Finding a Diverse Set of Decision Variables in Evolutionary Many-Objective Optimization

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ABSTRACT

In this paper, we modify an evolutionary many-objective optimization algorithm so that it can find a diverse set of solutions in the decision variable space. The modification is based on considering the Euclidean distance in the decision variable space. The effect of our modification is examined by using benchmark test problems. From computational experiments, we can say that a diverse set of solutions in the decision variable space is searched by the modification.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Heuristic Methods*

General Terms

Algorithms

Keywords

Diversity; decision variable; evolutionary multi-objective optimization (EMO); many-objective optimization

1. INTRODUCTION

In the field of evolutionary computation, a vast number of evolutionary algorithms have been proposed for multi-objective optimization [1] in which the target is to find the Pareto front in the objective space. Therefore, the task of evolutionary multi-objective optimization (EMO) algorithms is to obtain solutions that approximate the Pareto front very well. They are often evaluated by their convergence to the Pareto front, diversity, and uniformity in the objective space. However, the diversity of solutions in the decision variable space is also important from the decision maker's point of view. Although a number of approaches for improving the diversity in the decision variable space have been proposed for optimization problems with up to three objectives (e.g., [6]), there are few approaches for optimization problems with more than three objectives (i. e., many-objective optimization problems). In this paper, we modify an evolutionary many-objective optimization algorithm (i.e., CDAS [5]) so that it can find a diverse set of solutions in the decision variable space for many-objective optimization problems.

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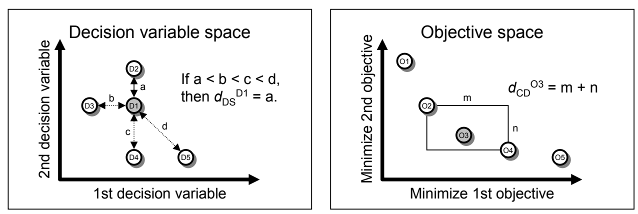
2. DIVERSITY OF DECISION VARIABLES

In order to obtain a diverse set of solutions in the decision variable space, the distance d_{DS} of each solution to its nearest neighbor in the decision variable space is taken into account. We also consider the crowding distance d_{CD} [2] in the objective space with the aim of avoiding a deterioration of the diversity in the objective space. These distances d_{DS} and d_{CD} are illustrated in Fig. 1. Either d_{DS} or $d_{\mathrm{DS}} \cdot d_{\mathrm{CD}}$ is used instead of d_{CD} as a second criterion to compare solutions with the same rank in our study for CDAS [5]. CDAS is the same as NSGA-II [2] except for its relaxed definition of the Pareto dominance.



(a) Distance d_{DS} (b) Crowding distance d_{CD}

Figure 1: Diversity measures in (a) the decision variable space and (b) the objective space

3. COMPUTATIONAL EXPERIMENTS

We use DTLZ1 and DTLZ2 with six objectives (denoted by DTLZ1-6 and DTLZ2-6 in results, respectively) as benchmark test problems [3]. We also include a relatively new test problem [4] and call it IHTN in this paper. In IHTN, multiple Pareto optimal regions, each of which gives the same Pareto front in the objective space, are represented in the two-dimensional decision variable space as a polygonal shape. Six-objective IHTN instances with one, nine, and 81 Pareto optimal regions are used in our study (denoted by IHTN-6-1, IHTN-6-9, and IHTN-6-81, respectively).

In order to compare obtained solutions, we use two performance indicators. One is the mean of d_{DS}. This performance indicator can measure the diversity of solutions in the decision variable space (denoted by "DIV"). Larger values of DIV mean better diversity in the decision variable space. Next, we use a relative hypervolume (denoted by "RHV") for DTLZ1 and DTLZ2. For DTLZ1 and DTLZ2, the hypervolume of the Pareto front can be obtained analytically. Therefore, we can calculate the relative hypervolume of obtained solutions [7]. It should be noted that the rela-

