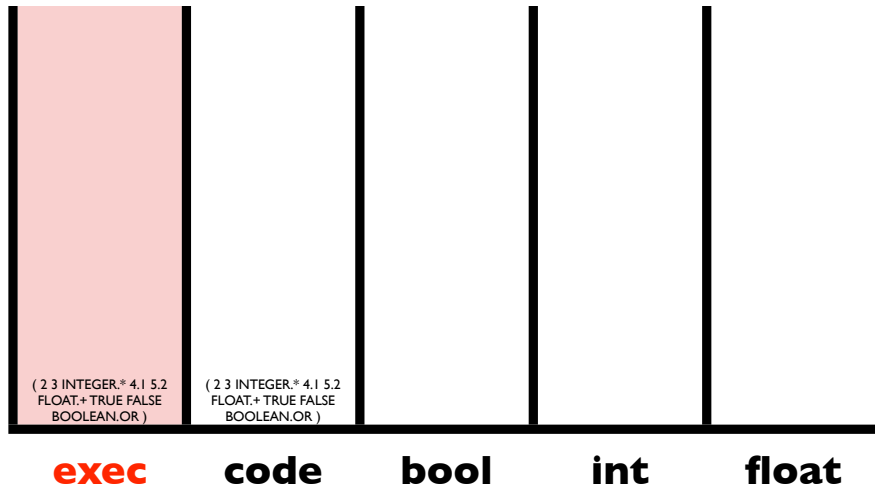
Sample Push Instructions

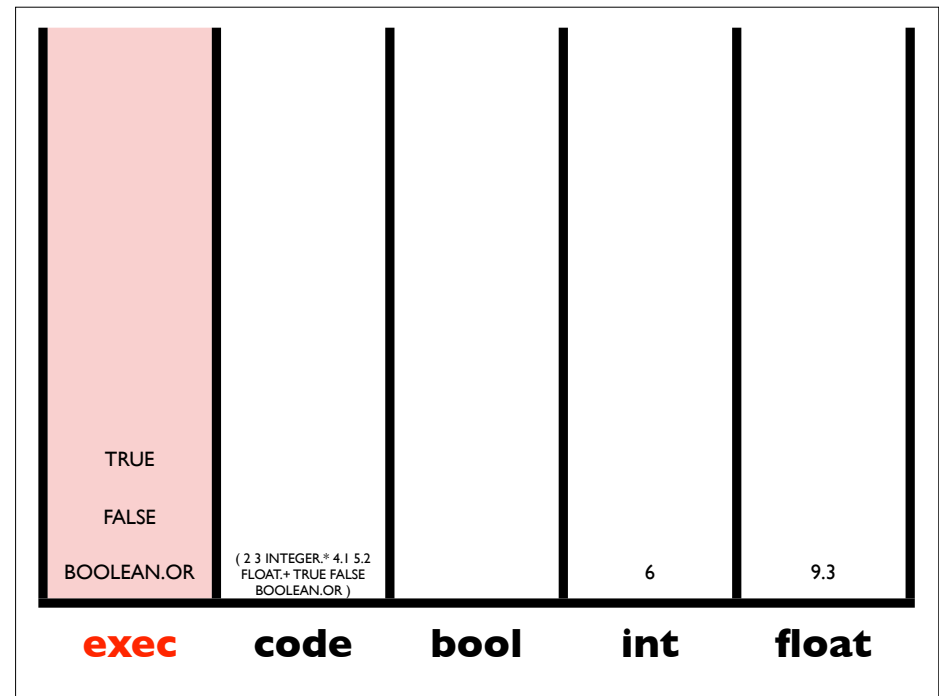
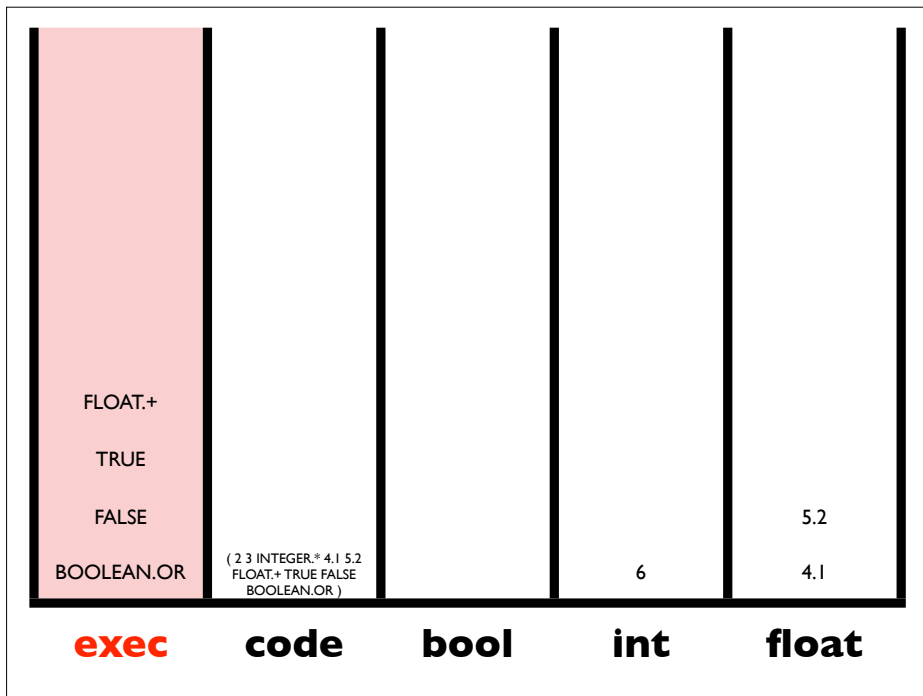
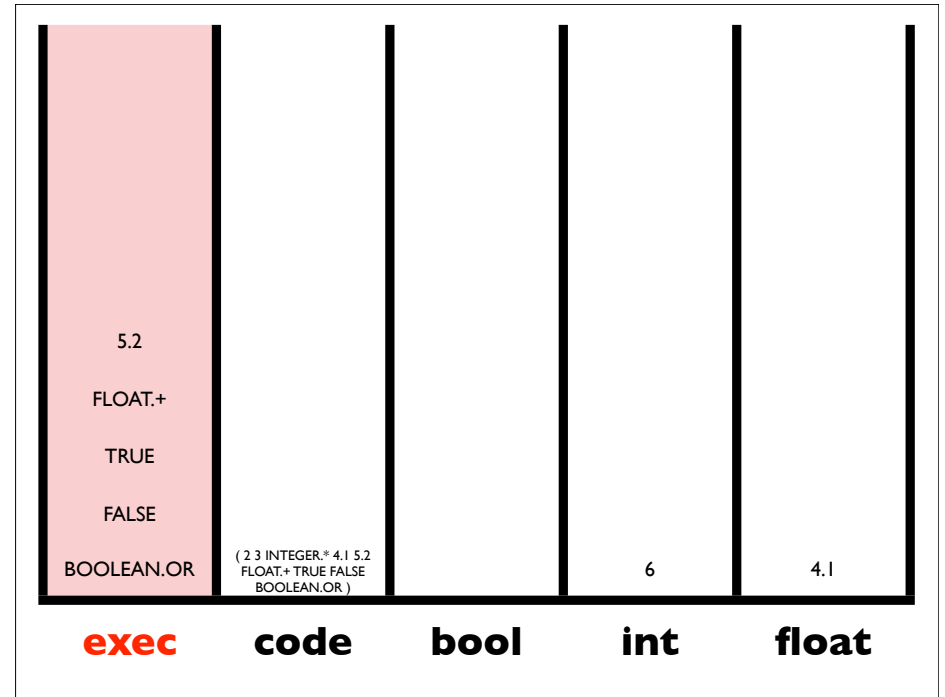
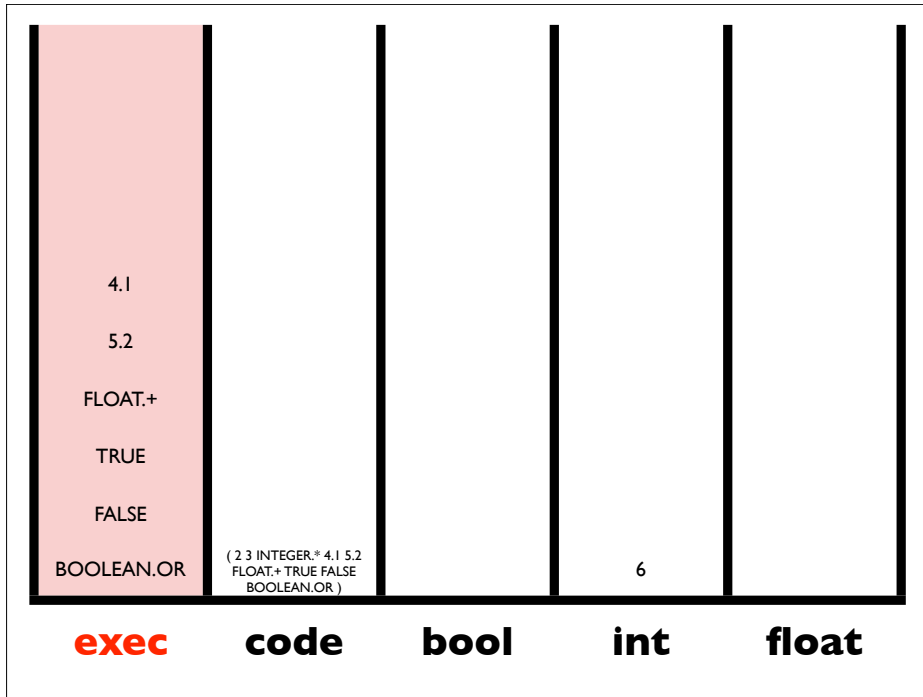
Stack manipulation instructions (all types)	POP, SWAP, YANK, DUP, STACKDEPTH, SHOVE, FLUSH, =
Math (INTEGER and FLOAT)	+, -, /, *, >, <, MIN, MAX
Logic (BOOLEAN)	AND, OR, NOT, FROMINTEGER
Code manipulation (CODE)	QUOTE, CAR, CDR, CONS, INSERT, LENGTH, LIST, MEMBER, NTH, EXTRACT
Control manipulation (CODE and EXEC)	DO*, DO*COUNT, DO*RANGE, DO*TIMES, IF

