A Statistical Study of Discrete Differential Evolution Approaches for the Capacitated Vehicle Routing Problem.

Andre Luis Silva
Graduate Program in Electrical Engineering
Universidade Federal de Minas Gerais
Belo Horizonte, MG, Brazil
andreluismg@gmail.com

Jaime Arturo Ramírez
Dep. Engenharia Elétrica
Universidade Federal de Minas Gerais
Belo Horizonte, MG, Brazil
jramirez@ufmg.br

Felipe Campelo
Dep. Engenharia Elétrica
Universidade Federal de Minas Gerais
Belo Horizonte, MG, Brazil
fcampelo@ufmg.br

ABSTRACT

We examine the performance of four discrete differential evolution (DE) algorithms for the solution of capacitated vehicle routing problems (CVRPs). Twenty seven test instances are employed in the experimental analysis, with comparisons of final solution quality and time to convergence. The results indicate that two approaches presented significantly better results, but that all algorithms are still lacking in their ability to converge to the vicinity of the global optimum.

Categories and Subject Descriptors

G.1 [**Combinatorics**]: Combinatorial optimization—*VRP*; G.3 [**Probability and Statistics**]: Experimental design—*performance comparisons*

General Terms

Algorithms; Experimentation

Keywords

Differential evolution; CVRP; algorithm comparison.

1. INTRODUCTION

The vehicle routing problem (VRP) is one of the most important and widely studied within the context of combinatorial optimization, since it encompasses many applied problems in transportation logistics and other pickup and delivery tasks [1, 5]. VRPs can be defined in a number of ways, but tend to coalesce around the concept of defining economically interesting routes between depot(s) and customers, with different variants emerging from specific constraints added to the problem [7, 4]. Among these, the present work focuses on what is called the Capacitated Vehicle Routing Problem (CVRP) [13].

Techniques for obtaining solutions for the CVRP can be classified in two categories: exact methods and heuristics [13, 7]. In practical applications, the approach using heuristics is often an interesting one, since it can lead to solutions to problems otherwise unsolvable by exact methods. Among the heuristics, Differential Evolution (DE) approaches [12] have been used with varying degrees of success [10, 11, 6, 8].

GECCO'13 Companion, July 6–10, 2013, Amsterdam, The Netherlands.
ACM 978-1-4503-1964-5/13/07.

In this paper we present a statistical comparison of these four DE approaches for the solution of the CVRP, in terms of solution quality and time to convergence. A set of twenty-seven test instances is employed for the comparison. The results obtained indicate that even the two best approaches still require some improvement, and also suggest directions in which such improvements could be made.

2. EXPERIMENTAL COMPARISON

The statistical tests were performed independently for two quality metrics, namely *final solution quality* and *time until convergence*. Thirty randomized independent runs were performed for each test problem. A factorial design [9, 3] was used with a 95% confidence level, having algorithms and problems as experimental factors. Afterwards, a pairwise comparison of the algorithms was performed to pinpoint the significant differences. In all experiments, the algorithms are referred to by the following acronyms: **EDE** [10], **RDM** [11], **MDE** [6], **IDE** [8]. Twenty-seven problem instances with Euclidean distances were considered [2, 14], with 32 to 80 customers with fixed positions and randomly generated demands.

For the solution quality results, significant differences ($p < 2 \times 10^{-16}$) were detected for the algorithms, with the MDE approach significantly outperformed by the other three methods. Figure 1(a) presents the average performance for the algorithms on each problem.

The comparison of time until convergence was performed on the three algorithms that were detected as non-significantly different in terms of solution quality, i.e., RDM, EDE, and IDE. For each problem, the "worse" final value obtained from these three algorithms was recorded, and the time it took each method to reach this value was taken as the response variable. A permutational analysis of variance detected statistically significant differences among the algorithms ($p = 2.1 \times 10^{-12}$), with the EDE presenting significantly longer times than the other two approaches.Figure 1(b) presents the average times to convergence.

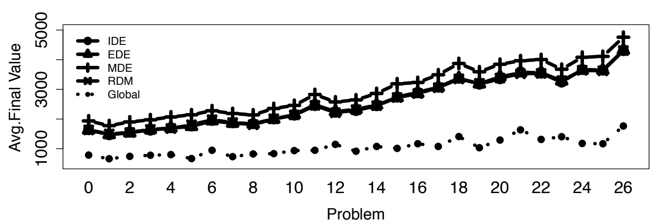
It is worth to notice that none of the DE approaches included in this comparison was able to approximate the true global optimum of the problems within the 1000 iterations defined as the stop criterion in this experiment, as shown in Fig. 1(a). Possible reasons for this performance include the need for longer execution times; the inadequacy of the discrete operators employed by the existing techniques to adequately explore the discrete search space of the CVRP;

or the inadequacy of the algorithm itself for this particular class of problems. The investigation of these three possibilities is the topic of ongoing research, dealing particularly with the tuning of techniques for the solution of the CVRP, the development of novel operators specific aimed at promoting an intelligent exploration of discrete search spaces, and the execution of more comprehensive comparisons of DE-based methods for vehicle routing problems against other heuristics and exact methods.

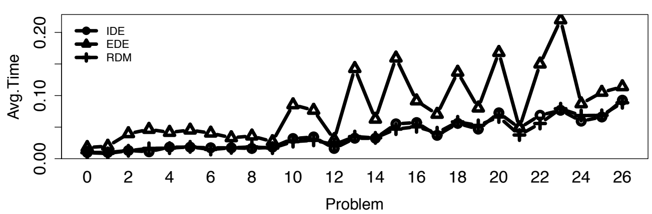
3. CONCLUSIONS

A statistical comparison of DE-based methods for the CVRP was performed, in terms of final solution quality and time to convergence. A factorial design with post-hoc pairwise comparisons was employed.

The results obtained indicated that the EDE and IDE approaches presented similar performance in terms of both metrics, and can be considered superior to the other methods. However, all techniques yielded results far from the known optima for all instances employed, which suggest that the current DE-based techniques for the solution of the CVRP are still not competitive in terms of the solutions returned. This indicates the need for improvement, which can be achieved by means of better tuning of the tools or the implementation of operators capable of a more efficient exploration of the search space, possibly on neighborhood structures more appropriate for the problem under consideration.



(a)



(b)

Figure 1: Comparison of the algorithms: (a) Average final solution of the algorithms for each problem; (b) Average time to stagnation (in seconds) of the algorithms on each problem.

4. ACKNOWLEDGMENTS

This work was supported by FAPEMIG(APQ-04611-10) and CNPq (306910/2006-3;475763/2012-2), Brazil.

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