Exploring Some Scalarization Techniques for EMOAs

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ABSTRACT

When solving a multi-objective problem Pareto based evolutionary algorithms are the preferred choice. They are able to find a good approximation of the Pareto front and assure good diversity. But Pareto dominance scales badly with the number of objectives. Decomposition based algorithms represent a good choice for many-objective problems, their performance is not affected in such a severe way because they solve multiple one-objective problems. The preferred methods for scalarizing all objectives into one single objective are *weighted sum* and *weighted Tchebycheff*. With some modifications to the *Tchebycheff* approach some drawbacks, such as obtaining weak Pareto optimal solutions, can be avoided. We study the *augmented, modified Tchebycheff* and L_p decomposition techniques as an alternative. Numerical results on test problems indicate an in crease in performance over *weighted sum* and *weighted Tchebycheff* when applied to many-objective optimization problems.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search - Heuristic Methods

General Terms

Algorithms, Experimentation, Performance, Theory

Keywords

Many-objective optimization, Scalability, Scalarization techniques, Decomposition methods

1. INTRODUCTION

Almost every real life problem requires the simultaneous optimizations of two or more conflicting objectives. Evolutionary multiobjective optimization algorithms represent a popular technique for solving such problems. Recently there has been a growing interest in studying the scalability and applicability of these techniques to many-objective optimization problems.

Decomposition based EMOAs rely on a decomposition technique [1]: a scalarizing function and a set of weight vectors decompose the optimization problem in many single-objective problems. The main advantages of this approach are computational efficiency and good scalability with the number of objectives. By decomposing the problem the selection pressure is lover than within Pareto based algorithms. The selection pressure is transferred to weight vector diversity - one needs to generate sufficient weight vectors and assure their. But the number of sub-problems required to approximate the Pareto front also grows exponentially with the number of objectives.

Decomposition approaches seem to be a good choice when solving a many-objective optimization problem. It is easier to solve multiple single-objective problems. The main scalarization function used as decomposition technique are: Weighted Sum and Weighted Tchebycheff [3]. But there are are other scalarization techniques such as Weighted L_p, augmented Tchebycheff, and modified Tchebycheff [2], [3] that can be used as a decomposition technique. This paper explores the performance and scalability of these techniques to many-objective problems.

2. SCALARIZATION TECHNIQUES

By aggregating all objectives, under some conditions, a Pareto optimal solution to the defined Multi-Objective Problem (MOP) can be found. By repeatedly solving the scalar problem a subset of efficient solutions for the MOP can be found. The most common scalarization techniques are weighted sum and weighted Tchebycheff [3]. Other scalarization techniques can be used: weighted L_p, augmented and modified Tchebycheff. Regardless of the scalarization technique it must meet the following conditions [5]: *(i)* an optimal solution of the scalar problem needs to be an efficient solution for the MOP, *(ii)* all efficient solutions of the MOP need to be found using the scalarization.

Weighted L_p. Pareto optimal solutions ca be obtained using the global criterion method. Here the objective is to minimize the distance between the feasible objective region and a reference point x^*. For obtaining different weakly Pareto optimal solutions this metrics can be weighted.

For a set of weights $\Omega = \{w \in \mathbb{R}^m | w > 0, \sum_{i=1}^{m} w_i = 1\}$ the weighted L_p metric is - $min\|F(x) - z^*\|_p^w$ for $1 \leq p < \infty$. $z^* = (z_1^*, ..., z_m^*)$ represents an ideal objective vector: $z_i^* \leq f_i(x), \forall i = 1, ..., m \; and \; x \in S$. If $w \in \Omega$ then the solution of weighted L_p problem is also Pareto optimal. Better solutions than the ideal vector cannot be found with this metric. Here parameter p defines the importance of each deviation.

With the weighted L_p problem all Pareto optimal solutions can be found only if the MOP in convex.

Improvements on Weighted Tchebycheff. If $p \to \infty$ the relative weighted deviation of only one objective is im-

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portant [3], this scalatization technique is called the Weighted Tchebycheff (WTCH) problem.

If this solution provided by WTHC is not unique then weak Pareto optimal solutions can be obtained. This disadvantage can be overcome by giving a slope to the metric [3]. The *augmented Tchebycheff* problem is: $min \|F(x) - z^*\|_\infty^w + \rho\|F(x) - z^*\|_1$ where $\rho \geq 0$ is a small scalar.