Push(3) Semantics

- To execute program P :
 1. Push P onto the EXEC stack.
 2. While the EXEC stack is not empty, pop and process the top element of the EXEC stack, E :
 - (a) If E is an instruction: execute E (accessing whatever stacks are required).
 - (b) If E is a literal: push E onto the appropriate stack.
 - (c) If E is a list: push each element of E onto the EXEC stack, in reverse order.

(2 3 INTEGER.* 4.1 5.2 FLOAT.+
TRUE FALSE BOOLEAN.OR)





FALSE				
BOOLEAN.OR	(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)	TRUE	6	9.3
exec	code	bool	int	float

		FALSE		
BOOLEAN.OR	(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)	TRUE	6	9.3
exec	code	bool	int	float

	(2 3 INTEGER.* 4.1 5.2 FLOAT.+ TRUE FALSE BOOLEAN.OR)	TRUE	6	9.3
exec	code	bool	int	float

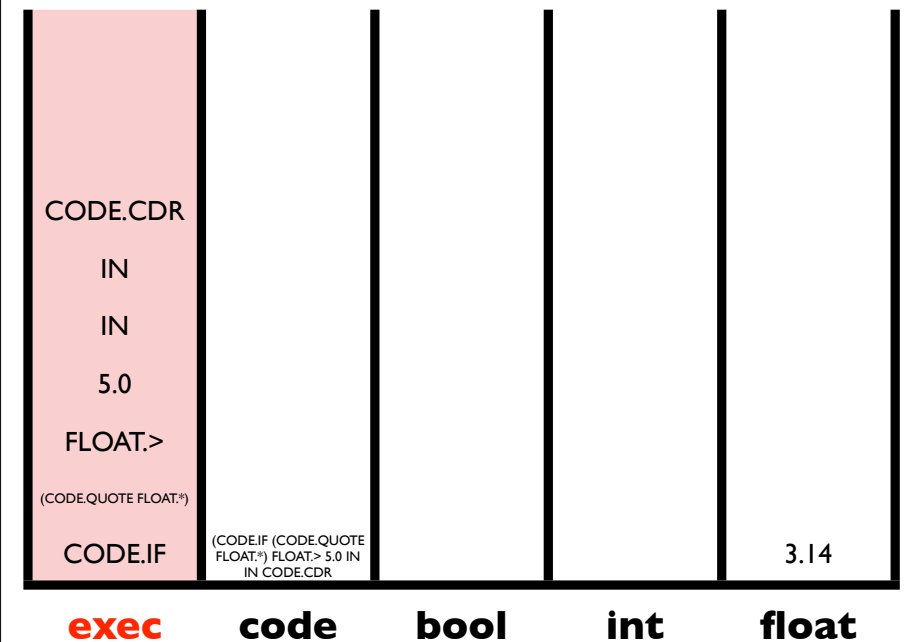
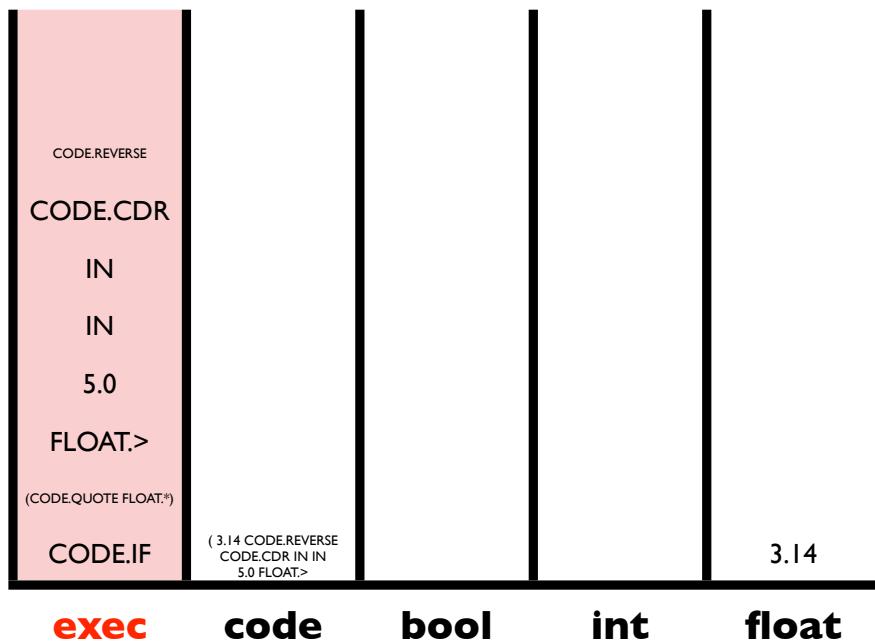
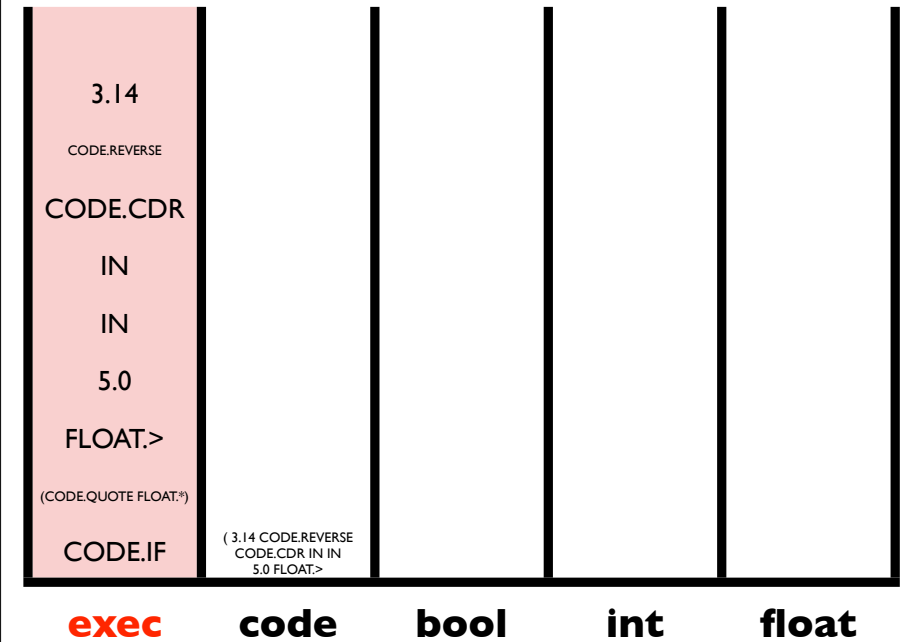
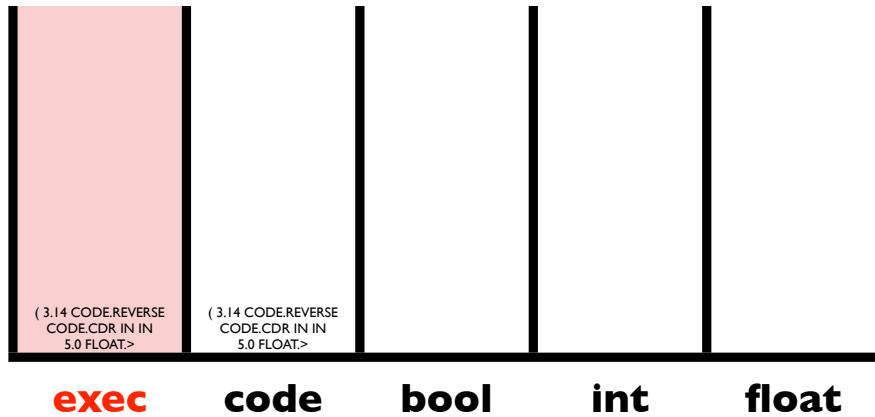
Same Results

