A Speed-Up and Speed-Down Strategy for Swarm Optimization

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ABSTRACT

In this paper, inspired by speed-up and speed-down (SUSD) mechanism observed by the fish swarm avoiding light, an SUSD strategy is proposed to develop new swarm intelligence based optimization algorithms to enhance the accuracy and efficiency of swarm optimization algorithms. By comparing with the global best solution, each particle adaptively speeds up and speeds down towards the best solution. Specifically, a new directed speed term is added to the original particle swarm optimization (PSO) algorithm or other PSO variations. Due to the SUSD mechanism, the algorithm shows a great improvement of the accuracy and convergence rate compared with the original PSO and other PSO variations. The numerical evaluation is provided by solving recent benchmark functions in IEEE CEC 2013.

Categories and Subject Descriptors

G.1.6 [Optimization]: Unconstrained optimization

Keywords

Swarm intelligence

1. INTRODUCTION

Particle swarm optimization (PSO), firstly proposed in [1], has been widely used in various disciplines. As a swarm intelligence based optimization algorithm, a particle in PSO updates its velocity by comparing the difference between current position and local best position found by itself and the difference between current position and the global best position among all the particles, and then moves into the next position, after which the local best position and the global position will be updated simultaneously.

GECCO'14, July 12–16, 2014, Vancouver, BC, Canada. ACM 978-1-4503-1964-5/14/07. http://dx.doi.org/10.1145/2598394.2602285. The update formulas for the PSO algorithm are shown in the following equation.

$$\mathbf{v}_{k+1}^{i,j} = w\mathbf{v}_{k}^{i,j} + b_{1}r_{1}(\mathbf{g}_{loc,i}^{j} - \mathbf{x}_{k}^{i,j}) + b_{2}r_{2}(\mathbf{g}_{k}^{j} - \mathbf{x}_{k}^{i,j})
\mathbf{x}_{k+1}^{i,j} = \mathbf{x}_{k}^{i,j} + \mathbf{v}_{k+1}^{i,j}, \quad i = 1, \dots, q, \quad j = 1, \dots, n \quad (1)$$

where $\mathbf{x}_{k}^{i,j}$ and $\mathbf{v}_{k}^{i,j}$ are the *j*th position and velocity element of particle *i* at iteration *k*, respectively, \mathbf{g}_{k}^{j} is the *j*th element of the global best solution among all the particles and $\mathbf{g}_{loc,i}^{j}$ is the *j*th element of local best solution found by particle *i*, and *w*, *b*₁, *b*₂, *r*₁ and *r*₂ are positive constants.

Due to the great success of PSO, lots of research efforts have been conducted to improve the performance in both accuracy and efficiency. In this research, observed from the fish swarm behavior [3], a speed-up and speed-down (SUSD) mechanism is proposed for PSO as an extra force for the velocity towards both local and global best solutions. Moreover, this SUSD mechanism can be easily integrated into other PSO variants for performance enhancement. As an example, multiagent coordination optimization (MCO) [4] is introduced and illustrated by using the SUSD mechanism.

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2. SPEED-UP AND SPEED-DOWN MECH-ANISM

Based on the observation that a fish in a group speeds up when the light intensity at its current position is relatively high and slows down as the light intensity decreases [3], a SUSD mechanism is proposed as follows.

$$SUSD = \lambda_1 (f_j(\mathbf{g}_k^j) - f_j(\mathbf{x}_k^{i,j})) \operatorname{sign}(\mathbf{g}_k^j - \mathbf{x}_k^{i,j})$$

where $f_j(z) = f(x_1, \ldots, x_n)|_{x_j=z}$, $i = 1, \ldots, q$, $j = 1, \ldots, n$, q is the number of particles, n is the dimension of the objective function $f : \mathbb{R}^n \to \mathbb{R}$, $\lambda_1 < 0$ is a constant, $\operatorname{sign}(x) = 1$ if x > 0, $\operatorname{sign}(x) = -1$ if x < 0, and $\operatorname{sign}(0) = 0$.

2.1 SUSD for the PSO algorithm

Now this SUSD mechanism is applied to the PSO algorithm, and the update formulas for position and velocity are

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proposed in Equations (2-3).

$$\mathbf{v}_{k+1}^{i,j} = \mathbf{v}_{k}^{i,j} + b_{1}r_{1}(\mathbf{g}_{loc,i}^{j} - \mathbf{x}_{k}^{i,j}) + b_{2}r_{2}(\mathbf{g}_{k}^{j} - \mathbf{x}_{k}^{i,j}) \\
+ \lambda_{1}(f_{j}(\mathbf{g}_{k}^{j}) - f_{j}(\mathbf{x}_{k}^{i,j})) \mathbf{sign}(\mathbf{g}_{k}^{j} - \mathbf{x}_{k}^{i,j}) \quad (2) \\
\mathbf{x}_{k+1}^{i,j} = \mathbf{x}_{k}^{i,j} + \mathbf{v}_{k+1}^{i,j} \quad (3)$$

2.2 SUSD for the MCO algorithm

Multiagent coordination optimization (MCO) [4] is a novel swarm optimization algorithm by introducing the velocity consensus protocol and communication topology between particles into a PSO-like algorithm. The update formulas for position and velocity in MCO are shown in Equations (4–6).