For different weighting vectors $w \in \Omega$ and different values for ρ all Pareto optimal solutions could be found.

Another way to overcome the weakness of weighted Tchebycheff and modify the slope of the metric contour is the *modified weighted Tchebycheff*: $min\ max[w_i(|f_i(x) - z^*| + \rho\|F(x) - z_i^*\|_1)]$, where $\rho > 0$ and $i = 1, ..., m$.

In the *augmented Tchebycheff* problem the slope depends on parameter ρ and the weighting coefficients, the slope is different for each objective function. For the *modified Tchebycheff* problem the slope depends only on the value of parameter ρ, the slope is the same for each objective function.

3. NUMERICAL EXPERIMENTS

We want to test the scalability of the above scalarization techniques to many-objective problems. For this purpose we use Differential Evolution [4] as an underlying evolutionary technique. We intend to observe the behavior of the scalarization techniques not to analyze the effectiveness of DE for our purpose.

As basis for the comparison the WFG, DTLZ, and ZDT test problem suites are used. All problems are real valued unconstrained and require the minimization of objective functions. These test problems provide different difficulties such as: different geometry for the Pareto optimal front - linear, convex, concave, disconnected, DTLZ and WFG offer scalability with the number of objectives. We compute the hypervolume and inverted generational distance for the obtained solution sets.

We test the performance of weighted sum, L_p, weighted Tchebycheff and variants of Tchebycheff for 2-5 objectives. For L_p technique different values for parameter p are tested $\{1, 2, 5, 10, 20, 50, 75, 100, 1000\}$. For *augmented* and *modified Tchebycheff* the slope is varied for different values $\rho = \{0.1, 0.3, 0.5, 0.7, 1\}$.

For the Hypervolume indicator the best results are obtained for the *weighted sum* and *modified Tchebycheff* techniques. For the HV on average we ca see a 5% increase in performance for the *modified Tchebycheff* technique compared to the other techniques. All techniques are able to find good approximations of the Pareto front for 2 objectives. In terms of spread and closeness to the true Pareto front *weighted Tchebycheff* and *modified Tchebycheff* give the best results (the lowest IGD values).

For three objective problems the best results are obtained with *weighted Tchebycheff* and *modified Tchebycheff* for $\rho = \{0.3, 0.5, 0.7, 1\}$. As for two objectives an average increase of 5% in performance over other techniques. IGD results are similar to those obtained for two objectives.

For many-objective problems the best results are obtained with *augmented* and *modified Tchebycheff* techniques for the Hypervolume metric for $\rho = \{0.1, 0.5, 0.7, 1\}$. On average a 15% increase in performance over *weighted sum* and *weighted Tchebycheff* scalarization techniques can be observed. But results between the improved *Tchebycheff* and L_p techniques are very similar. For the L_p scalarization higher values for p give better results when the number of objectives increases. We also performed experiments for 7 and 10 objective problems and computed the average Hypervolume over 50 runs, the results are similar to those for 4-5 objective problems.

4. CONCLUSIONS

Decomposition approaches represent a good choice for solving a many-objective optimization problem, because it is much easier solve a single objective problem. By solving an optimization problem in this way the exponential increase of non-dominated solutions is avoided.

Most evolutionary approaches based on decomposition rely on scalarization techniques for decomposing a problem into many single objective problems and solve these problems independently. Weighted sum and weighted Tchebycheff are the preferred scalarization techniques. This paper explores the *augmented Tchebycheff*, *modified Tchebycheff*, and L_p techniques for decomposition as alternatives to *weighted sum* and *weighted Tchebycheff.*

Numerical experiments are performed for continuous optimization problems that are scalable with the number of objectives and have different difficulties. For 2 and 3 objectives experiments indicate that *modified* and *augmented Tchebycheff* techniques performe better than *weighted sum* and *weighted Tchebycheff.* For many-objective problems we can observe a bigger increase in performance of *augmented, modified Tchebycheff*, and L_p techniques. L_p norm with p sufficiently large $p = \{100, 1000\}$ is a good choice for many-objective problems.

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