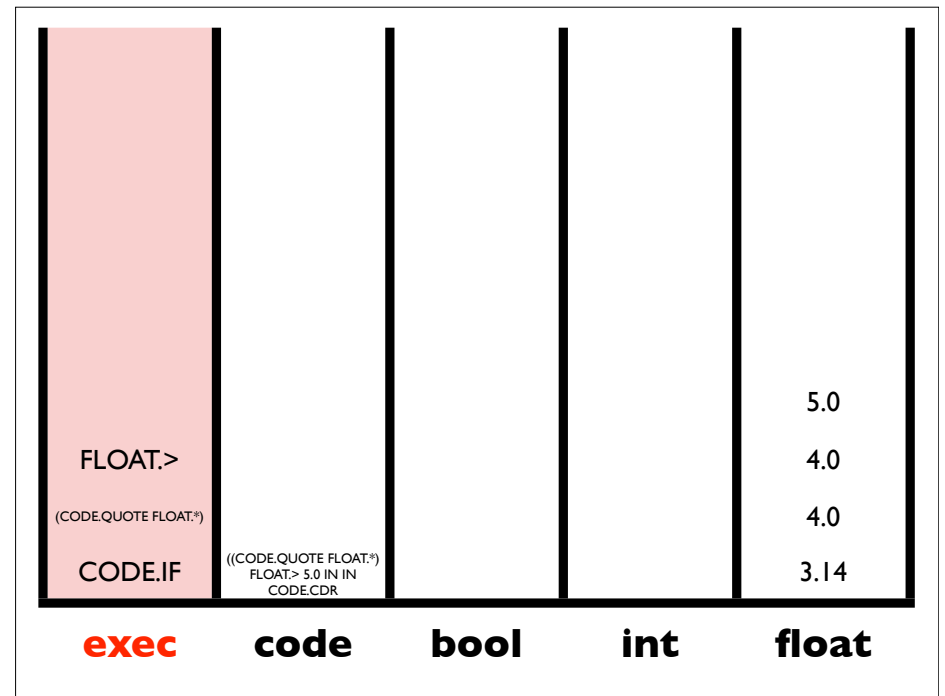
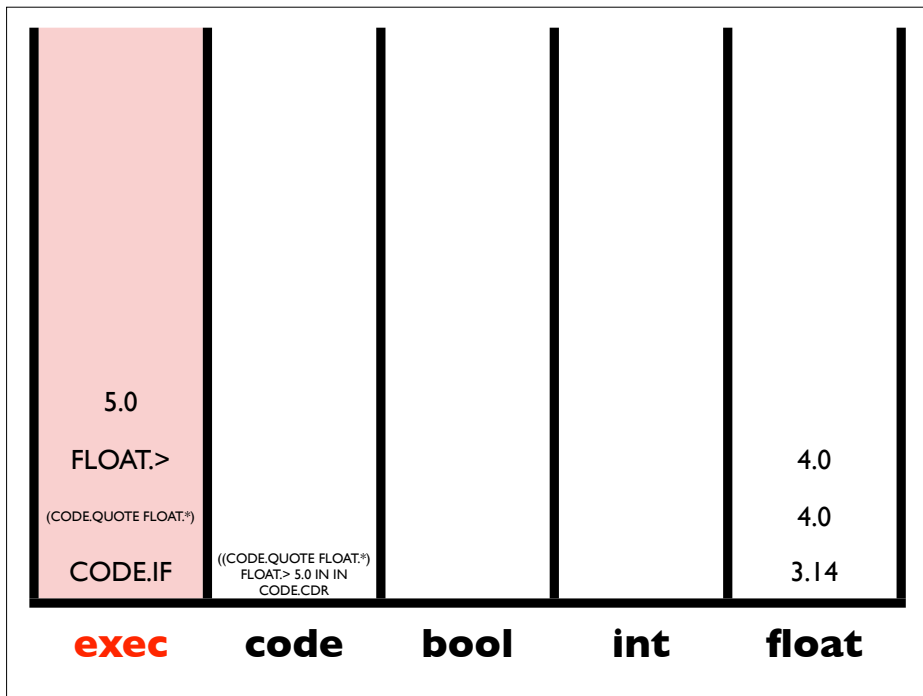
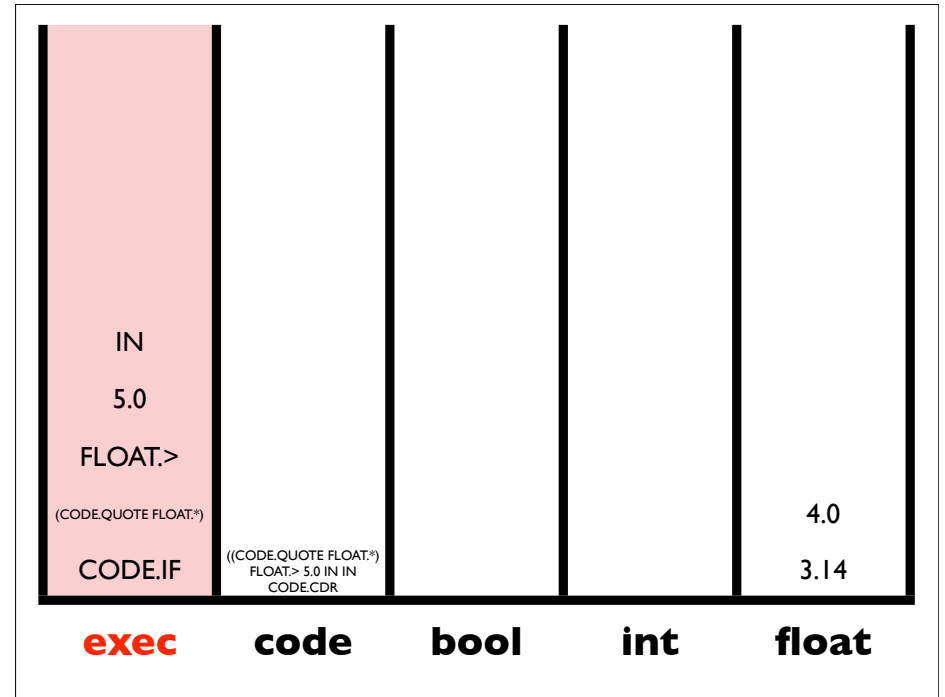
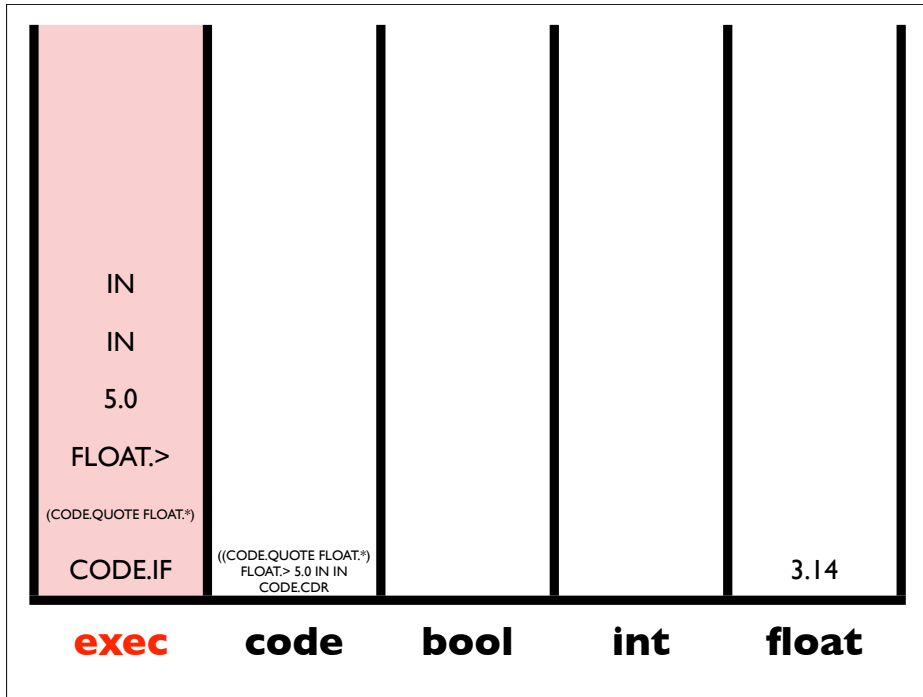
```
( 2 3 INTEGER.* 4.1 5.2 FLOAT.+  
  TRUE FALSE BOOLEAN.OR )
```

```
( 2 BOOLEAN.AND 4.1 TRUE INTEGER./ FALSE  
  3 5.2 BOOLEAN.OR INTEGER.* FLOAT.+ )
```

```
( 3.14 CODE.REVERSE CODE.CDR IN IN 5.0
FLOAT.> (CODE.QUOTE FLOAT.*) CODE.IF )
```