$$\mathbf{v}_{k+1}^{i,l} = w\mathbf{v}_k^{i,j} + \check{\mu} \sum_{j \in \mathcal{N}_i} (\mathbf{x}_k^{j,l} - \mathbf{x}_k^{i,l}) + \check{\eta} \sum_{j \in \mathcal{N}_i} + (\mathbf{v}_k^{j,l} - \mathbf{v}_k^{i,l})$$

$$\check{\kappa}(\mathbf{g}_k^l - \mathbf{x}_k^{i,l}) \tag{4}$$

$$\mathbf{x}_{k+1}^{i} = \mathbf{x}_{k}^{i} + \mathbf{v}_{k+1}^{i} \tag{5}$$

$$\mathbf{g}_{k+1}^{l} = \mathbf{g}_{k}^{l} + \check{\kappa}(\mathbf{g}_{loc,i}^{l} - \mathbf{g}_{k}^{l})$$

$$\tag{6}$$

where $i = 1, \ldots, q, l = 1, \ldots, n, \check{\eta} \sim U(0, 2\eta), \check{\mu} \sim (0, 2\mu),$ $\check{\kappa} \sim (0, 2\kappa), \eta, \mu, \kappa > 0$ are constants, $U(\cdot, \cdot)$ is the uniform distribution, $\mathcal{N}_i = \{j \in \mathcal{V} : \{i, j\} \in \mathcal{E}\}$ is the set of neighbors of node i and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the set of edges, which is the communication links between two particles, and the set of nodes $\mathcal{V} = \{1, \ldots, q\}$ denotes the index of the particles. Inspired by the SUSD mechanism, the SUSD-MCO algorithm is proposed in Equations (7-9).

$$\mathbf{v}_{k+1}^{i,l} = w\mathbf{v}_{k}^{i,j} + \check{\mu} \sum_{j \in \mathcal{N}_{i}} (\mathbf{v}_{k}^{j,l} - \mathbf{v}_{k}^{i,l}) \\ + \check{\mu} \sum_{j \in \mathcal{N}_{i}} (\mathbf{x}_{k}^{j,l} - \mathbf{x}_{k}^{i,l}) + \check{\kappa}(\mathbf{g}_{k}^{l} - \mathbf{x}_{k}^{i,l}) \\ + \lambda_{1}(f_{l}(\mathbf{g}_{k}^{l}) - f_{l}(\mathbf{x}_{k}^{i,l})) \mathbf{sign}(\mathbf{g}_{k}^{l} - \mathbf{x}_{k}^{i,l})$$
(7)

$$\mathbf{x}_{k+1}^i = \mathbf{x}_k^i + \mathbf{v}_{k+1}^i \tag{8}$$

$$\mathbf{g}_{k+1}^{l} = \mathbf{g}_{k}^{l} + \check{\kappa}(\mathbf{g}_{loc,i}^{l} - \mathbf{g}_{k}^{l})$$
(9)

where $l = 1, \cdots, n$ and $i = 1, \ldots, q$.

3. NUMERICAL EVALUATION

For numerical evaluation, 1000 particles are used to solve the 30-dimension benchmark functions in [2]. Specifically, we use the shifted sphere function, rotated Rosenbrock function, and rotated Griewank function to test the proposed SUSD mechanism. The shifted optimal solution is shown as X^* , where $X^* = [98.7900 \ 17.0400 \ 25.7800 \ 39.6800 \ 7.4000$ 68.4100 40.2400 98.2800 40.2200 62.0700 15.4400 38.1300 16.1100 75.8100 87.1100 35.0800 68.5500 29.4100 53.0600 83.2400 59.7500 33.5300 29.9200 45.2600 42.2600 35.9600 55.8300 74.2500 42.4300 42.9400]. 20 executions of both PSO and SUSD-PSO algorithms solving the three benchmark functions have been conducted, and the results are shown in Table 1. It follows from the simulation results that the SUSD mechanism can largely improve the accuracy of PSO. Moveover, based on the results in Table 2, the SUSD-MCO algorithm also improves the accuracy of MCO. Due to the page limitation, only the searching trajectories of PSO and SUSD-PSO algorithms solving the shifted sphere function is provided in Figure 1.

Table 1: Comparison between PSO and SUSD-PSO

Function	Min	Max
	PSO SUSD-PSO	PSO SUSD-PSO
Sphere	1.64E-2 5.9263E-6	2.266 7.49E-2
Rosenbrock	2.10E-2 4.5727E-7	8.429E-1 3.1E-3
Griewank	2.0379E2 2.306E-1	2.3292E3 5.4249
Function	Median	Average
	PSO SUSD-PSO	PSO SUSD-PSO
Sphere	2.135E-1 2.1E-3	4.543E-1 1.07E-2
Rosenbrock	1.0811E-1 1.7272E-4	1.597E-1 4.1827E-4
Griewank	8.5715E2 1.2871	1.0002E3 2.1007

Table 2: Comparison between MCO and SUSD-MCO

Function	Min	Max
	MCO SUSD-MCO	MCO SUSD-MCO
Sphere	2.739E-1 8.6E-3	7.845E-1 2.56E-2
Rosenbrock	6.5E-3 4.0371E-4	7.32E-2 6.3E-3
Griewank	8.0663E3 9.2577E2	1.4405E4 1.0827E4
Function	Median	Average
	MCO SUSD-MCO	MCO SUSD-MCO
Sphere	6.334E-1 1.55E-2	5.857E-1 1.45E-2
Rosenbrock	1.62E-2 2.3E-3	2.09E-2 2.8E-3
Griewank	1.0690E4 6.8156E3	1.1243E4 8.5082E3

4. **REFERENCES**

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Figure 1: Test function: Shifted sphere function