Table 1: Results of computational experiments

Problem	Measure	CD	CD-M1	CD-M2
DTLZ1-6	RHV	0.992	0.963	0.980
	DIV	0.057	0.100	0.095
DTLZ2-6	RHV	0.636	0.687	0.619
	DIV	0.082	0.111	0.117
IHTN-6-1	HV ($\times 10^4$)	7.389	8.365	8.234
	DIV ($\times 10^2$)	1.305	1.805	1.836
IHTN-6-9	HV ($\times 10^7$)	1.837	1.993	1.993
	DIV ($\times 10^3$)	5.969	15.427	15.473
IHTN-6-81	HV ($\times 10^{10}$)	1.577	1.673	1.679
	DIV ($\times 10^3$)	5.486	32.888	32.807

tive hypervolume is one for the Pareto front. On the other hand for IHTN, we use a standard hypervolume (denoted by "HV") to assess obtained solutions. Larger values of RHV or HV indicate better convergence to the Pareto front and better diversity in the objective space.

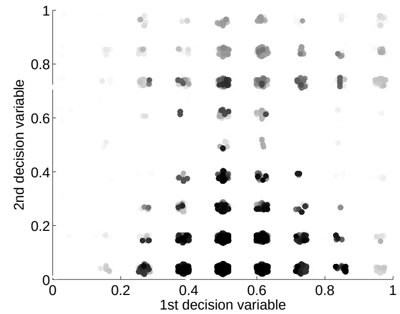
In computational experiments, the population size is set as 100. The stopping condition is specified as 30,100 fitness evaluations (including the initialization). The parameter S in CDAS is set as $S = 0.4$. The other settings are the same as in [7]. In order to calculate the relative hypervolume for DTLZ1 and DTLZ2, and a hypervolume for IHTN, a reference point $\mathbf{r}$ needs to be specified. [1]

CDAS together with either d_{DS} or $d_{\mathrm{DS}} \cdot d_{\mathrm{CD}}$ is applied to test problems in order to show the effect of our proposals. The average of each performance indicator over 20 runs is summarized in Table 1 where CDAS is denoted by CD, CDAS with d_{DS} by CD-M1, CDAS with $d_{\mathrm{DS}} \cdot d_{\mathrm{CD}}$ by CD-M2, respectively. From Table 1, we can see that the diversity of solutions in the decision variable space is improved by our modification. For all problems except for DTLZ1-6, both the RHV or HV and DIV are improved. In order to show that a diverse set of solutions in the decision variable space is searched by our modification to CDAS, we plot decision variables of IHTN-6-81 during optimization in Fig. 2 for CDAS and CDAS with d_{DS}, respectively, where initial solutions are in white, middle ones in gray, finally obtained ones in black. From Fig. 2, we can see that all the Pareto optimal regions of IHTN-6-81 can be found by our modification to CDAS.

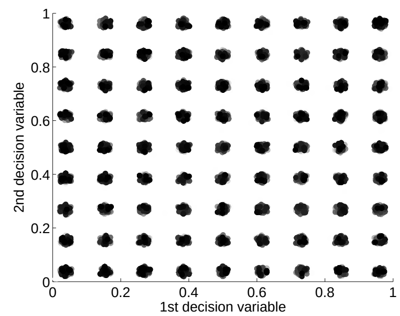
4. SUMMARY

In this paper, we proposed to consider a diversity distance in the decision variable space. The diversity distance is used as a second criterion to compare solutions with the same rank in CDAS. From computational experiments for all test problems, we can say that the diversity of obtained solutions in the decision variable space is dramatically improved. In a relatively new test problem IHTN, our modification to CDAS improves not only the diversity in the decision variable space but also both the convergence to the Pareto front and the diversity in the objective space. We can also see that the decision variable space is searched evenly during optimization by our modification.

[1] The reference point $\mathbf{r}$ is specified as $\mathbf{r} = 0.7^6$, $\mathbf{r} = 1.1^6$, $\mathbf{r} = 0.5^6$, $\mathbf{r} = 0.125^6$, $\mathbf{r} = 0.0384615^6$ for DTLZ1-6, DTLZ2-6, IHTN-6-1, IHTN-6-9, IHTN-6-81, respectively.



(a) CDAS



(b) CDAS with d_{DS}

Figure 2: Solutions obtained by CDAS and CDAS with d_{DS} for IHTN-6-81

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