IN=4.0



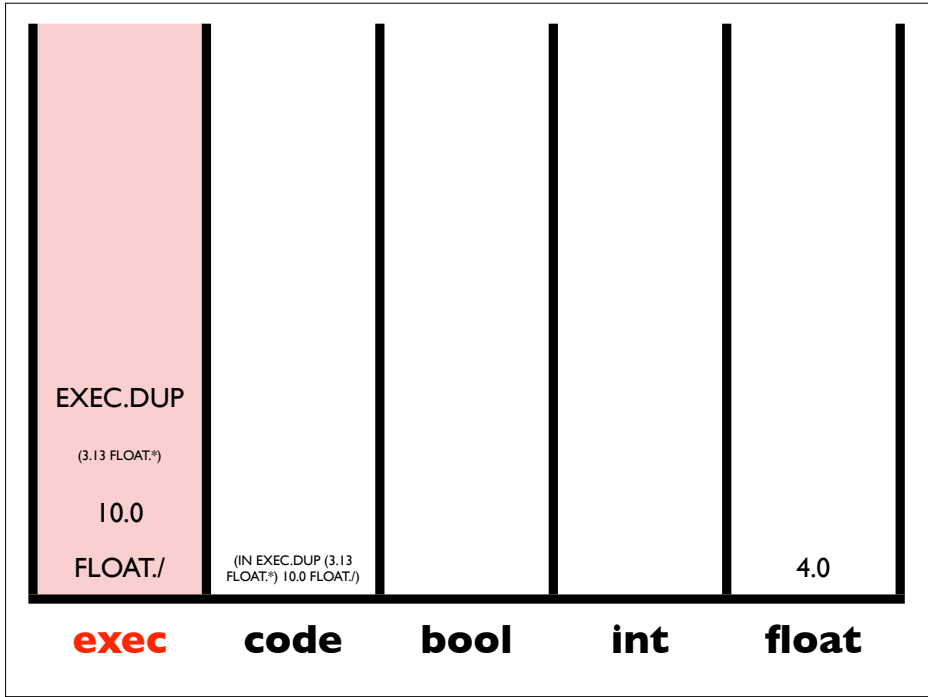
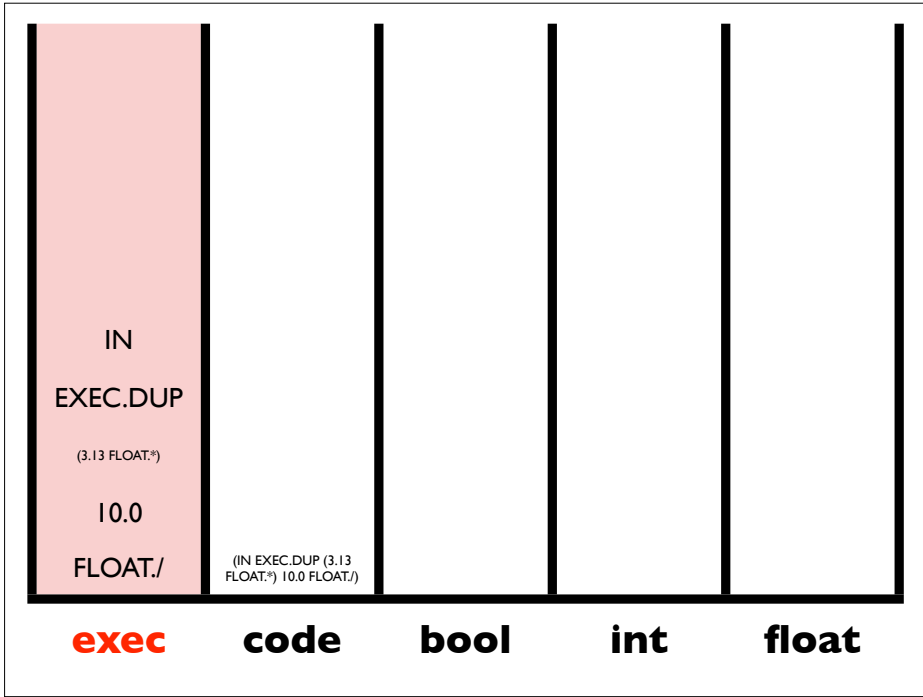
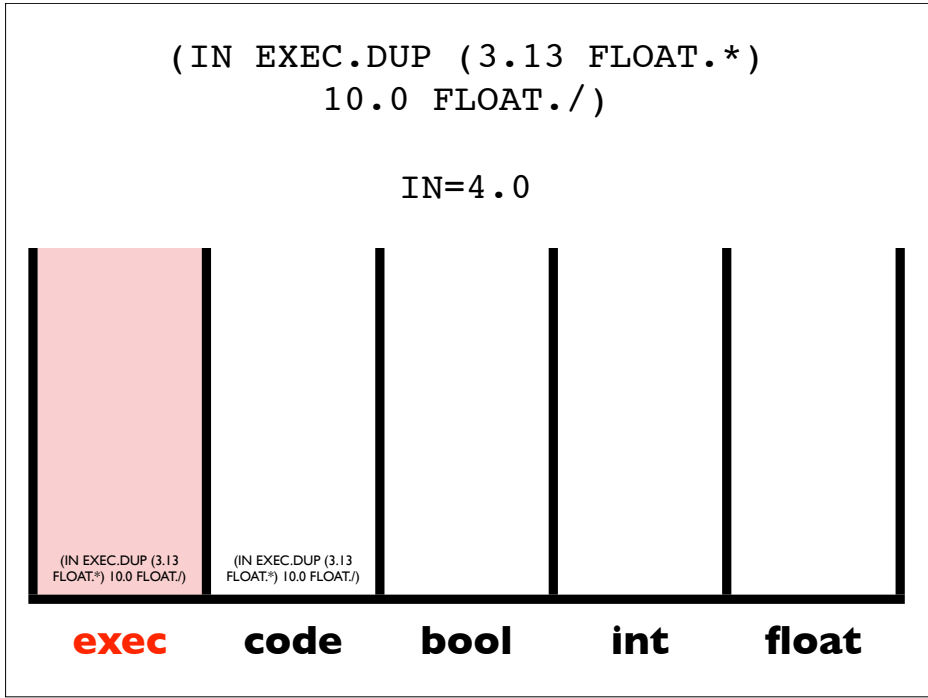
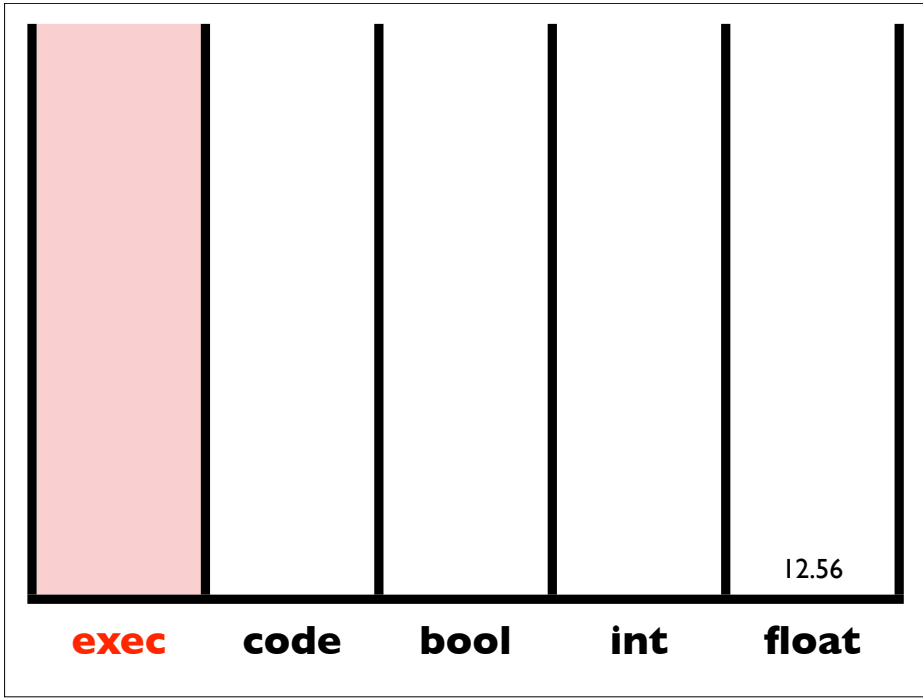


