Influence of Relaxed Dominance Criteria in Multiobjective Evolutionary Algorithms

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ABSTRACT

This work explores the influence of three different dominance criteria, namely the Pareto-, ϵ-, and cone ϵ-dominance, on the performance of multiobjective evolutionary algorithms. The approaches are incorporated into two different algorithms, which are then applied to the solution of twelve benchmark problems from the ZDT and DTLZ families. The final results of the algorithms are compared in terms of cardinality, convergence, and diversity of solutions using a statistical methodology designed to indicate whether any of the criteria provides significantly better results over the whole test set. The results obtained suggest that the cone ϵ-approach is an interesting alternative for finding well-distributed fronts without the loss of efficient solutions usually presented by the ϵ-dominance.

Categories and Subject Descriptors

G.1.6 [**Optimization**]: Global Optimization—*multiobjective optimization*; G.3 [**Probability and Statistics**]: Experimental design—*performance comparisons*

General Terms

Algorithms, Experimentation

Keywords

Evolutionary algorithms, multiobjective optimization

1. INTRODUCTION

When dealing with a multi-objective problem, one must cope with the fact that it may have more than one, possibly infinite, optimal solutions [5]. They are called *Pareto-optimal set* in the variable space, which map to the *Pareto front* in the objective space. Due to this reality, the task usually attributed to multi-objective algorithms is to find a finite set of points which correspond to a good representation of the Pareto front. A set with a "good representation" is usually translated to [11] a collection of points which are as close as possible to the front (a *convergence* idea), and well distributed to cover its extension (an exigence of *diversity*). Among the classes of methods to solve these problems are the Multi-Objective Evolutionary Algorithms (MOEA), which are able to approximate a whole set of non-dominated points within one single run [9].

In order to guide the population to promising regions, one must find a way to compare each solution. This is accomplished by the so called *dominance criteria*. It may be reasonable to suspect that different dominance criteria may lead to different population results. This is the purpose of this work: to study the influence of dominance criteria in the results of MOEAs. The design of the experiments is described in the following.

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2. EXPERIMENTAL DESIGN

The dominance criteria were incorporated into two evolutionary algorithms, namely, the Differential Evolution for Multiobjective Optimization[1] (DEMO) [8] and the steady-state MOEA proposed by Deb et al. [2]. Moreover, two well known algorithms were considered to provide a comparison baseline for the performance of the methods: the NSGA-II [3] and SPEA2 [11] approaches.

The algorithms were applied to the five two-objective continuous ZDT problems (ZDT 1-4 and ZDT 6) [10] and the seven three-objective unconstrained DTLZ problems (DTLZ 1-7) [4]. The internal parameters of each algorithm were chosen as originally proposed in the respective original references. The characteristics of the tests performed for all problems were Population size (DEMO) and maximum size of the archive (MOEA): $\mu = 100$; stopping condition: 30,000 function evaluations.

To evaluate the performance of the methods regarding the convergence and diversity of the solutions found, we have used the convergence and diversity metrics [3].

Twenty independent runs were performed for each test problem, after which the performance metrics were calculated for the final fronts returned. A factorial design [6] was employed in all cases to detect whether any of the algorithms presented significantly different results from the others. Whenever the usual assumptions were satisfied, a parametric analysis of covariance [6, 1] was performed, with *Criterion*, *Algorithm* and *Problem* as the experimental factors, and the *number of objectives* of each problem as a covariate of the model. In cases where large violations of the

[1]Originally, the DEMO was proposed with the Pareto-dominance in mind, and it does not use an archive population, which is a prerequisite of the ϵ- and cone ϵ-criteria. However, adaptation is quite straightforward.

normality or variance assumptions were detected, transformation of the response variable or permutational methods [1] were employed for the statistical inference.

After the significance tests were performed, the problem effects were estimated and removed, allowing the derivation of problem-independent confidence intervals on the effect sizes of the algorithms. All analyses of the data were conducted using the statistical platform R [7], using a baseline significance level of 95% and LSD corrections for the pairwise comparisons of means [1].

3. RESULTS AND CONCLUSIONS

Figs. 1 - 3 illustrate the significant results in terms of effect size of the differences between algorithms and dominance criteria. While the convergence metric had significant differences only between the algorithms, the delta metric had significant differences both for algorithms and criteria, albeit at a small magnitude of effects.

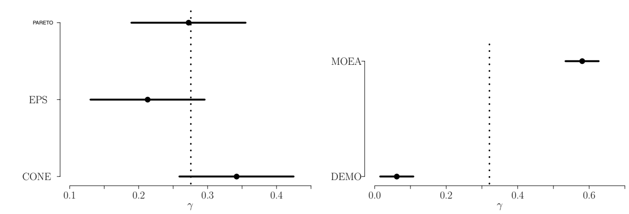

Figure 1: Effect sizes for the comparison among the dominance criteria (left) and the algorithms (right), γ metric.

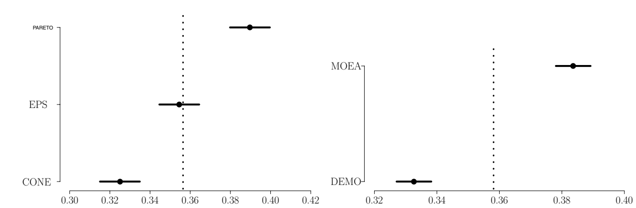

Figure 2: Effect sizes for the comparison among the dominance criteria (left) and the algorithms (right), Δ metric.

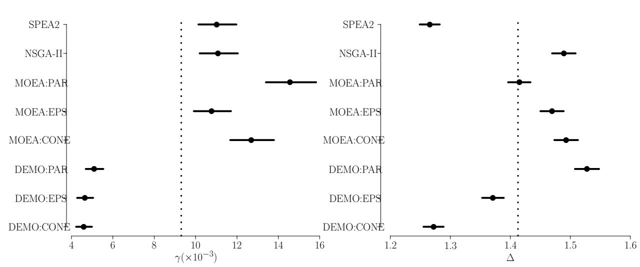

Figure 3: Effect sizes for the comparison among all algorithms: γ metric (left) and Δ metric (right).

When comparing all algorithms, we had very small significant effects for the convergence metric, and more considerable ones for the diversity one (Fig. 3). For this metric, the SPEA2 and DEMO:cone presented the best values for overall average diversity, followed by the DEMO:ϵ and the MOEA with Pareto dominance.

From the comparisons we could note that no significant differences were detected for the γ measure. Nevertheless, the cone ϵ-approach achieved the overall best average result for the Δ indicator. Moreover, the difference between the algorithm means showed significant effects for both quality metrics, with the DEMO presenting reasonably better values than the MOEA, mainly for the Δ measure. Indeed, by combining the good convergence properties of the DEMO with the diversity promotion of the cone ϵ-relation, this strategy presented robust performance indicators, with results regarding the γ metric not inferior to any other approach, and sound effects for the Δ measure, with a performance similar to the one of SPEA2, a method known to handle diversity efficiently.

4. ACKNOWLEDGMENTS

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