An Iterative Model Refinemen Approach for MOEA Computation Time Reduction

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1. INTRODUCTION

Optimization problems may present different solutions depending on the chosen model representing the system to optimize; in particular, from an initial coarse model, more refined ones can be derived with additional decision variables which may present better performances. However, in the context of generic algorithms, the computation time required to generate individuals and evaluate their fitnesses increases accordingly.

In this abstract, we present an Iterative Model Refinement Approach (IMRA) used to solve a Multi-Objective Optimization Problem (MOP) based on NSGA-II. It is aimed at decreasing the computation time and identifying the optimal number of decision variables to achieve the best performances.

This abstract will introduces a state of the art of related methods in Section 2. Section 3 presents the algorithm which is applied to a simple 2D geometric problem in Section 4.

2. STATE OF THE ART

The approach presented in this abstract builds upon prior works regarding design variables expansion [1] and model refinement in the field of Free-Form-Deformation [2]. Those works showed the benefits of an iterative model refinement in the case of monocriterion, non-evolutionary based optimization. More refined models presented better performances while the refinement strategy allowed a significant improvement in the computation time.

The proposed Iterative Model Refinement Approach (IMRA) extends such methods in the case of a Multi-Objective Evolutionary Algorithm (MOEA). The Hypervolume Estimator is therefore used to assess the global quality of a population regarding the MOP hence determining the need for further refinements of the model.

3. PROPOSED ALGORITHM

The IMRA can be used with any Genetic Algorithm (GA). The refinement consists in the introduction of new Decision Variables (DV) in the model, thus allowing better performances (assessed through hypervolume estimation).

It relies on a Trigger Operator ($\mathfrak{T}$) to identify a suitable moment for the refinement to happen, with regards to the computation time. $\mathfrak{T}$ operates on the basis of an Hypervolume Estimator (HVE) average slope and takes two parameters as inputs: an averaging window (w) and a threshold (t) under which the refinement is launched.

Algorithm 1: General algorithm

Data: MOP, initial_population
Result: Pareto Front Estimation, Optimized Population

```
P = initial_population
for i < MOP.num_gen do
    fast_non_dominated_search(P)
    P = make_new_pop(P)
    if 𝔗(P,w,t) then
        P = MOP.refine(P)
Return fast_non_dominated_search(P), P
```

The refinement function for P is problem dependant and is uniformely done over the entire population. In the case of a geometric problem, this refinement is most likely the addition of vertices in the model.

4. CASE STUDY

4.1 Description

The case study is an MOP that can be described as follows:

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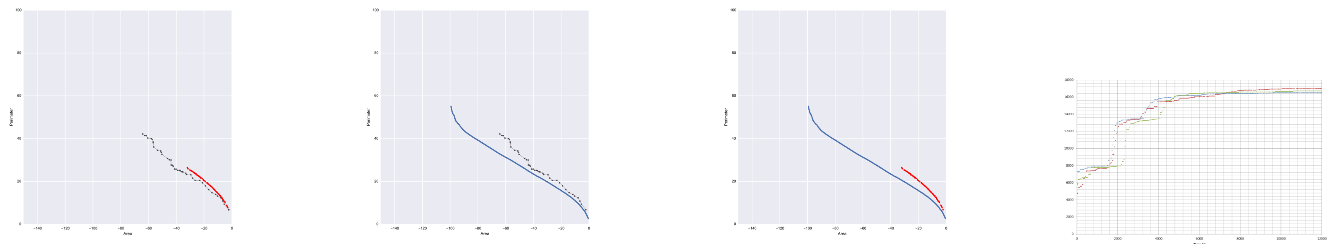
http://dx.doi.org/10.1145/2598394.2602284.



(a) 6 DV vs 12 DV (b) 12 DV vs 24 DV (c) 6 DV vs 24 DV (d) Threshold effect

Figure 1: Effect of the refinement on the Pareto-Front and the computation time

Algorithm 2: Trigger operator

Data: P, w, t
Result: Trigger decision: True if the model needs to be refined

```
HVE = compute HVE(P)
Append HVE to HVE_list
Average HVE = sum(HVE_list[:-w])/w
Append Average HVE to Average HVE List
if Average HVE < t * max(Average HVE List[:-w]) then
    Empty HVE List
    Empty Average HVE List
    Return True
Return False
```

$$
\begin{array}{rl}
\text{Maximize} & f_1(X) = Area(X) \\
\text{Minimize} & f_2(X) = Perimeter(X) \\
\text{Subject to} & \Omega = [-5;5] \times [-5;5] \\
 & [(2,2);(3,3)]^T \in X \\
 & [(0,0);(3,2);(4,4)]^T \notin X
\end{array}
$$

with X representing the polygon.

At first, X consists of three vertices (i.e. six DV). This MOP is solved with IMRA using NSGA-II, with the number of DV doubling with each refinement.

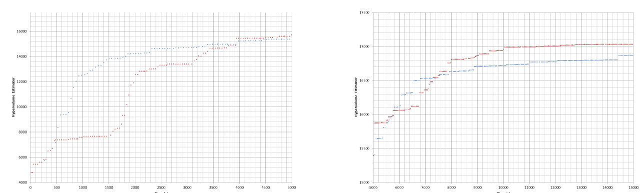
The computing times are compared to the one required by NSGA-II when optimizing directly the refined model (12 vertices i.e. 24 DV).

4.2 Optimization results

The optimization results of the IMRA are shown in Figure 1 After the first model performance stops improving it is refined, thus leading to a second step of performance enhancement. Those two optimization phase are compared in Figure 1a (with red being the coarse model). This behaviour continues upon reaching the Pareto-Front plotted in blue in Figure 1b and 1c.

4.3 Performances

Figure 2 shows the speedup provided by the IMRA with a suitable Trigger. The speedup happens in the second part of the optimization (Figure 2b), when the IMRA reaches its final model. The initial coarse model allows to explore a first part of the decision space while presenting a reduced computation time. The first part of the optimization gives the feeling of the IMRA being slower; however it is due to the fact that only a small portion of the objective space is reachable with a coarse model thus impacting the HVE.



(a) First part: IMRA still not at its final model (b) Second part: IMRA and NSGA-II use the same 12 vertices model

Figure 2: Performance comparison: NSGA-II (blue dots) IMRA (red triangles)

The reachable space is being expanded with each refinement. In this case study, first triangles optimize low-area low-perimeter polygons. Then, the hexagons expands the decision space beyond the constraints points from already optimized triangles. Finally, the dodecagons complete the optimization while exploring high-area high-perimeter parts of the Pareto-Front from already optimized hexagons. Those phenomena explain the 3x speedup upon final convergence.

5. CONCLUSIONS AND FUTURE WORK

As can be seen in the different experimentations, the IMRA can yield satisfying results both in terms of computation speed and final estimation of the Pareto-Front. However, this algorithm seems to be very dependant of the Trigger operator, which takes two inputs: its sensibility and its averaging window. Future work will explore the possibility to handle 3D geometric problems, which is a class of real-world problems for which IMRA is likely to be suitable.

6. REFERENCES

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[2] Han, X., and Zingg, D. W. An adaptive geometry parameterization for aerodynamic shape optimization. *Journal of Optimization and Engineering* (2013).