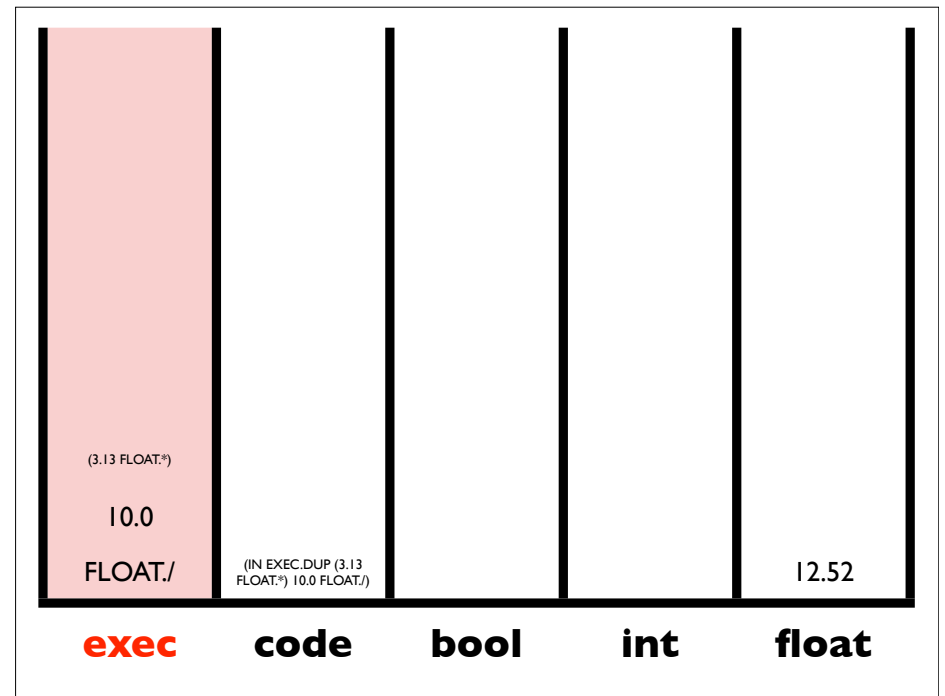
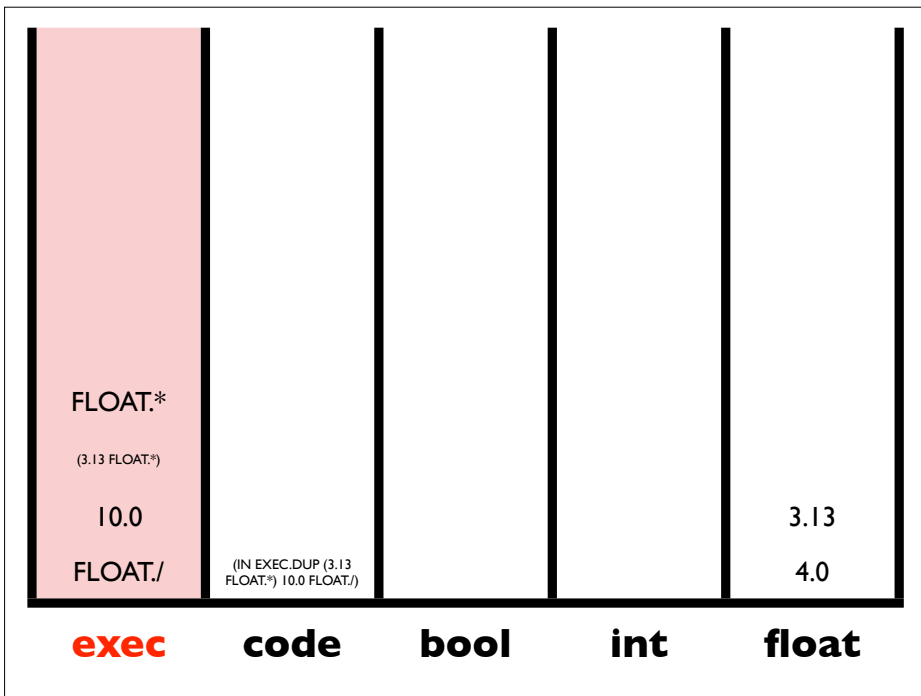
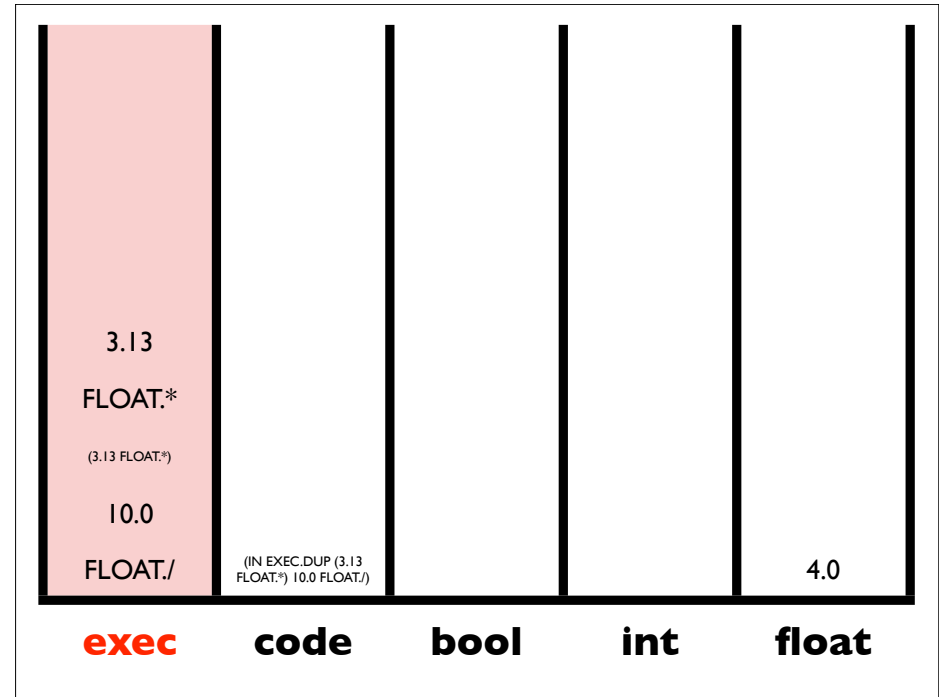
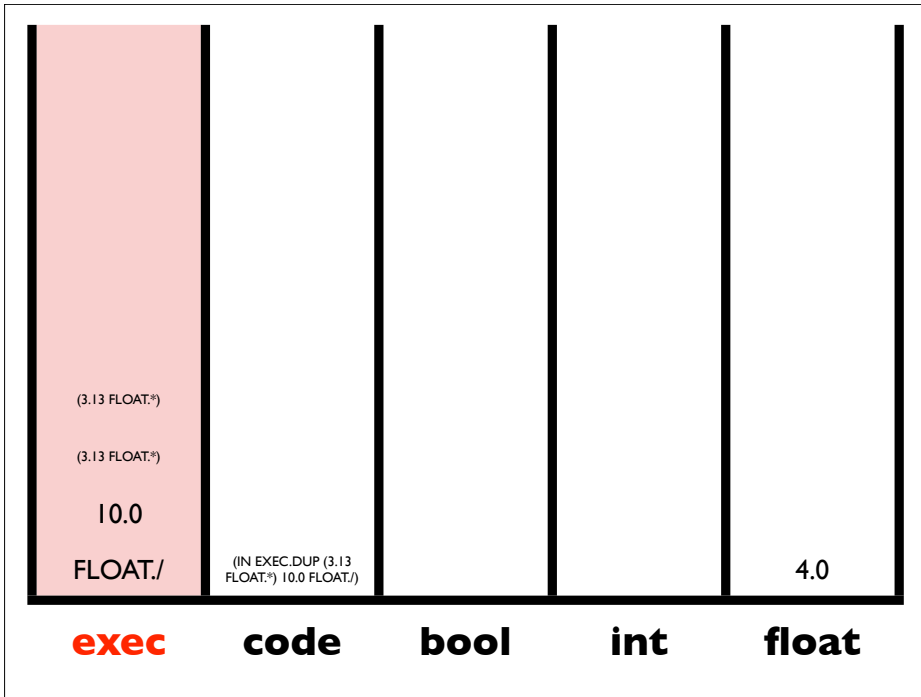
<div>(CODE.QUOTE FLOAT.?)</div> <div>CODE.IF</div>	<div>((CODE.QUOTE FLOAT.?) FLOAT.> 5.0 IN IN CODE.CDR)</div>	FALSE		<div>4.0</div> <div>3.14</div>
exec	code	bool	int	float

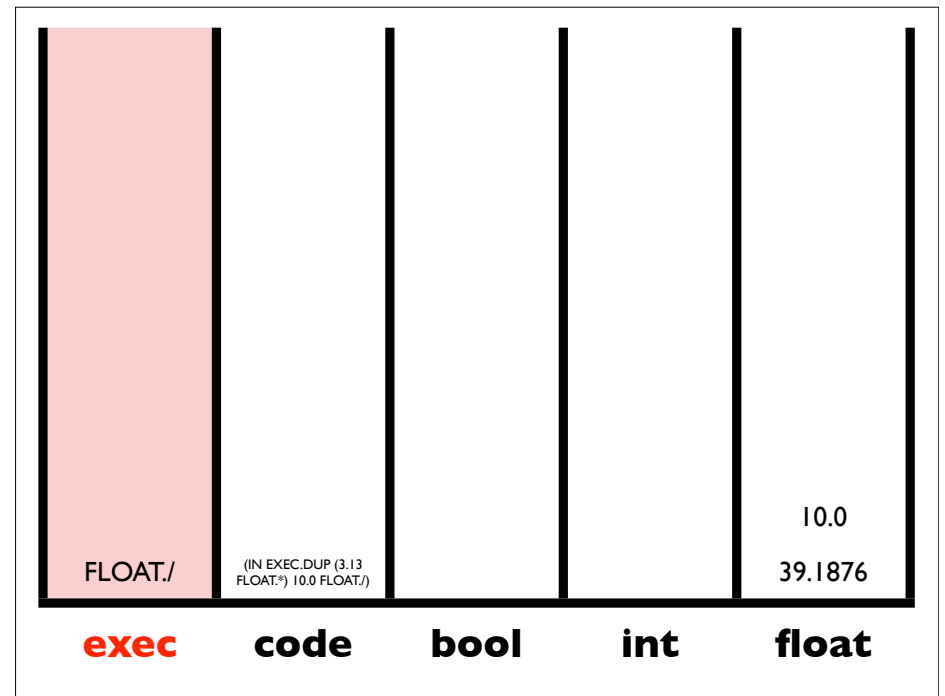
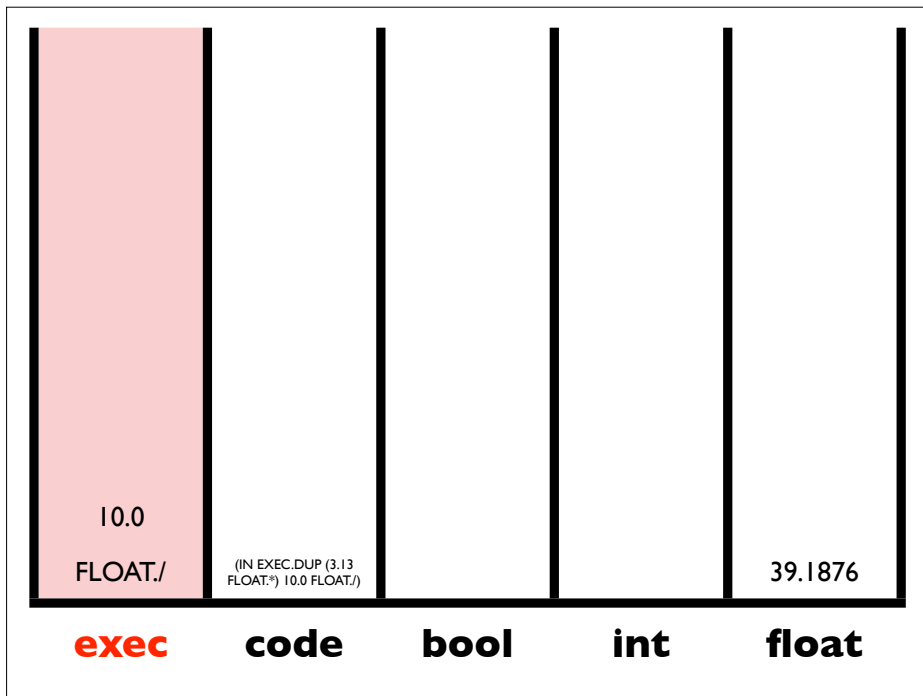
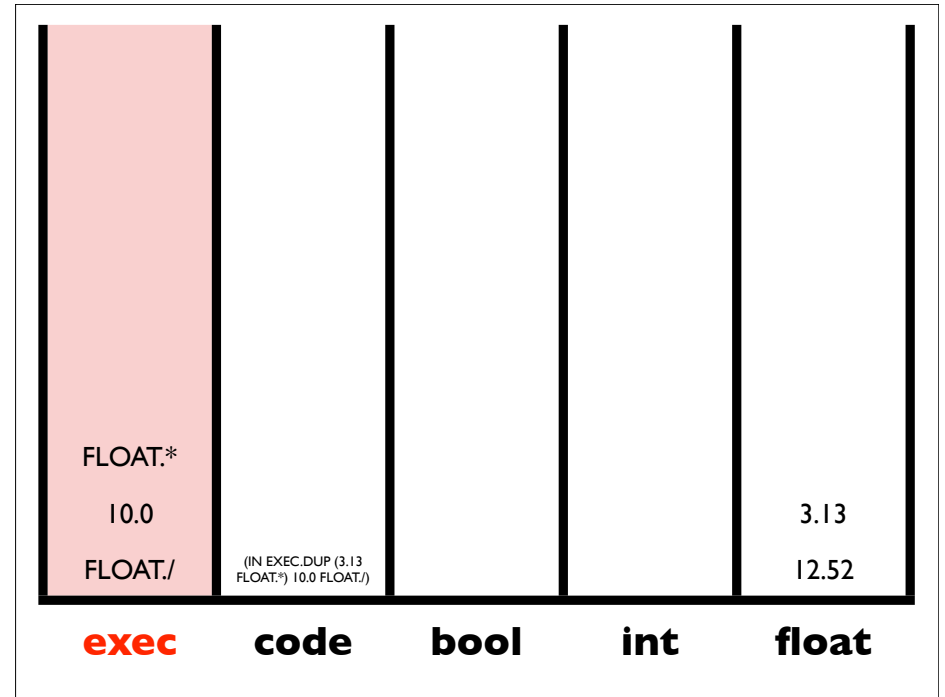
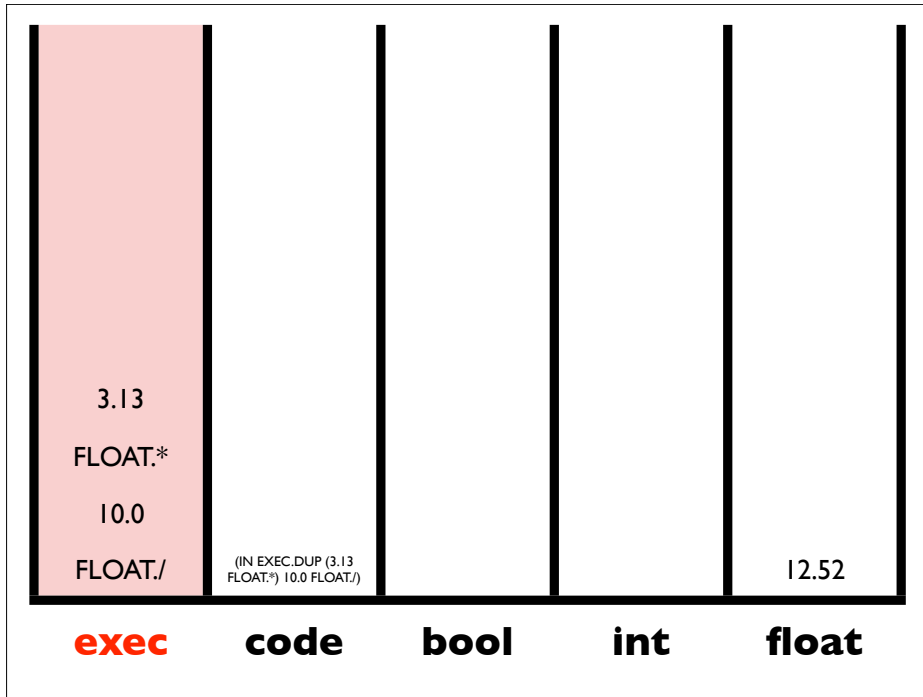
<div>CODE.QUOTE</div> <div>FLOAT.*</div> <div>CODE.IF</div>	<div>((CODE.QUOTE FLOAT.?) FLOAT.> 5.0 IN IN CODE.CDR)</div>	FALSE		<div>4.0</div> <div>3.14</div>
exec	code	bool	int	float

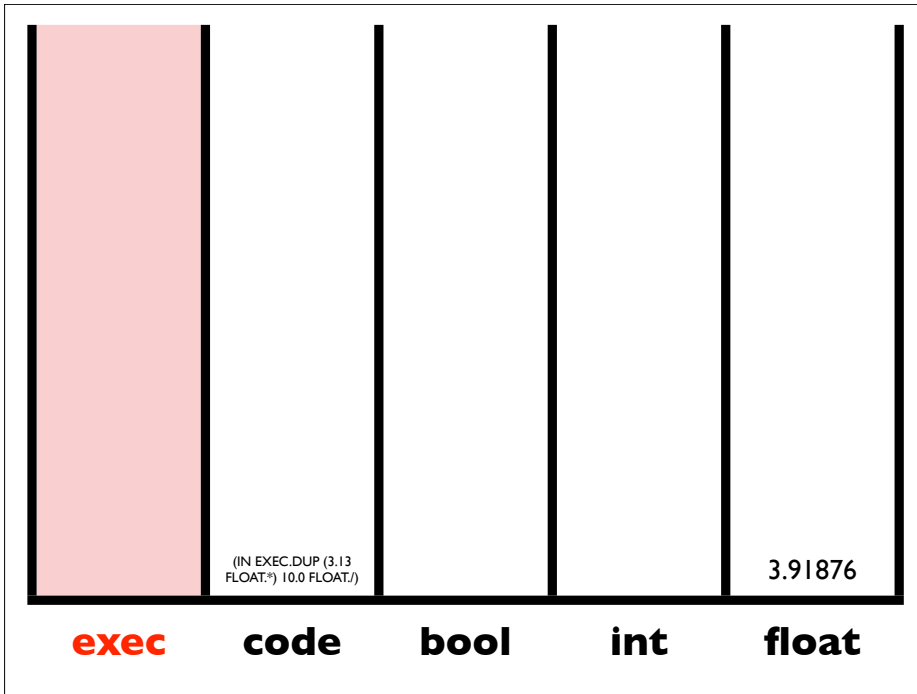
<div>CODE.IF</div>	<div>FLOAT.*</div> <div>((CODE.QUOTE FLOAT.?) FLOAT.> 5.0 IN IN CODE.CDR)</div>	FALSE		<div>4.0</div> <div>3.14</div>
exec	code	bool	int	float

<div>FLOAT.*</div>				<div>4.0</div> <div>3.14</div>
exec	code	bool	int	float









The Odd Problem

- Integer input
- Boolean output
- Was the input odd?
- `((code.nth) code.atom)`

Combinators

- Standard *K*, *S*, and *Y* combinators:
 - `EXEC.K` removes the second item from the `EXEC` stack.
 - `EXEC.S` pops three items (call them *A*, *B*, and *C*) and then pushes `(B C)`, *C*, and then *A*.
 - `EXEC.Y` inserts `(EXEC.Y T)` under the top item (*T*).
- A *Y*-based “while” loop:


```
( EXEC.Y
  ( <BODY/CONDITION> EXEC.IF
  ( ) EXEC.POP ) )
```

Iterators

`CODE.DO*TIMES`, `CODE.DO*COUNT`,
`CODE.DO*RANGE`

`EXEC.DO*TIMES`, `EXEC.DO*COUNT`,
`EXEC.DO*RANGE`

Additional forms of iteration are supported through code manipulation (e.g. via `CODE.DUP CODE.APPEND CODE.DO`)

Named Subroutines

```
( TIMES2 EXEC.DEFINE ( 2 INTEGER.* ) )
```

Auto-simplification

Loop:

Make it randomly simpler

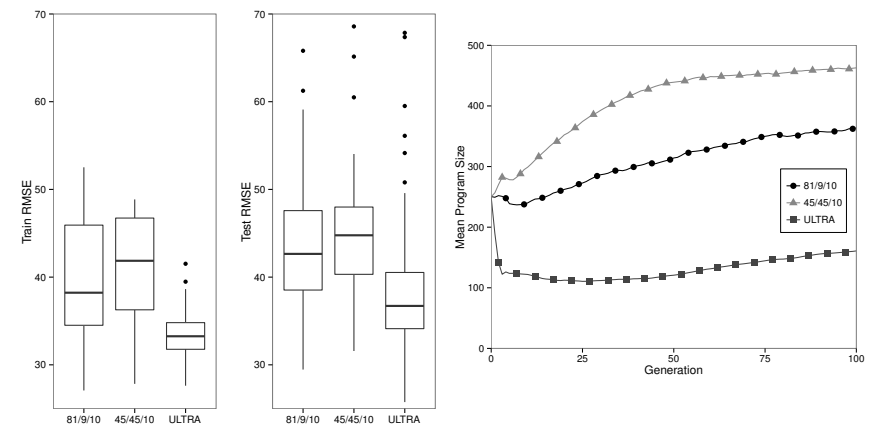
If it's as good or better: keep it

Otherwise: revert

The ULTRA Operator

- Uniform Linear Transformation with Repair and Alternation
- Linearize 2 parents, treating “(” and “)” as ordinary tokens
- Start at the beginning of one parent and copy tokens to the child, switching parents stochastically (according to the *alternation rate*, and subject to an *alignment deviation*)
- Post-process with uniform mutation (according to a *mutation rate*) and repair

ULTRA on the bioavailability problem



Problems Solved by PushGP in the GECCO-2005 Paper on Push3

- Reversing a list
- Factorial (many algorithms)
- Fibonacci (many algorithms)
- Parity (any size input)
- Exponentiation
- Sorting

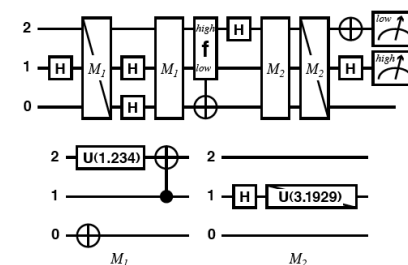
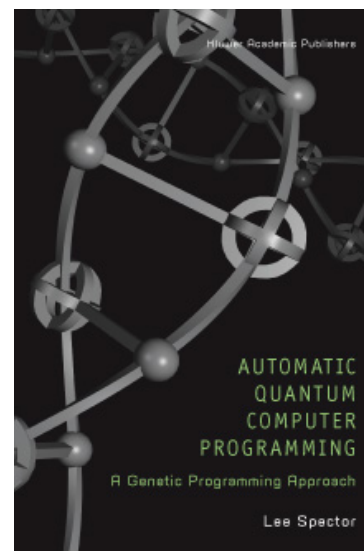


Figure 8.7. A gate array diagram for an evolved version of Grover's database search algorithm for a 4-item database. The full gate array is shown at the top, with M_1 and M_2 standing for the smaller gate arrays shown at the bottom. A diagonal line through a gate symbol indicates that the matrix for the gate is transposed. The "f" gate is the oracle.

**Humies 2004
GOLD MEDAL**

Genetic Programming for Finite Algebras

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**Humies 2008
GOLD MEDAL**

Autoconstructive Evolution

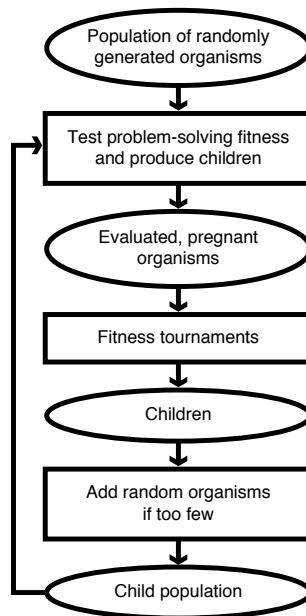
- Individuals make their own children
- Agents thereby control their own mutation rates, sexuality, and reproductive timing
- The machinery of reproduction and diversification (i.e., the machinery of evolution) evolves
- Radical self-adaptation

Related Work

- MetaGP: but (1) programs and reproductive strategies dissociated and (2) generally restricted reproductive strategies
- ALife systems such as Tierra, Avida, SeMar: but (1) hand-crafted ancestors, (2) reliance on cosmic ray mutation, and (3) weak problem solving
- Evolved self-reproduction: but generally exact reproduction, non-improving (exception: Koza, but very limited tools for problem solving *and* for construction of offspring)

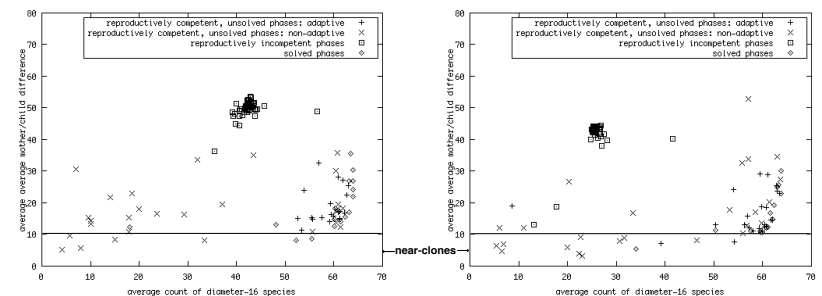
Pushpop

- A soup of evolving Push programs
- Reproductive procedures emerge ex nihilo:
 - No hand-designed “ancestor”
 - Children constructed by any computable process
 - No externally applied mutation procedure or rate
 - Exact clones are prohibited, but near-clones are permitted.
- Selection for problem-solving performance



Species vs. Mother/Child Differences

Note distribution of “+” points: adaptive populations have many species and mother/daughter differences in a relatively high, narrow range (above near-clone levels).



Runs including sexual instructions

Runs without sexual instructions

Pushpop Results

- In adaptive populations:
 - Species are more numerous
 - Diversification processes are more reliable
- Selection can promote diversity
- Provides a possible explanation for the evolution of diversifying reproductive systems

SwarmEvolve 2.0

- Behavior (**including reproduction**) controlled by evolved Push programs
- Color, color-based agent discrimination controlled by agents
- Energy conservation
- Facilities for communication, energy sharing
- Ample user feedback (e.g. diversity metrics, agent energy determines size)

Instruction(s)	Description
DUP, POP, SWAP, REP, =, NOOP, PULL, PULLDUP, CONVERT, CAR, CDR, QUOTE, ATOM, NULL, NTH, +, *, /, >, <, NOT, AND, NAND OR, NOR, DO*, IF	Standard Push instructions (See [11])
VectorX, VectorY, VectorZ, VPlus, VMinus, VTimes, VDivide, VectorLength, Make-Vector	Vector access, construction, and manipulation
RandI, RandF, RandV, RandC	Random number, vector, and code generators
SetServoSetpoint, SetServoGain, Servo	Servo-based persistent memory
Mutate, Crossover	Stochastic list manipulation (parameters from stacks)
Spawn	Produce a child with code from code stack
ToFood	Vector to energy source
FoodIntensity	Energy of energy source
MyAge, MyEnergy, MyHue, MyVelocity, MyLocation, MyProgram	Information about self
ToFriend, FriendAge, FriendEnergy, FriendHue, FriendVelocity, FriendLocation, FriendProgram	Information about closest agent of similar hue
ToOther, OtherAge, OtherEnergy, OtherHue, OtherVelocity, OtherLocation, OtherProgram	Information about closest agent of non-similar hue
FeedFriend, FeedOther	Transfer energy to closest agent of indicated category

SwarmEvolve 2.0



Winner, Best Paper Award, AAAA Track, GECCO-2003

AutoPush

- Goals:
 - Superior problem-solving performance
 - Tractable analysis
- Push3
- Asexual
- Children produced on demand (not during fitness testing)
- Constraints on selection and birth
- Still work in progress

Evolving Modular Programs

With Code Manipulation

- Transform code as data on “code” stack
- Execute transformed code with `code.do`, etc.
- Simple uses of modules can be evolved easily
- Does not scale well to large/complex systems

Evolving Modular Programs

With Execution Stack Manipulation

- Code queued for execution is stored on an “execution stack”
- Allow programs to duplicate and manipulate code that on the stack
- Example: `(3 exec.dup (1 integer.+))`
- More parsimonious, but same scaling issue

Evolving Modular Programs

With Named Modules

- Uses Push’s “name” stack
- Example:

```
(plus1 exec.define (1 integer.+))  
...  
plus1
```
- Coordinating definitions/references is tricky
and this never arises in evolution!

Module Identity

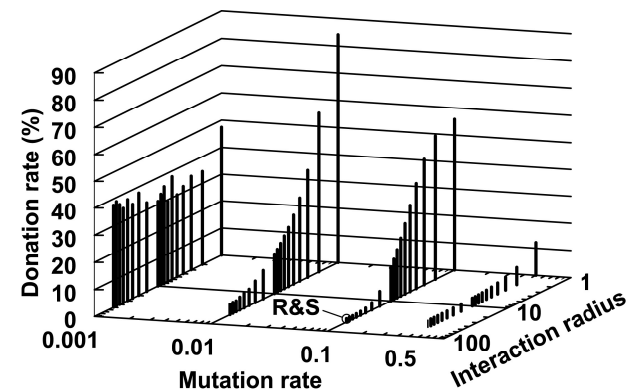
- How are modules recognized by other components of a system?
- Where do module identities come from?
- How can module identity co-evolve with modular architecture?

Holland's Tags

- Initially arbitrary identifiers that come to have meaning over time
- Matches may be inexact
- Appear to be present in some form in many different kinds of complex adaptive systems
- Examples range from immune systems to armies on a battlefield
- A general tool for the support of emergent complexity

Tag-Based Altruism

- Individuals have tags and tag-difference tolerances
- Donate when $\Delta\text{tags} \leq \text{tolerance}$
- Riolo et al. (*Nature*, 2001) showed that tag-based altruism can evolve; Roberts & Sherratt (*Nature*, 2002) claimed it would not evolve under more realistic conditions



Spector, L., and Klein, J. Genetic stability and territorial structure facilitate the evolution of tag-mediated altruism. In *Artificial Life*.

Evolving Modular Programs

With tags

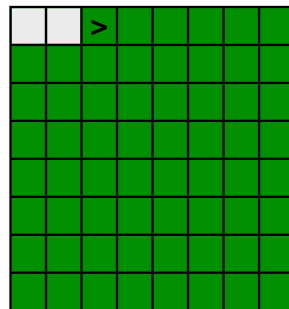
- Include instructions that tag code (modules)
- Include instructions that recall and execute modules by *closest matching tag*
- If a single module has been tagged then all tag references will recall modules
- The number of tagged modules can grow incrementally over evolutionary time
- **Expressive and evolvable**

Tags in Push

- Tags are integers embedded in instruction names
- Instructions like `tag.exec.123` tag values
- Instructions like `tagged.456` recall values by *closest matching tag*
- If a single value has been tagged then all tag references will recall (and execute) values
- The number of tagged values can grow incrementally over evolutionary time

Lawnmower Problem

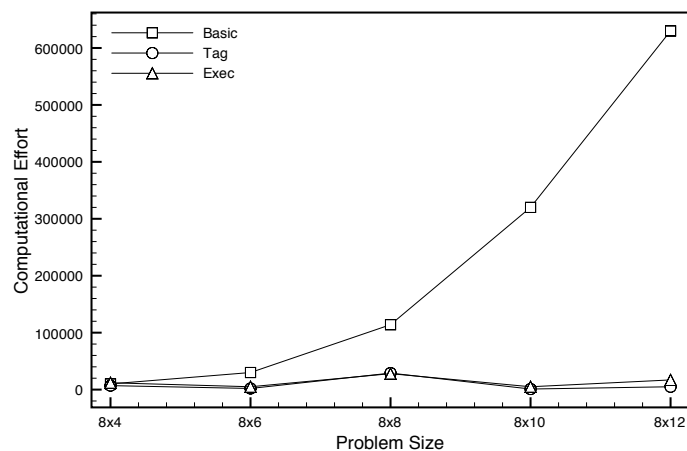
- Used by Koza to demonstrate utility of ADFs for scaling GP up to larger problems



Lawnmower Instructions

Condition	Instructions
Basic	left, mow, v8a, frog, \mathcal{R}_{v8}
Tag	left, mow, v8a, frog, \mathcal{R}_{v8} , tag.exec.[1000], tagged.[1000]
Exec	left, mow, v8a, frog, \mathcal{R}_{v8} , exec.dup, exec.pop, exec.rot, exec.swap, exec.k, exec.s, exec.y

Lawnmower Effort

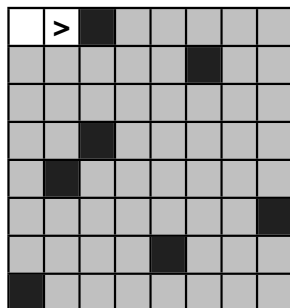


Lawnmower Effort

	problem size				
	8x4	8x6	8x8	8x10	8x12
instr set					
basic	10000	30000	114000	320000	630000
tag	7000	2000	29000	<1000	5000
exec	12000	5000	28000	5000	17000

Dirt-Sensing, Obstacle-Avoiding Robot Problem

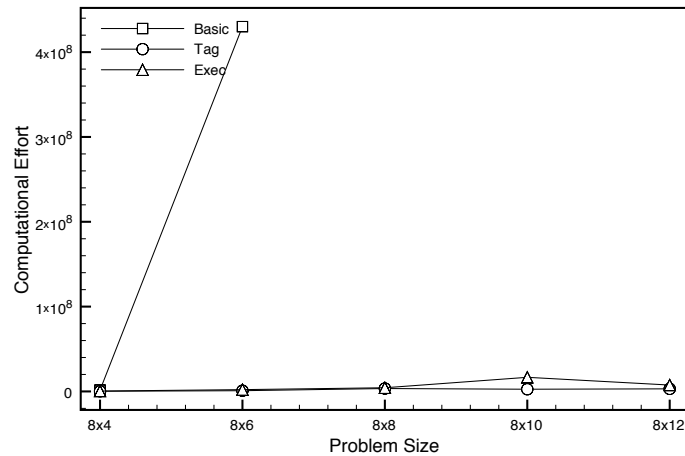
Like the lawnmower problem but harder and less uniform



DSOAR Instructions

Condition	Instructions
Basic	if-dirty, if-obstacle, left, mop, v8a, frog, \mathcal{R}_{v8}
Tag	if-dirty, if-obstacle, left, mop, v8a, frog, \mathcal{R}_{v8} , tag.exec.[1000], tagged.[1000]
Exec	if-dirty, if-obstacle, left, mop, v8a, frog, \mathcal{R}_{v8} , exec.dup, exec.pop, exec.rot, exec.swap, exec.k, exec.s, exec.y

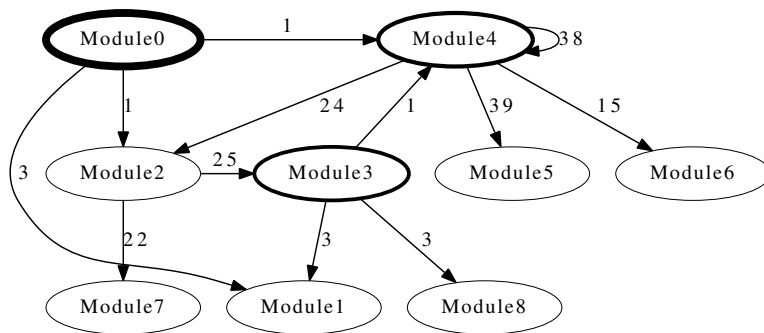
DSOAR Effort



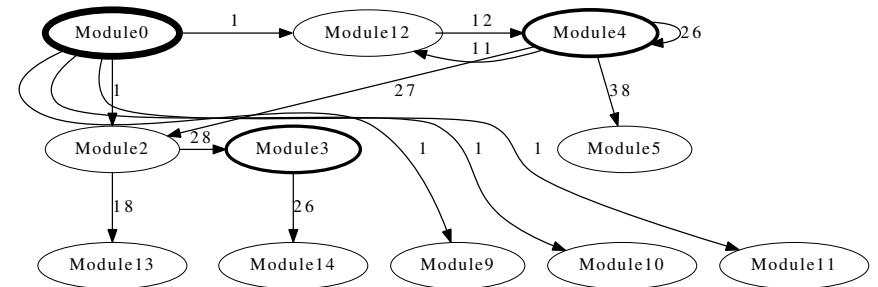
DSOAR Effort

	problem size				
	8x4	8x6	8x8	8x10	8x12
instr set					
basic	1584000	430083000	inf	inf	inf
tag	216000	864000	3420000	2599000	3051000
exec	450000	2125000	4332000	16644000	7524000

Evolved DSOAR Architecture (in one environment)



Evolved DSOAR Architecture (in another environment)



Tags in Trees

- Example:

```
(progn (tag.123 (+ a b))  
      (+ tagged.034 tagged.108))
```
- Must do something about endless recursion
- Must do something about return values of tagging operations and references prior to tagging
- Non-trivial to support arguments in a general way
- Utility not clear from experiments conducted to date

Expressiveness and Assessment

- Expressive languages ease representation of programs that over-fit training sets
- Expressive languages ease representation of programs that work only on subsets of training sets
- Lexicase selection may help: Select parents by starting with a pool of candidates and then filtering by performance on individual fitness cases, considered one at a time

Future Work

- Expression of variable scope and local environments (implemented in Push, but not yet studied systematically)
- Expression of concurrency, parallelism, and time-based structures
- Applications for which expressiveness is likely to be essential, e.g. complete software applications and programs for agents in complex, dynamic, heterogeneous environments

Conclusions

- GP in expressive languages may allow for the evolution of complex software
- Minimal-syntax languages can be expressive, and GP systems that evolve programs in such languages can be simple
- Push is expressive, evolvable, successful, and extensible
- Tags appear to allow for the evolvable expression of program modularity

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